Unit 8 Review Key

1. $x^2 = -12y$ parabola

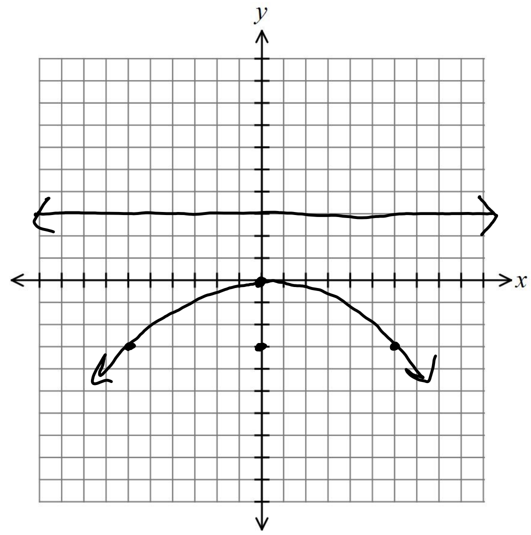
$-4a = -12$

$a = 3$

vertex: $(0,0)$

focus: $(0,-3)$

directrix: $y = 3$



2. $\frac{x^2}{16} - \frac{y^2}{4} = 1$ hyperbola

$a = 4$ ($\leftrightarrow$) $b = 2$ ($\updownarrow$)

center: $(0,0)$

vertices: $(4,0)$ & $(-4,0)$

foci: $(2\sqrt{5},0)$ & $(-2\sqrt{5},0)$

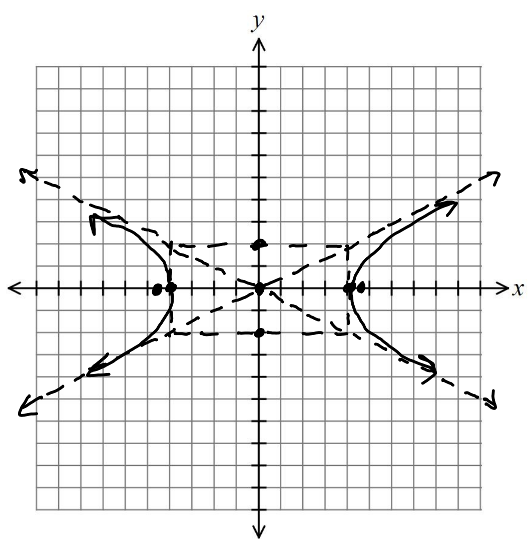
$c^2 = a^2 + b^2 = 16 + 4 = 20$

$c = \sqrt{20} = 2\sqrt{5} \approx 4.5$

asymptotes: $y = \pm\frac{1}{2}x$

$m = \frac{\updownarrow}{\leftrightarrow} = \pm\frac{2}{4} = \pm\frac{1}{2}$

transverse axis: $y = 0$
(x-axis)

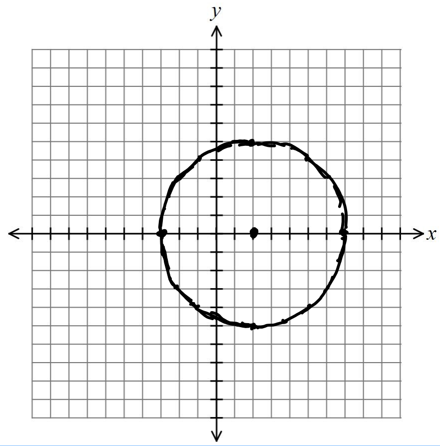


3. $(x-2)^2 + y^2 = 25$

circle

center: $(2,0)$

$r = 5$



4. $\dfrac{(x-1)^2}{49} + \dfrac{(y+5)^2}{9} = 1$ Ellipse

$a = 7 (\leftrightarrow)$ $b = 3 (\updownarrow)$

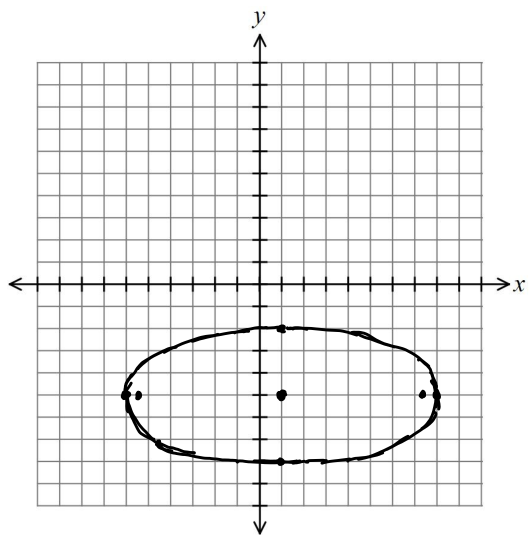
center: $(1,-5)$

vertices: $(-6,-5)$ & $(8,-5)$

foci: $(1-2\sqrt{10},-5)$ & $(1+2\sqrt{10},-5)$

$c^2 = a^2 - b^2 = 49 - 9 = 40$

$c = \sqrt{40} = 2\sqrt{10} \approx 6.3$



5. $\dfrac{4x^2 + y^2}{64} = \dfrac{64}{64}$

$\dfrac{x^2}{16} + \dfrac{y^2}{64} = 1$ ellipse

$b = 4 (\leftrightarrow)$ $a = 8 (\updownarrow)$

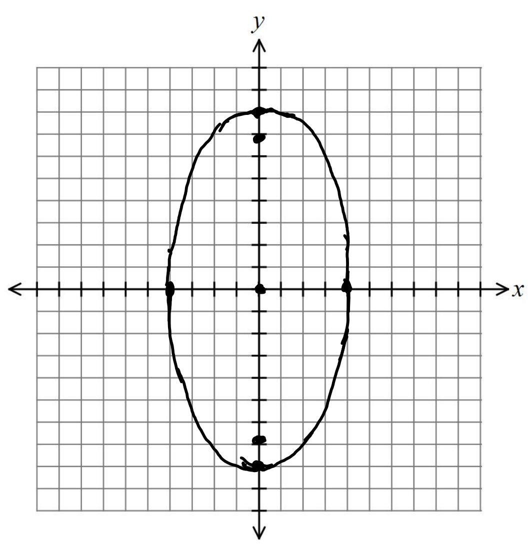
center: $(0,0)$

vertices: $(0,-8)$ & $(0,8)$

foci: $(0,-4\sqrt{3})$ & $(0,4\sqrt{3})$

$c^2 = a^2 - b^2 = 64 - 16 = 48$

$c = \sqrt{48} = 4\sqrt{3} \approx 6.9$



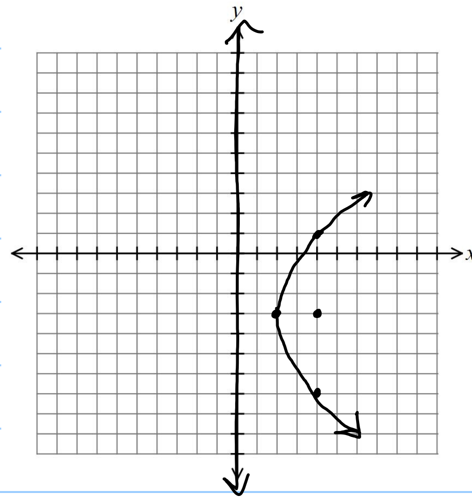
6. $(y+3)^2 = 8(x-2)$ parabola

$4a = 8 \quad a = 2$

vertex: $(2,-3)$

focus: $(4,-3)$

directrix: $x = 0$

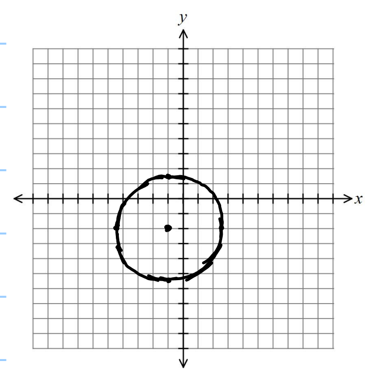


7. $(x+1)^2 + (y+2)^2 = 12$

circle

center: $(-1,-2)$

radius: $\sqrt{12} = 2\sqrt{3} \approx 3.5$



8. $\dfrac{4(y-3)^2 - 36(x-4)^2}{36} = \dfrac{36}{36}$ hyperbola

$\dfrac{(y-3)^2}{9} - (x-4)^2 = 1$

$a = 3 (\updownarrow) \quad b = 1 (\leftrightarrow)$

center: $(4,3)$

vertices: $(4,0)$ & $(4,6)$

foci: $(4, 3-\sqrt{10})$ & $(4, 3+\sqrt{10})$

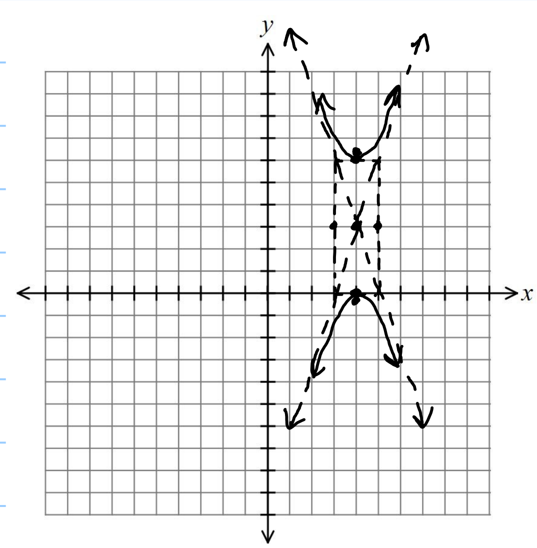
$c^2 = a^2 + b^2 = 9 + 1 = 10$

$c = \sqrt{10} \approx 3.2$

asymptotes: $y - 3 = \pm 3(x-4)$

$m = \dfrac{\updownarrow}{\leftrightarrow} = \dfrac{3}{1} = 3$

transverse axis: $x = 4$



9. $x^2+y^2+8x-33=0$

$(x^2+8x\quad)+y^2=33$ $\qquad \frac{8}{2}=4 \quad 4^2=16$

$(x^2+8x+16)+y^2=33+16$

$\boxed{(x+4)^2+y^2=49}$ $\boxed{\text{circle}}$

10. $25x^2+9y^2+250x-36y-239=0$

$(25x^2+250x)+(9y^2-36y)=239$

$25(x^2+10x\quad)+9(y^2-4y\quad)=239$ $\qquad \frac{10}{2}=5 \quad 5^2=25$

$\underbrace{25(x^2+10x+25)}_{+625}+\underbrace{9(y^2-4y+4)}_{+36}=239+625+36$ $\qquad \frac{-4}{2}=-2 \quad (-2)^2=4$

$$\frac{25(x+5)^2+9(y-2)^2}{900}=\frac{900}{900}$$

$$\boxed{\frac{(x+5)^2}{36}+\frac{(y-2)^2}{100}=1}$$ $\boxed{\text{ellipse}}$

11. $9x^2-4y^2-108x+8y-4=0$

$(9x^2-108x)+(-4y^2+8y)=4$

$9(x^2-12x\quad)-4(y^2-2y\quad)=4$ $\qquad \frac{-12}{2}=-6 \quad (-6)^2=36$

$\underbrace{9(x^2-12x+36)}_{+324}-\underbrace{4(y^2-2y+1)}_{-4}=4+324-4$ $\qquad -\frac{2}{2}=-1 \quad (-1)^2=1$

$$\frac{9(x-6)^2-4(y-1)^2}{324}=\frac{324}{324}$$

$$\boxed{\frac{(x-6)^2}{36}-\frac{(y-1)^2}{81}=1}$$ $\boxed{\text{hyperbola}}$

12. $y^2+4x+20y+64=0$

$y^2+20y\quad=-4x-64$ $\qquad \frac{20}{2}=10 \quad 10^2=100$

$y^2+20y+100=-4x-64+100$

$(y+10)^2=-4x+36$

$\boxed{(y+10)^2=-4(x-9)}$ $\boxed{\text{parabola}}$

13. circle $r=4$ center: $(2,-5)$

$$\boxed{(x-2)^2+(y+5)^2=16}$$

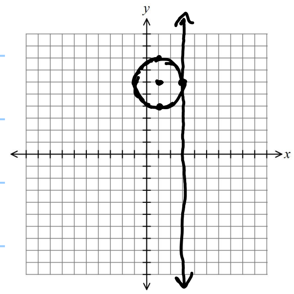
14. circle

center: $(1,6)$

tangent to $x=3$

$r=2$

$$\boxed{(x-1)^2+(y-6)^2=4}$$



15. Circle

$(2,-5)$ & $(6,1)$ are endpoints of a diameter

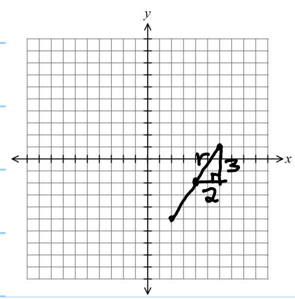
center: midpoint of $(2,-5)$ & $(6,1)$

$\left(\frac{2+6}{2}, \frac{-5+1}{2}\right)=(4,-2)$

radius: distance from $(4,-2)$ to $(6,1)$

$r=\sqrt{2^2+3^2}=\sqrt{13}$

$$\boxed{(x-4)^2+(y+2)^2=13}$$



16. Parabola

Focus: $(1,-1)$ Directrix: $y=-5$

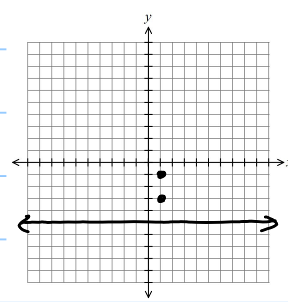
Vertex: $(1,-3)$

opens up

$a=2$

$(x-h)^2=4a(y-k)$

$$\boxed{(x-1)^2=8(y+3)}$$



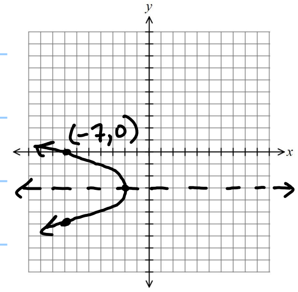
17. parabola

vertex $(-2,-3)$

x-int $(-7,0)$

axis of symm: $y=-3$

opens left



$(y-k)^2=-4a(x-h)$

$(y+3)^2=-4a(x+2)$

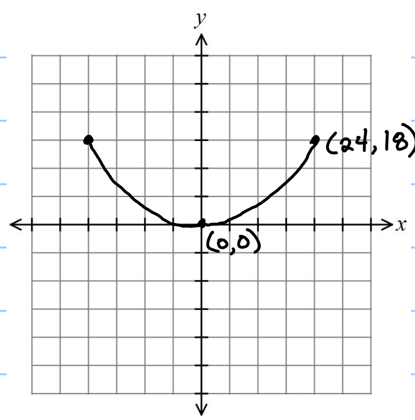
$(0+3)^2=-4a(-7+2)$

$9=20a$

$a=\frac{9}{20}$

$\boxed{(y+3)^2=-\frac{9}{5}(x+2)}$

18.



$a=?$

$x^2=4ay$

$24^2=4a(18)$

$576=72a$

$a=8$

$\boxed{\text{8 inches from the base}}$

19. Ellipse

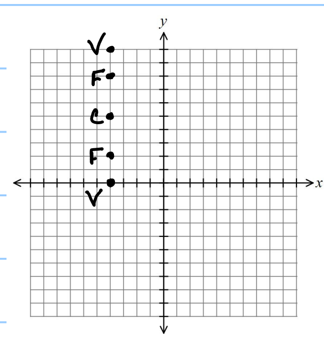
Foci: $(-4,2)$ & $(-4,8)$

Vertex: $(-4,10)$

Center: $(-4,5)$ (midpoint of foci)

$a=5$ (center to vertex) $\updownarrow$

$c=3$ (center to focus)



$a^2-b^2=c^2$

$25-b^2=9$

$b^2=16$

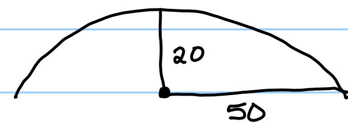
$b=4$ $\leftrightarrow$

$$\boxed{\frac{(x+4)^2}{16}+\frac{(y-5)^2}{25}=1}$$

20. Whispering gallery (ellipse)

100 ft long, 20 feet high

Center to focus?



$a = 50 \qquad b = 20 \qquad c = ?$

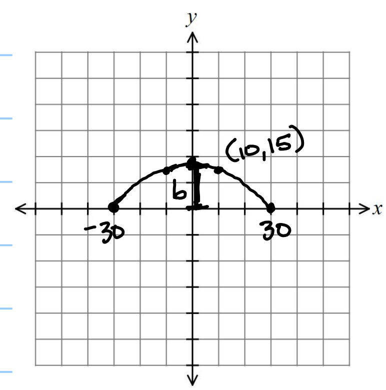
$$c^2 = a^2 - b^2 = 50^2 - 20^2 = 2100$$

$$c = \sqrt{2100} \approx \boxed{45.8 \text{ ft}}$$

21. Span = 60 ft

$a = 30 \ (\leftrightarrow)$

Goes through $(10,15)$ $\qquad b = ?$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{30^2} + \frac{y^2}{b^2} = 1$$

$$\frac{10^2}{30^2} + \frac{15^2}{b^2} = 1$$

$$\frac{1}{9} + \frac{225}{b^2} = 1$$

$$\frac{225}{b^2} = \frac{8}{9}$$

$$8b^2 = 2025$$

$$b^2 = 253.125$$

$$b \approx \boxed{15.9 \text{ ft}}$$

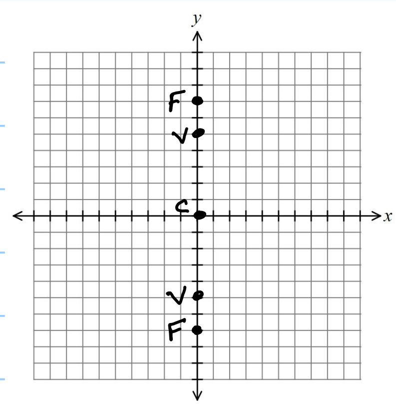
22. Hyperbola

Vertices: $(0, \pm 5)$ $\qquad a = 5 \ (\updownarrow)$

Foci: $(0, \pm 7)$ $\qquad c = 7$

opens

center: $(0,0)$



$$a^2 + b^2 = c^2$$

$$25 + b^2 = 49$$

$$b^2 = 24$$

$$b = \sqrt{24} = 2\sqrt{6} \ (\leftrightarrow)$$

$$\boxed{\frac{y^2}{25} - \frac{x^2}{24} = 1}$$

23. Hyperbola

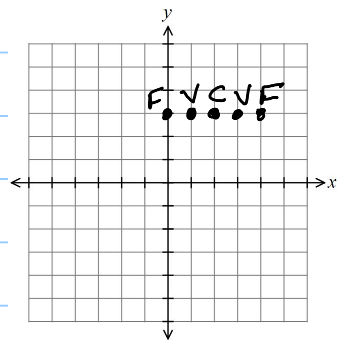
Center: (2,3)

Focus: (0,3) $\quad c = 2$

Vertex: (1,3) $\quad a = 1\ (\leftrightarrow)$

opens

$$a^2 + b^2 = c^2$$
$$1 + b^2 = 4$$
$$b^2 = 3$$
$$b = \sqrt{3}\ (\updownarrow)$$



$$\boxed{(x-2)^2 - \frac{(y-3)^2}{3} = 1}$$

8.1 # 16

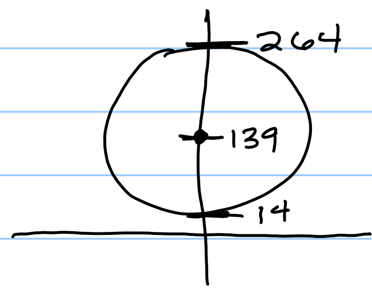
Ferris wheel

max height = 264 ft

diameter = 250 ft → radius = 125 ft

center on y-axis $\quad 264 - 125 = 139$

center @ (0,139)



$$x^2 + (y-139)^2 = 125^2$$
$$\boxed{x^2 + (y-139)^2 = 15{,}625}$$

8.2 # 21

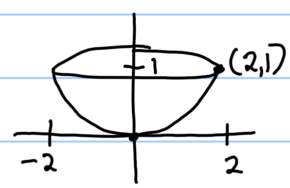
Flashlight reflector

Paraboloid of revolution

4 in. diameter

1 in. deep

How far from vertex should bulb be? ← Where's the focus?



$$x^2 = 4ay$$
$$2^2 = 4a(1)$$
$$4 = 4a$$
$$a = 1$$

$\boxed{\text{1 inch above the vertex}}$

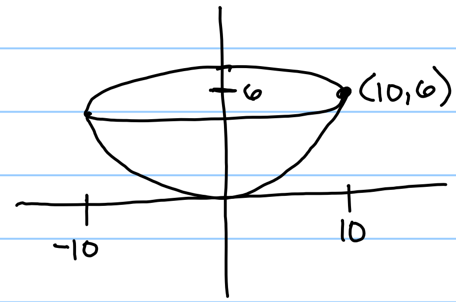
8.2 #22.

Mirror-paraboloid

20 feet across

6 feet deep

Heat source at focus ← $a = ?$



$x^2 = 4ay$

$10^2 = 4a(6)$

$100 = 24a$

$a = \frac{100}{24}$

$\approx$ $\boxed{\text{4.2 ft above base}}$

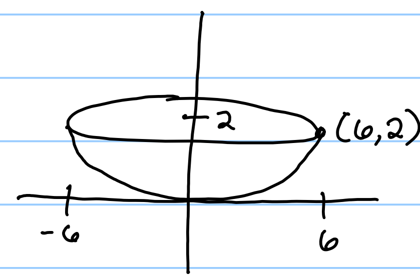
8.2 #23.

Satellite dish-paraboloid

12 ft diameter

2 ft deep

Receiver at focus ← $a = ?$



$x^2 = 4ay$

$6^2 = 4a(2)$

$36 = 8a$

$a = \frac{36}{8} =$ $\boxed{\text{1.5 ft above base}}$

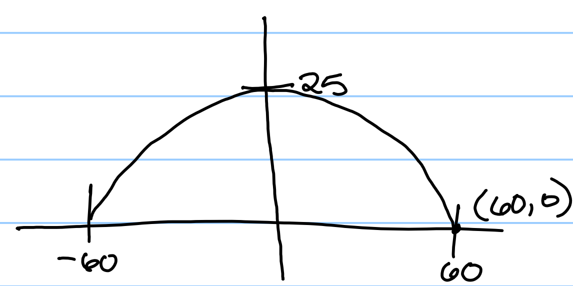
8.2 #25

Bridge-parabolic arch

Span 120 feet

Max height 25 feet

Height 10, 30, & 50 ft from center?



Vertex: $(\overset{h}{0}, \overset{k}{25})$, opens down

$(x-h)^2 = -4a(y-k)$

$x^2 = -4a(y-25)$

$60^2 = -4a(0-25)$

$3600 = 100a$

$a = 36$

$x^2 = -144(y-25)$

Solve for y: $\frac{x^2}{-144} = y - 25$

$y = -\frac{x^2}{144} + 25$

Plug in 10, 30, & 50:

10 ft: height $\approx$ 24.31 ft

30 ft: height = 18.75 ft

50 ft: height $\approx$ 7.64 ft

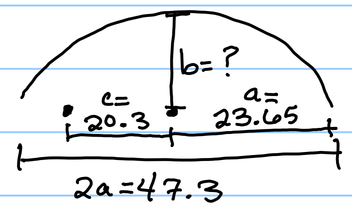
8.3 # 18:

Whispering gallery (ellipse)

47.3 ft. long

foci 20.3 ft. from center

How high? ← what is b?



$$a^2 - b^2 = c^2$$

$$\Rightarrow b^2 = a^2 - c^2$$

$$b^2 = 23.65^2 - 20.3^2 = 147.2325$$

$$b \approx \boxed{12.1 \text{ ft.}}$$

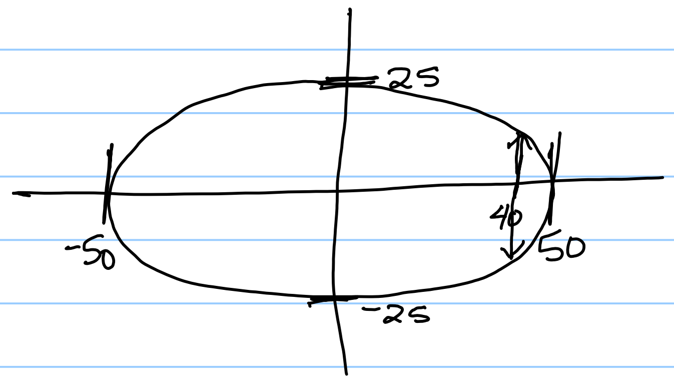
8.3 # 20:

Elliptical racetrack

100 ft. long

50 ft. wide

Width 10 feet from vertex?



width = 2y for x = 40

$$\frac{x^2}{50^2} + \frac{y^2}{25^2} = 1$$

$$\frac{40^2}{50^2} + \frac{y^2}{25^2} = 1$$

$$\frac{1600}{2500} + \frac{y^2}{625} = 1$$

$$0.64 + \frac{y^2}{625} = 1$$

$$\frac{y^2}{625} = 0.36$$

$$y^2 = 225$$

$$y = 15 \text{ ft}$$

$$\text{width} = 2(15) = \boxed{30 \text{ ft}}$$