Precalculus - Unit 10 Review

1. $\frac{7!}{4!} = \frac{7\cdot 6\cdot 5\cdot \cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 7\cdot 6\cdot 5 = \boxed{210}$

2. a) $\{s_n\} = \{4n-2\}$

$s_1 = 4(1)-2 = 2$
$s_2 = 4(2)-2 = 6$
$s_3 = 4(3)-2 = 10$
$s_4 = 4(4)-2 = 14$
$s_5 = 4(5)-2 = 18$
$\boxed{2, 6, 10, 14, 18, \ldots}$

b) $\{a_n\} = \{2n^2 + n\}$

$a_1 = 2(1)^2 + 1 = 3$
$a_2 = 2(2)^2 + 2 = 10$
$a_3 = 2(3)^2 + 3 = 21$
$a_4 = 2(4)^2 + 4 = 36$
$a_5 = 2(5)^2 + 5 = 55$
$\boxed{3, 10, 21, 36, 55, \ldots}$

3. a) $a_1 = 8$
$a_n = 5a_{n-1} + 2$

$a_1 = 8$
$a_2 = 5a_1 + 2 = 5(8) + 2 = 42$
$a_3 = 5a_2 + 2 = 5(42) + 2 = 212$
$a_4 = 5a_3 + 2 = 5(212) + 2 = 1062$
$\boxed{8, 42, 212, 1062 \ldots}$

b) $a_1 = -15$
$a_n = 2n + 3a_{n-1}$

$a_1 = -15$
$a_2 = 2(2) + 3a_1 = 4 + 3(-15) = -41$
$a_3 = 2(3) + 3a_2 = 6 + 3(-41) = -117$
$a_4 = 2(4) + 3a_3 = 8 + 3(-117) = -343$
$\boxed{-15, -41, -117, -343, \ldots}$

4. $\sum_{k=1}^{n} (3k-1) = [3(1)-1] + [3(2)-1] + [3(3)-1] + \ldots + [3n-1]$
$= \boxed{2 + 5 + 8 + \ldots + 3n-1}$

5. $4^3 + 5^3 + 6^3 + \ldots + 13^3 = \boxed{\sum_{k=4}^{13} k^3}$

6th through 20th = All 20 − First 5

6. $\sum_{k=6}^{20} 3k^2 = 3\sum_{k=6}^{20} k^2 = 3\left(\sum_{k=1}^{20} k^2 - \sum_{k=1}^{5} k^2\right) = 3\left[\frac{20(20+1)(2(20)+1)}{6} - \frac{5(5+1)(2(5)+1)}{6}\right]$

$= 3(2870 - 55) = \boxed{8445}$

7. 37th term of 15, 11, 7, 3, ... $a_1 = 15$ $d = -4$

$a_n = a_1 + (n-1)d$ $a_{37} = 15 + (37-1)(-4) = \boxed{-129}$

8. $a_1 = 5$ $d = 3$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)3$

$a_n = 5 + 3n - 3$

$\boxed{a_n = 3n + 2}$

$a_{18} = 3(18) + 2 = \boxed{56}$

−or−

$a_{18} = 5 + (18-1)(3) = \boxed{56}$

9. 28 rows ($\uparrow n$) 18 seats in row 1 ($\uparrow a_1$) 18, 20, 22, ...

$d = 2$

$S_n = \frac{n}{2}(a_1 + a_n)$ $a_{28} = 18 + (28-1)(2) = 72$ seats on 28th row

$S_{28} = \frac{28}{2}(18 + 72) = \boxed{1260 \text{ seats}}$

10. $\underbrace{7^{th} \text{ term} = -47,\ 13^{th} \text{ term} = -101}_{-54 \text{ over } 6 \text{ terms}}$

$d = -\frac{54}{6}$

$\boxed{d = -9}$

$a_n = a_1 + (n-1)d$

$-47 = a_1 + (7-1)(-9)$

$-47 = a_1 - 54 \longrightarrow \boxed{a_1 = 7}$

Recursive Formula:

$\boxed{a_1 = 7,\ a_n = a_{n-1} - 9}$

11. $(-5) + (-2) + 1 + 4 + \ldots + 76$

$a_1 = -5$ $a_n = 76$ $d = 3$ $n = ?$

$76 = -5 + (n-1)(3)$

$76 = -5 + 3n - 3$

$76 = 3n - 8$

$84 = 3n$

$n = 28$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{28} = \frac{28}{2}(-5 + 76) = 14(71) = \boxed{994}$

12. $\sum_{k=1}^{64} (3k-5)$ arithmetic $n=64$

$S_n = \frac{n}{2}(a_1+a_n)$ $a_1 = 3(1)-5 = -2$

$S_{64} = \frac{64}{2}(-2+187)$ $a_{64} = 3(64)-5 = 187$

$= \boxed{5920}$

13. 10^{th} term of $-1, \frac{1}{2}, -\frac{1}{4}, \ldots$

$a_1 = -1$ $r = -\frac{1}{2}$

$a_n = a_1 r^{n-1}$

$a_{10} = (-1)(-\frac{1}{2})^{10-1} = (-1)(-\frac{1}{2})^9 = \boxed{\frac{1}{512}}$

14. $a_1 = 3$ $r=4$

$\boxed{a_n = 3(4)^{n-1}}$

$a_6 = 3(4)^{6-1} = 3(4)^5 = 3(1024) = \boxed{3072}$

15. n^{th} term of $2, 6, 18, 54, 162, \ldots$

$a_1 = 2$ $r=3$

$\boxed{a_n = 2(3)^{n-1}}$

16. a) $\sum_{k=1}^{5} (\frac{1}{2})(2)^k$ - OR - write it out

$a_1 = (\frac{1}{2})(2)^1 = 1$ $(\frac{1}{2})(2)^1 + (\frac{1}{2})(2)^2 + (\frac{1}{2})(2)^3 + (\frac{1}{2})(2)^4 + (\frac{1}{2})(2)^5$

$r = 2$ $= 1 + 2 + 4 + 8 + 16 = \boxed{31}$

$n = 5$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = 1\left(\frac{1-2^5}{1-2}\right) = \boxed{31}$

b) $12 - 8 + \frac{16}{3} + \ldots + 12(-\frac{2}{3})^9$ $a_1 = 12$ $n=10$ $r = -\frac{2}{3}$

$12(-\frac{2}{3})^0$ $12(-\frac{2}{3})^1$ 10^{th} term

$S_{10} = 12\left(\frac{1-(-\frac{2}{3})^{10}}{1-(-\frac{2}{3})}\right) = \boxed{\frac{46,420}{6,561} \approx 7.075}$

17. geometric, $a_1 = 2.5$ mi $\quad r = 1.2 \quad a_n = 2.5(1.2)^{n-1}$

$a_7 = 2.5(1.2)^{7-1} = \boxed{7.46496 \text{ mi on } 7^{th} \text{ day}}$

$S_7 = 2.5\left(\dfrac{1-1.2^7}{1-1.2}\right) = \boxed{32.28967 \text{ mi total}}$

18. a) $\sum_{k=1}^{\infty} \frac{2}{3} \cdot 2^{k-1}$

$\uparrow$

$r = 2$

$\boxed{\text{diverges}}$

b) $4 - 2 + 1 - \frac{1}{2} + \ldots$

$a_1 = 4 \quad r = -\frac{1}{2}$

$\boxed{\text{converges}}$

$S = \dfrac{a_1}{1-r} = \dfrac{4}{1-(-\frac{1}{2})} = \dfrac{4}{3/2} = 4\left(\frac{2}{3}\right) = \boxed{\frac{8}{3}}$

19. Infinite geometric sequence

$a_1 = 90$ in $\quad r = .8 \quad$ converges

$S = \dfrac{90}{1-.8} = \boxed{450 \text{ in}}$

20. a) $\{-5n + 2\}$

linear $\rightarrow$ arithmetic

$a_n = -5n + 2$

$a_{n-1} = -5(n-1) + 2$

$= -5n + 5 + 2$

$= -5n + 7$

$a_n - a_{n-1} = (-5n+2) - (-5n+7)$

$= -5n + 2 + 5n - 7$

$= -5 \leftarrow$ constant

$\boxed{\text{arithmetic, } d = -5}$

b) $\{4n^2 + 7\}$

quadratic $\rightarrow$ neither

$a_n = 4n^2 + 7$

$a_{n-1} = 4(n-1)^2 + 7$

$= 4(n^2 - 2n + 1) + 7$

$= 4n^2 - 8n + 4 + 7$

$= 4n^2 - 8n + 11$

$a_n - a_{n-1} = (4n^2+7) - (4n^2 - 8n + 11)$

$= 4n^2 + 7 - 4n^2 + 8n - 11$

$= 8n - 4$

$\uparrow$

not constant $\Rightarrow$ not arithmetic

$\dfrac{a_n}{a_{n-1}} = \dfrac{4n^2+7}{4n^2-8n+11}$

$\nearrow$

not constant $\Rightarrow$ not geometric

$\boxed{\text{neither}}$

c) $\{3^{2n}\} = \{9^n\}$

exponential $\rightarrow$ geometric

$a_n = 9^n$

$a_{n-1} = 9^{n-1}$

$\dfrac{a_n}{a_{n-1}} = \dfrac{9^n}{9^{n-1}}$

$= 9^{n-(n-1)}$

$= 9 \leftarrow$ constant

$\boxed{\text{geometric, } r = 9}$

21. a) $\binom{10}{4} = \frac{10!}{4!\,6!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6!}}{4\cdot 3\cdot 2\cdot 1\cdot \cancel{6!}} = \boxed{210}$

b) $\binom{6}{5} = \frac{6!}{5!\,1!} = \frac{6\cdot \cancel{5!}}{\cancel{5!}\cdot 1} = \boxed{6}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

22. a) $(x-3)^5$ Use 1 5 10 10 5 1

$= 1(x)^5 + 5(x)^4(-3) + 10(x)^3(-3)^2 + 10(x)^2(-3)^3 + 5(x)(-3)^4 + 1(-3)^5$

$= \boxed{x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243}$

b) $(2x+5)^4$ Use 1 4 6 4 1

$= 1(2x)^4 + 4(2x)^3(5) + 6(2x)^2(5)^2 + 4(2x)(5)^3 + 1(5)^4$

$= \boxed{16x^4 + 160x^3 + 600x^2 + 1000x + 625}$

c) $(4x - y^2)^6$ Use 1 6 15 20 15 6 1

$= 1(4x)^6 + 6(4x)^5(y^2) + 15(4x)^4(y^2)^2 + 20(4x)^3(y^2)^3 + 15(4x)^2(y^2)^4 + 6(4x)(y^2)^5 + 1(y^2)^6$

$= \boxed{4096x^6 - 6144x^5y^2 + 3840x^4y^4 - 1280x^3y^6 + 240x^2y^8 - 24xy^{10} + y^{12}}$

23. a) Coefficient of x^2 in $(3x+2)^6$

$\binom{6}{6-2}(3x)^2(2)^{6-2} = \binom{6}{4}(3x)^2(2)^4$

$= 15(9x^2)(16)$

$= 2160x^2$ Coefficient: $\boxed{2160}$

b) 4th term in $(7x-2y)^7$ x^7 x^6y x^5y^2 $\underline{x^4y^3}$

$= \binom{7}{7-4}(7x)^4(-2y)^{7-4} = \binom{7}{3}(7x)^4(-2y)^3$

$= 35(2401x^4)(-8y^3)$

$= \boxed{-672{,}280x^4y^3}$