Objective: Combining Functions with Arithmetic

It can be useful to combine two functions to make a new function. For instance, you may have a function describing the revenue from a product and a function describing the costs of producing the product. By subtracting the two functions, you can create a function describing the profit made from the product.

Tips:
- Use parentheses carefully. Write each function in parentheses.
- Be careful with negatives. If there is a negative outside parentheses, it changes the sign of *everything* inside the parentheses.
- When adding or subtracting, combine like terms.
- When multiplying, distribute or FOIL and make sure to use exponent rules correctly.

Examples: Let $f(x)=3x-5$ and $g(x)=x^2+5x-2$. Perform the indicated operations.

a) $h(x)=f(x)+g(x)$

b) $h(x)=f(x)-g(x)$

c) $h(x)=g(x)-f(x)$

d) $h(x)=2f(x)+3g(x)$

e) $h(x)=-f(x)+4g(x)$

f) $h(x)=f(x)-5f(x)$

g) $h(x)=f(x)\cdot g(x)$

h) $h(x)=f(x)\cdot f(x)$

Evaluating Combined Functions

To evaluate a combined function for certain values of x, replace x with the specified number in each function, then add, subtract, multiply or divide. Make sure to follow order of operations!

Examples: Let $f(x) = 2x - 7$, and let $g(x) = -x^2 + 3$. Evaluate the following.

a) $f(2) + g(1)$ b) $f(0) - g(-3)$ c) $f(-2) \cdot 3g(2)$

Examples: Let $f(x) = 3x - 5$ and $g(x) = (x+3)(x-1)$ Perform the indicated operations and state the domain of the new function.

a) $r(x) = \dfrac{g(x)}{f(x)}$ b) $r(x) = \dfrac{f(x)}{g(x)}$

Domain: **Domain:**

c) $r(x) = \dfrac{2f(x)}{f(x)}$ d) $r(x) = \dfrac{g(x)}{-3g(x)}$

Domain: **Domain:**

Examples: Let $f(x) = 3x - 5$ and $g(x) = (x+3)(x-1)$ Evaluate the following functions with the given values and functions.

a) $\dfrac{f(2)}{g(-2)}$ b) $\dfrac{-2f(5)}{g(-1)}$

Story Problems Involving Combined Functions

a) A company estimates that its cost and revenue can be modeled by the functions $C(x) = 0.6x^2 + 49x + 150$ and $R(x) = 100x + 75$, where x is the number of items produced. The company's profit, P, can be modeled by $P(x) = R(x) - C(x)$. Find the profit equation and determine the profit when 60 items are produced.

b) A service committee is organizing a fundraising dinner. The cost of renting a facility is $250 plus $3 per person, or $C(x) = 3x + 250$, where x represents the number of people attending the fundraiser. The committee wants to charge attendees $20 each or $R(x) = 20x$. How many people must attend the fundraiser for the event to raise $500?