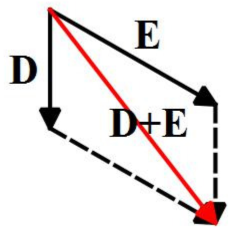
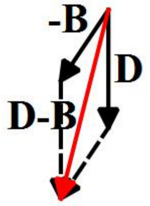
Precalculus
Unit 7 Review Key

1.



2.



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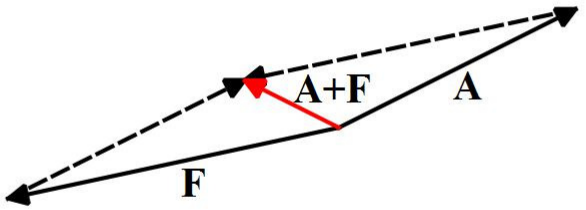

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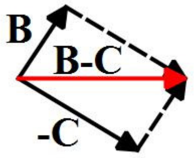

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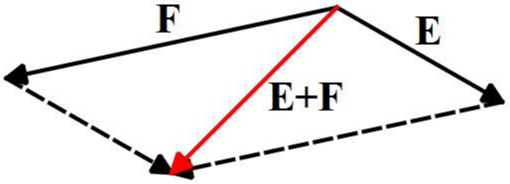

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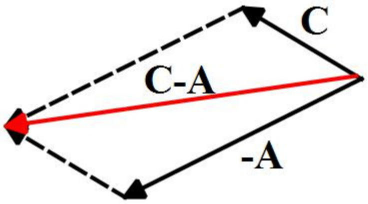
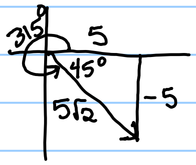


7.



$x = 5\sqrt{2}\cos 315°$

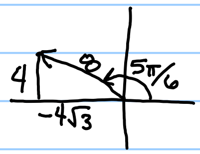
$= 5\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{5(2)}{2} = 5$

$y = 5\sqrt{2}\sin 315°$

$= 5\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{5(2)}{2} = -5$

$\boxed{\langle 5, -5 \rangle}$

8.



$x = 8\cos{}^{5\pi}/_{6}$

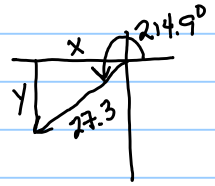
$= 8\left(-{}^{\sqrt{3}}/_{2}\right) = -4\sqrt{3}$

$y = 8\sin{}^{5\pi}/_{6}$

$= 8\left({}^{1}/_{2}\right) = 4$

$\boxed{\langle -4\sqrt{3}, 4 \rangle}$

9.

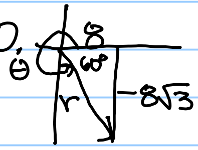


$x = 27.3\cos 214.9° = -22.4$

$y = 27.3\sin 214.9° = -15.6$

$\boxed{\langle -22.4, -15.6 \rangle}$

10.



$30°-60°-90° \ \triangle$

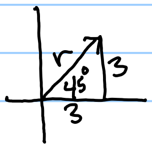
$r = 2 \times \text{short leg}$

$= 2(8) = 16$

$\theta = 300°$

$\boxed{r = 16 \quad \theta = 300°}$

11.



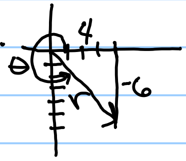
$45°-45°-90° \ \triangle$

$r = \text{leg} \times \sqrt{2} = 3\sqrt{2}$

$\theta = 45°$

$\boxed{r = 3\sqrt{2} \quad \theta = 45°}$

12.



$r = \sqrt{4^2 + 6^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$\tan\theta = -\frac{6}{4}$

$\tan^{-1}\left(-\frac{6}{4}\right) \approx -56.3°$

$\theta \approx -56.3° + 360° \approx 303.7°$

$\boxed{r = 2\sqrt{13} \quad \theta \approx 303.7°}$

13. $3\langle -1, 5 \rangle - \langle 4, -7 \rangle$

$= \langle -3, 15 \rangle - \langle 4, -7 \rangle$

$= \langle -3-4, 15-(-7) \rangle$

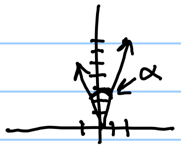
$= \boxed{\langle -7, 22 \rangle}$

14. $\langle -1, 5 \rangle \cdot \langle 4, -7 \rangle$

$= (-1)(4) + (5)(-7)$

$= -4 - 35 = \boxed{-39}$

15.



$\langle -1, 5 \rangle \cdot \langle 2, 7 \rangle$

$= (-1)(2) + (5)(7)$

$= -2 + 35 = 33$

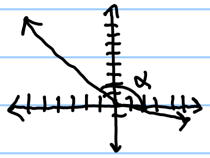
$|\langle -1, 5 \rangle| = \sqrt{1^2 + 5^2} = \sqrt{26}$

$|\langle 2, 7 \rangle| = \sqrt{2^2 + 7^2} = \sqrt{53}$

$\alpha = \cos^{-1}\left(\frac{33}{\sqrt{26}\cdot\sqrt{53}}\right)$

$\boxed{\alpha \approx 27.3°}$

16.



$\langle -6,8\rangle \cdot \langle 5,-1\rangle$
$= (-6)(5)+(8)(-1)$
$= -30-8 = -38$

$|\langle -6,8\rangle| = \sqrt{6^2+8^2} = 10$
$|\langle 5,-1\rangle| = \sqrt{5^2+1^2} = \sqrt{26}$

$\alpha = \cos^{-1}\left(\frac{-38}{10\sqrt{26}}\right)$
$\boxed{\alpha \approx 138.2^\circ}$

17. $\langle 2,-4\rangle \cdot \langle 6,3\rangle$
$= (2)(6)+(-4)(3)$
$= 12-12 = 0$
dot product = 0
$\boxed{\text{perpendicular}}$

OR

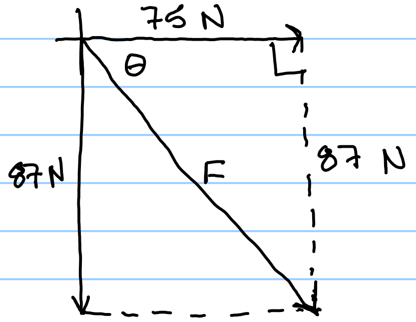
$\langle 2,-4\rangle$ $m = -\frac{4}{2} = -2$
$\langle 6,3\rangle$ $m = \frac{3}{6} = \frac{1}{2}$
slopes are negative reciprocals
$\boxed{\text{perpendicular}}$

18. $\langle 9,1\rangle$ & $\langle 1,9\rangle$
$m = \frac{1}{9}$ $m = \frac{9}{1}$
slopes are neither the same, nor negative reciprocals.
$\boxed{\text{neither}}$

19. $\langle -1,7\rangle$ & $\langle 3,-21\rangle$
$-3\langle -1,7\rangle = \langle 3,-21\rangle$
$\boxed{\text{parallel}}$

OR
$\langle -1,7\rangle$ $m = \frac{7}{-1} = -7$
$\langle 3,-21\rangle$ $m = \frac{-21}{3} = -7$
same slopes
$\boxed{\text{parallel}}$

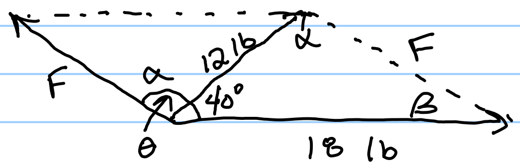
20.



right Δ → can use Pythagorean Thm & SOH-CAH-TOA
$|\vec{F}| = \sqrt{75^2+87^2} \approx \boxed{114.9\text{ N}}$

$\tan\theta = \frac{87}{75}$
$\theta \approx \boxed{49.2^\circ}$

21.



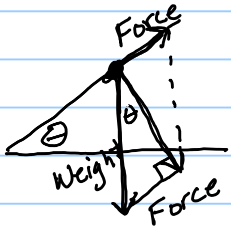
$|\vec{F}|^2 = 12^2+18^2-2(12)(18)\cos 40^\circ$
$|\vec{F}| \approx \boxed{11.7\text{ lb}}$

α is biggest ∠ in Δ (across from longest side), so might be obtuse. Don't use Law of Sines to find α.

$\frac{\sin 40^\circ}{11.7} = \frac{\sin\beta}{12} \Rightarrow \sin\beta = \frac{12\sin 40^\circ}{11.7} \Rightarrow \beta \approx 41.2^\circ$

$\theta \approx 180^\circ - 41.2^\circ$ ← consecutive angles in ▱ are supplementary.
$\boxed{\theta \approx 138.8^\circ}$

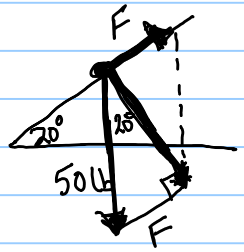
Inclined
Plane
Problems



$\sin\theta = \frac{\text{force}}{\text{weight}}$ ↙ opp ↖ hyp

Always right triangle!
Always sine!

22.

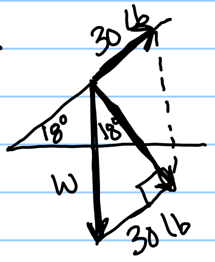


$\sin 20^\circ = \frac{|\vec{F}|}{50}$

$|\vec{F}| = 50\sin 20^\circ$

$|\vec{F}| \approx \boxed{17.1\text{ lb}}$

23.

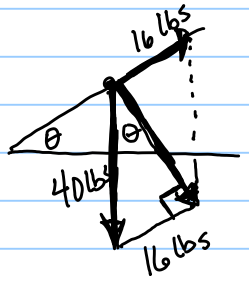


$\sin 18^\circ = \frac{30}{w}$

$w = \frac{30}{\sin 18^\circ}$

$w \approx \boxed{97.1\text{ lb}}$

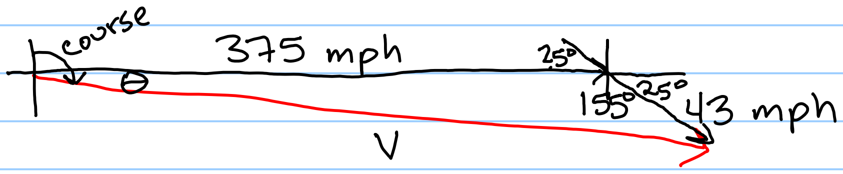
24.



$\sin\theta = \frac{16}{40}$

$\theta = \sin^{-1}\left(\frac{16}{40}\right) \approx \boxed{23.6\text{ lbs}}$

25.



$|\vec{v}|^2 = 375^2 + 43^2 - 2(375)(43)\cos 155^\circ$

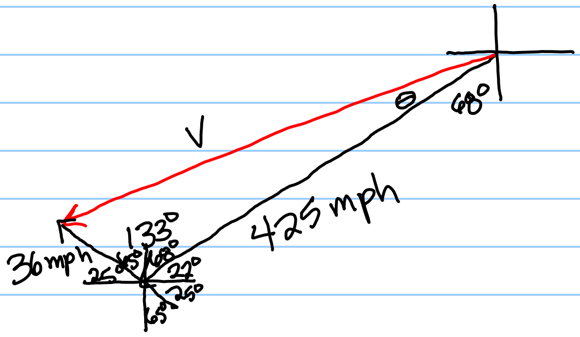
$|\vec{v}| \approx \boxed{414.4\text{ mph}}$

$\frac{\sin 155^\circ}{414.4} = \frac{\sin\theta}{43}$

$\sin\theta = \frac{43\sin 155^\circ}{414.4} \Rightarrow \theta \approx 2.5^\circ$

course $\approx 90^\circ + 2.5^\circ \approx \boxed{92.5^\circ}$

26.



$|\vec{v}|^2 = 36^2 + 425^2 - 2(36)(425)\cos 133^\circ$

$|\vec{v}| \approx \boxed{450.3\text{ mph}}$

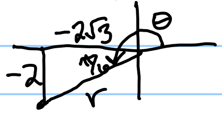
$\frac{\sin 133^\circ}{450.3} = \frac{\sin\theta}{36}$

$\sin\theta = \frac{36\sin 133^\circ}{450.3} \Rightarrow \theta \approx 3.4^\circ$

$68^\circ + 3.4^\circ = 71.4^\circ$

course: $\boxed{S\ 71.4^\circ W}$

27. $-2\sqrt{3} - 2i$



30°-60°-90° △

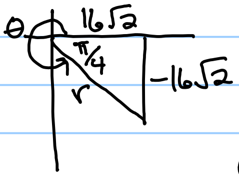
r = 2 × short leg

$r = 2(2) = 4$

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\boxed{4\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)}$

28. $16\sqrt{2} - 16i\sqrt{2}$



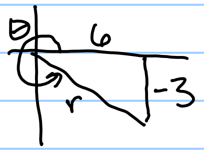
45°-45°-90° △

$r = \text{leg} \times \sqrt{2}$

$r = (16\sqrt{2})(\sqrt{2}) = 16(2) = 32$

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\boxed{32\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$

29. 

$r=\sqrt{6^2+3^2}=\sqrt{45}=3\sqrt{5}$

$\tan\theta=-\frac{3}{6}$

$\tan^{-1}(-3/6)\approx-0.46$ rad

$\theta\approx-0.46+2\pi\approx5.82$ rad

$\boxed{3\sqrt{5}(\cos 5.82+i\sin 5.82)}$

30. $9(\cos 240°+i\sin 240°)$

$=9\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$=\boxed{-\frac{9}{2}-\frac{9\sqrt{3}}{2}i}$

31. $24\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$

$=24\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

$=\boxed{-12\sqrt{2}+12i\sqrt{2}}$

32. $\sqrt{15}(\cos 312.8°+i\sin 312.8°)$

$\approx\boxed{2.63-2.84i}$

33. $4\text{ cis }80°\cdot3\text{ cis }130°$

$=(4\cdot3)\text{ cis }(80°+130°)$

$=12(\cos 210°+i\sin 210°)$

$=12(-\sqrt{3}/2-1/2\,i)$

$=\boxed{-6\sqrt{3}-6i}$

34. $\frac{7\text{ cis }\left(\frac{5\pi}{6}\right)}{2\text{ cis }(\pi/3)}=\frac{7}{2}\text{cis}\left(\frac{5\pi}{6}-\frac{\pi}{3}\right)$

$=\frac{7}{2}\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$

$=\frac{7}{2}(0+i)=\boxed{\frac{7}{2}i}$

35. $(2\text{ cis }225°)^7$

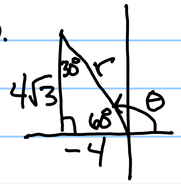
$=2^7\text{ cis }(225°\times7)$

$=128\text{ cis }1575°$ ← keep subtracting 360° until you get an angle you recognize

$=128(\cos 135°+i\sin 135°)$

$=128\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

$=\boxed{-64\sqrt{2}+64i\sqrt{2}}$

36. 

30°-60°-90°

$r=2(4)=8$

$\theta=120°$

$(-4+4i\sqrt{3})^4=(8\text{ cis }120°)^4$

$=8^4\text{ cis }(120°\times4)$

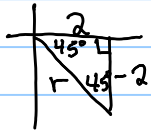
$=4096\text{ cis }480°$

$\begin{array}{r}480°\\-360°\\\hline120°\end{array}$

$=4096(\cos 120°+i\sin 120°)$

$=4096\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$=\boxed{-2048+2048i\sqrt{3}}$

37. 

45°-45°-90°

$r=2\sqrt{2}$

$\theta=-45°$

$2\sqrt{2}$ > 2
$2\sqrt{2}$
$2\sqrt{2}$ > 2
$2\sqrt{2}$
$2\sqrt{2}$
32

$(2-2i)^5=(2\sqrt{2}\text{ cis }(-45°))^5$

$=(2\sqrt{2})^5\text{ cis }(-45°\times5)$

$=128\sqrt{2}(\cos(-225°)+i\sin(-225°))$

$=128\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

$=-\frac{128(2)}{2}+\frac{128(2)}{2}i=\boxed{-128+128i}$

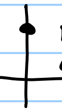
38. Fourth roots of 81 cis 280°

$81^{1/4}=3$ $(280°+360°k)/4$

$=70°+90k$

$\boxed{\begin{array}{l}3(\cos 70°+i\sin 70°)\\3(\cos 160°+i\sin 160°)\\3(\cos 250°+i\sin 250°)\\3(\cos 340°+i\sin 340°)\end{array}}$

39. Fifth roots of $32i$



r=32
θ=90°

Fifth roots of 32 cis 90°

$32^{1/5}=2$ $(90°+360°k)/5$

$=18°+72°k$

$\boxed{\begin{array}{l}2\text{ cis }18°,\ 2\text{ cis }90°,\\2\text{ cis }162°,\ 2\text{ cis }234°,\\2\text{ cis }306°\end{array}}$

40. $x^3+64=0$

$x^3=-64$

$x=$ cube roots of -64

$r=64$

$\theta=180^\circ$

cube roots of $64 \text{ cis } 180^\circ$

$64^{1/3}=4$ $\quad (180^\circ+360^\circ k)/3$

$= 60^\circ+120^\circ k$

$4(\cos 60^\circ+i\sin 60^\circ)=4\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\boxed{2+2i\sqrt{3}}$

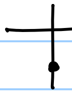
$4(\cos 180^\circ+i\sin 180^\circ)=4(-1+0i)=\boxed{-4}$

$4(\cos 300^\circ+i\sin 300^\circ)=4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=\boxed{2-2i\sqrt{3}}$

41. $x^2+18i=0$

$x^2=-18i$

$x=$ square roots of $-18i$



$r=18$

$\theta=270^\circ$

square roots of $18 \text{ cis } 270^\circ$

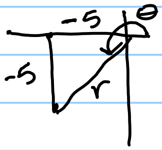
$18^{1/2}=\sqrt{18}=3\sqrt{2}$

$(270^\circ+360^\circ k)/2=135^\circ+180^\circ k$

$3\sqrt{2}(\cos 135^\circ+i\sin 135^\circ)$
$=3\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$
$=\boxed{-3+3i}$

$3\sqrt{2}(\cos 315^\circ+i\sin 315^\circ)$
$=3\sqrt{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)$
$=\boxed{3-3i}$

42.

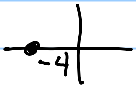


$45^\circ\text{-}45^\circ\text{-}90^\circ\ \triangle$

$r=5\sqrt{2}$ $\quad \theta=5\pi/4$

$\boxed{(5\sqrt{2}, 5\pi/4)}$

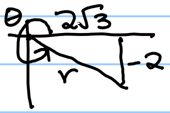
43.



$r=4$ $\quad \theta=\pi$

$\boxed{(4,\pi)}$

44.

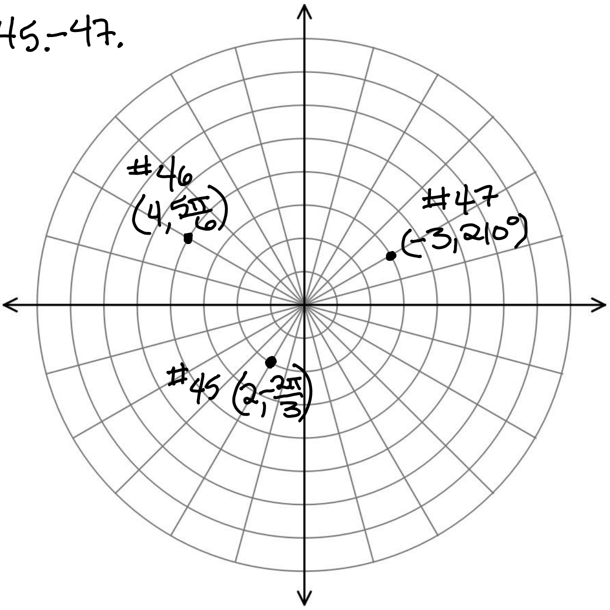


$30^\circ\text{-}60^\circ\text{-}90^\circ\ \triangle$

$r=2(2)=4$ $\quad \theta=\frac{11\pi}{6}$

$\boxed{\left(4,\frac{11\pi}{6}\right)}$

45.-47.



48. $\left(-2,\frac{3\pi}{4}\right)$

$x=-2\cos\left(\frac{3\pi}{4}\right)=-2\left(-\frac{\sqrt{2}}{2}\right)=\sqrt{2}$

$y=-2\sin\left(\frac{3\pi}{4}\right)=-2\left(\frac{\sqrt{2}}{2}\right)=-\sqrt{2}$

$\boxed{(\sqrt{2},-\sqrt{2})}$

49. $\left(3,-\frac{1}{2}\pi\right)=\left(3,-\frac{\pi}{2}\right)$

$x=3\cos\left(-\frac{\pi}{2}\right)=3(0)=0$

$y=3\sin\left(-\frac{\pi}{2}\right)=3(-1)=-3$

$\boxed{(0,-3)}$

50. $\left(-4,\frac{4\pi}{3}\right)$

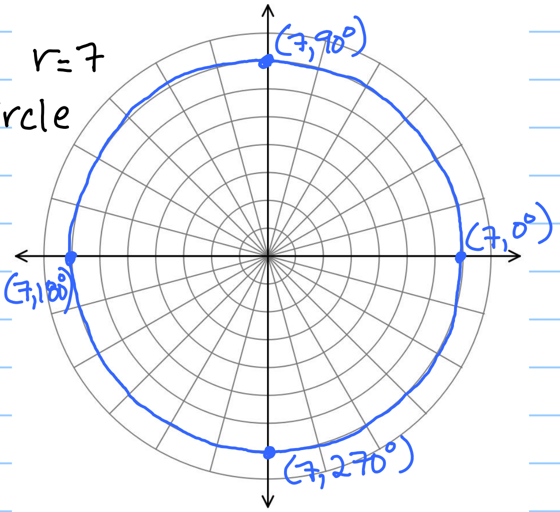
$x=-4\cos\left(\frac{4\pi}{3}\right)=-4\left(-\frac{1}{2}\right)=2$

$y=-4\sin\left(\frac{4\pi}{3}\right)=-4\left(-\frac{\sqrt{3}}{2}\right)=2\sqrt{3}$

$\boxed{(2,2\sqrt{3})}$

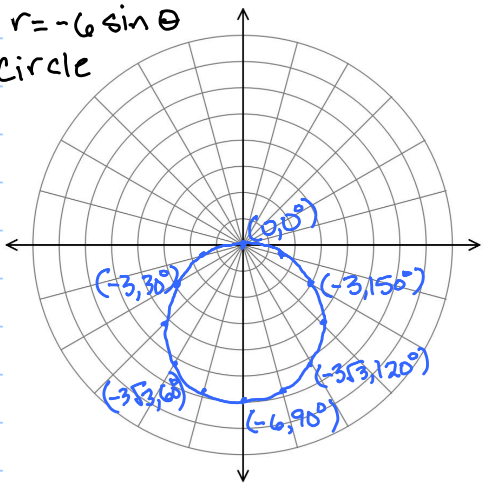
51. $r=7$

circle



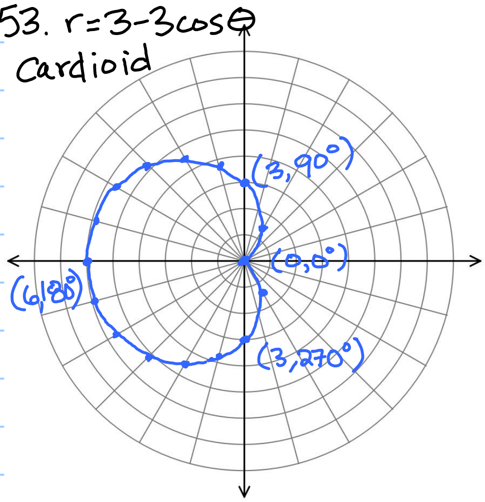
52. $r=-6\sin\theta$

circle



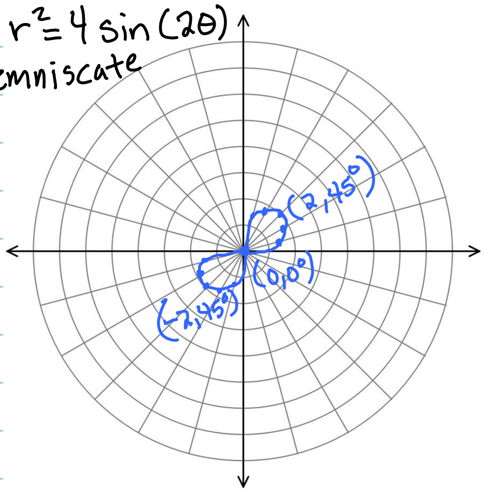
53. $r=3-3\cos\theta$

cardioid



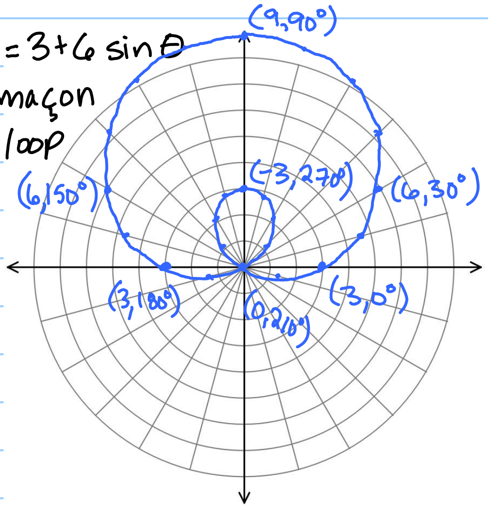
54. $r^2=4\sin(2\theta)$

lemniscate



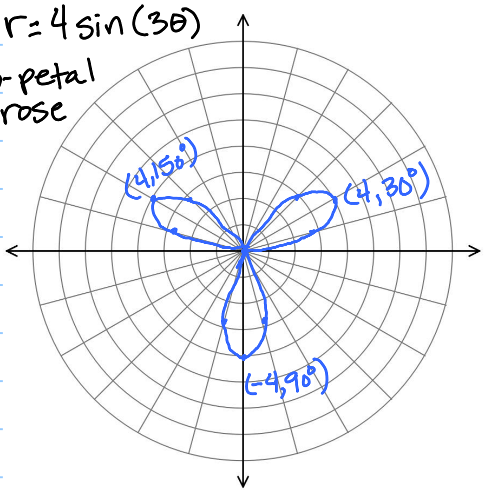
55. $r=3+6\sin\theta$

limaçon w/loop



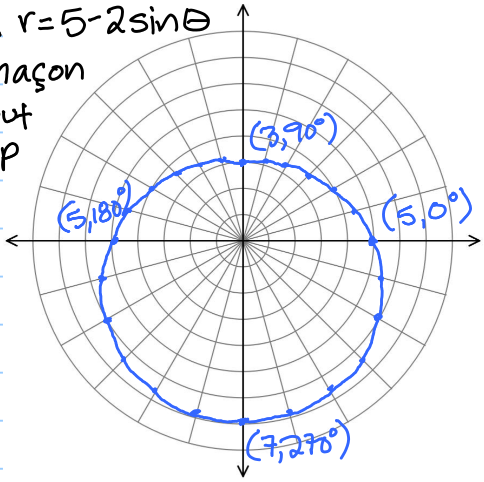
56. $r=4\sin(3\theta)$

3-petal rose



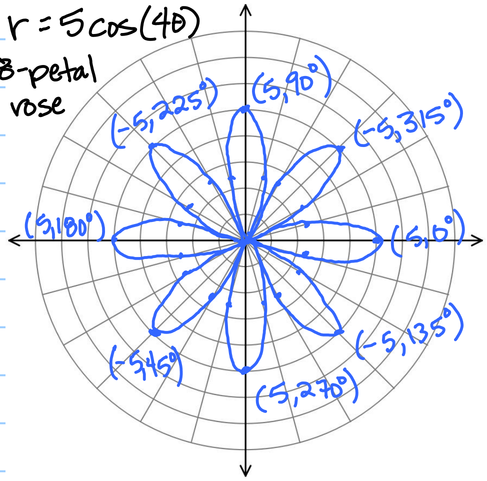
57. $r=5-2\sin\theta$

limaçon w/out loop



58. $r = 5\cos(4\theta)$

8-petal rose



59. $r = 7$

$r^2 = 49$

$\boxed{x^2 + y^2 = 49}$

60. $r = 5\cos\theta$

Mult. both sides by r:

$r^2 = 5r\cos\theta$

$\boxed{x^2 + y^2 = 5x}$

61. $r = 7\csc\theta$

$r = \frac{7}{\sin\theta}$

$r\sin\theta = 7$

$\boxed{y = 7}$

62. $x^2 + y^2 = 64$

$r^2 = 64$

$\boxed{r = 8}$

63. $x = 5$

$r\cos\theta = 5$

$r = \frac{5}{\cos\theta}$

$\boxed{r = 5\sec\theta}$

64. $x^2 + y^2 + 5y = 0$

$r^2 + 5r\sin\theta = 0$

$r(r + 5\sin\theta) = 0$

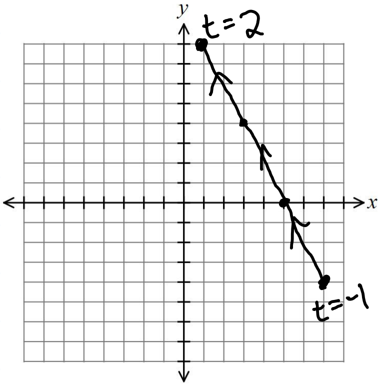
$r = 0$ $\boxed{r = -5\sin\theta}$

This is a point on the other graph. Ignore it.

65. $x = -2t + 5$

$y = 4t$ [-1,2]

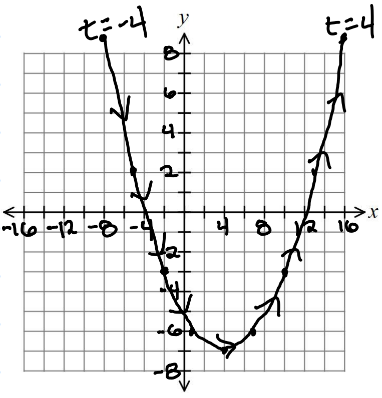
t	x	y
-1	7	-4
0	5	0
1	3	4
2	1	8



66. $x = 3t + 4$

$y = t^2 - 7$ [-4,4]

t	x	y
-4	-8	9
-3	-5	2
-2	-2	-3
-1	1	-6
0	4	-7
1	7	-6
2	10	-3
3	13	2
4	16	9



67. $x = 8t$

$t = \frac{x}{8}$

$y = 4t + 9$

$y = 4\left(\frac{x}{8}\right) + 9$

$\boxed{y = \frac{1}{2}x + 9}$

68. $x = t + 5$

$t = x - 5$

$y = t^2 + 3$

$\boxed{y = (x-5)^2 + 3}$ or

$y = x^2 - 10x + 25 + 3$

$\boxed{y = x^2 - 10x + 28}$

69. $x = 4\cos\theta \qquad y = \sin\theta$

$\cos\theta = \frac{x}{4}$

$\cos^2\theta = \frac{x^2}{16} \qquad \sin^2\theta = y^2$

$\underbrace{\cos^2\theta + \sin^2\theta}_{1} = \frac{x^2}{16} + y^2$

$\boxed{\frac{x^2}{16} + y^2 = 1}$

70. $r = -3\cos^2\theta$

$x = r\cos\theta \qquad y = r\sin\theta$

$x = (-3\cos^2\theta)\cos\theta \qquad y = (-3\cos^2\theta)\sin\theta$

$\boxed{x = -3\cos^3\theta \qquad y = -3\cos^2\theta\sin\theta}$

71. $x = mt + b$

$t=0, x=1: \; 1 = m(0) + b \Rightarrow b = 1$

$t=3, x=13: \; 13 = m(3) + b$

$13 = 3m + 1$

$12 = 3m$

$m = 4$

$\boxed{x = 4t + 1}$

$y = mt + b$

$t=0, y=-2: \; -2 = m(0) + b \Rightarrow b = -2$

$t=3, y=1: \; 1 = m(3) + b$

$1 = 3m - 2$

$3 = 3m$

$m = 1$

$\boxed{y = t - 2}$

72. $x = mt + b$

$t=3, x=2: \; -(2 = 3m + b)$

$t=7, x=-10: \; -10 = 7m + b$

$-2 = -3m - b$

$\frac{-12}{4} = \frac{4m}{4}$

$m = -3$

$2 = 3(-3) + b$

$2 = -9 + b$

$b = 11$

$\boxed{x = -3t + 11}$

$y = mt + b$

$t=3, y=5: \; -(5 = 3m + b)$

$t=7, y=13: \; 13 = 7m + b$

$-5 = -3m - b$

$\frac{8}{4} = \frac{4m}{4}$

$m = 2$

$5 = 3(2) + b$

$5 = 6 + b$

$b = -1$

$\boxed{y = 2t - 1}$