

Date:

Section: 10.4

SM 2

Objective: Use AA, SAS, and SSS similarity criteria to decide whether triangles are similar.

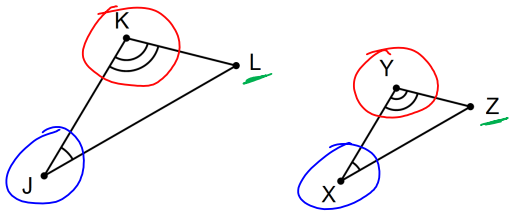
We learned last time that to show two figures are similar, we've had to show that **all** of the corresponding angles are congruent and **all** of the corresponding sides are proportional. Luckily, there are some shortcuts for triangles.

1. Angle-Angle Similarity Postulate (AA Similarity): If two angles in one triangle are congruent (have the same measures as) two

Write an if-then statement for the triangles.

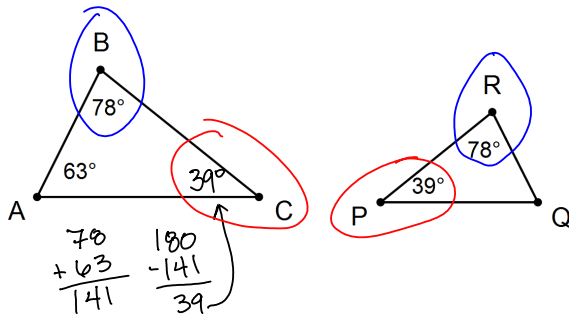
IF $\angle J \cong \angle X$ and $\angle K \cong \angle Y$,
then $\triangle JKL \sim \triangle XYZ$

measures in another triangle, then the two triangles are similar.



Examples: Determine whether the triangles are similar. **Explain** your reasoning. If they are similar, write a similarity statement.

a)



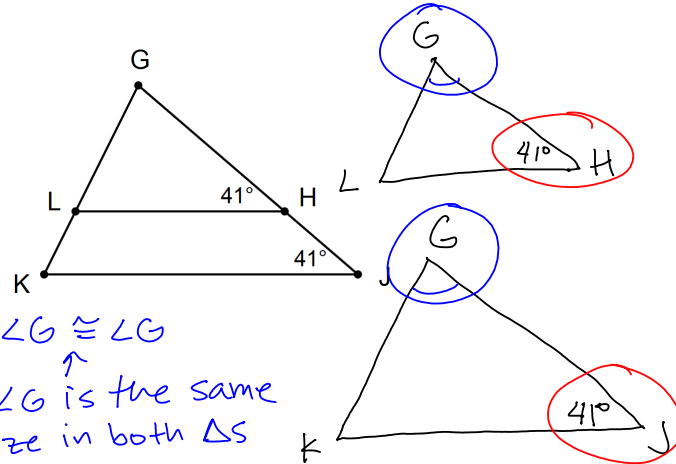
Angles in a triangle add to 180

$$\begin{array}{r} 78 \\ + 63 \\ \hline 141 \end{array} \quad \begin{array}{r} 180 \\ - 141 \\ \hline 39 \end{array}$$

$\angle B \cong \angle R$
 $\angle C \cong \angle Q$

$\triangle BCA \sim \triangle RPQ$ by AA

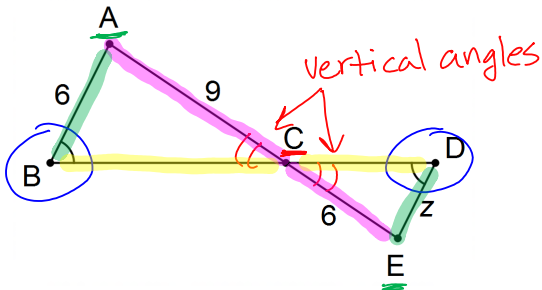
b)



$\angle G \cong \angle G$
 $\angle H \cong \angle J$
 $\angle L \cong \angle K$
 $\angle G$ is the same size in both \triangle s

$\triangle GHL \sim \triangle GJK$ by AA

Example: Use the diagram to fill in the statements.



a) $\angle B \cong \angle D$ (marked on diagram)

b) $\angle ACB \cong \angle ECD$ because they are vertical angles.

c) $\triangle ACB \sim \triangle ECD$ by the AA Similarity postulate.

d) What is the scale factor? $\frac{9}{6} = \frac{3}{2}$

e) $\frac{AB}{DE} = \frac{AC}{EC}$
 $\frac{6}{?} = \frac{9}{6}$

f) $\frac{6}{z} = \frac{9}{6}$

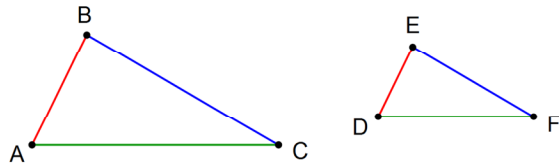
g) $z = ?$

$\frac{6}{z} = \frac{9}{6}$

$\frac{9z}{9} = \frac{36}{9}$

$z = 4$

2. **Side-Side-Side Similarity Theorem (SSS Similarity):**



Write an if-then statement for the triangles.

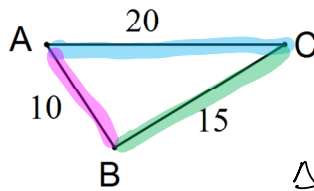
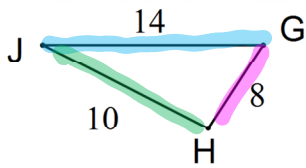
If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$,
 then $\triangle ABC \sim \triangle DEF$

If the sides of one triangle are proportional to the corresponding sides of the other triangle (the ratios of the sides are all equal),

then the triangles are similar.

★ **TIP:** When testing for SSS similarity, make fractions out of the shortest sides, longest sides, and medium sides.

Example: Which two triangles are similar? Write a similarity statement when you find out.



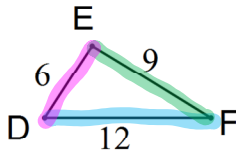
$\triangle JGH$ vs. $\triangle CAB$

$\frac{\text{longest}}{\text{longest}} = \frac{14}{20} = \frac{7}{10}$
 $\frac{\text{middle}}{\text{middle}} = \frac{10}{15} = \frac{2}{3}$ } not the same, not similar

$\triangle JGH$ vs. $\triangle FDE$

$\frac{\text{longest}}{\text{longest}} = \frac{14}{12} = \frac{7}{6}$

$\frac{\text{middle}}{\text{middle}} = \frac{10}{9}$ } not the same, not similar



$\triangle CAB$ vs. $\triangle FDE$

$\frac{\text{longest}}{\text{longest}} = \frac{20}{12} = \frac{5}{3}$ $\frac{\text{middle}}{\text{middle}} = \frac{15}{9} = \frac{5}{3}$

$\frac{\text{shortest}}{\text{shortest}} = \frac{10}{6} = \frac{5}{3}$ } Similar
 $\triangle CAB \sim \triangle FDE$ by SSS

3. **Side-Angle-Side Similarity Theorem (SAS Similarity):**

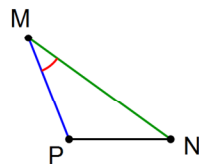
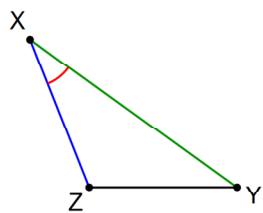
If two sides of one triangle are proportional to two sides in another triangle (ratios of the sides are equal),

Write an if-then statement for the triangles.

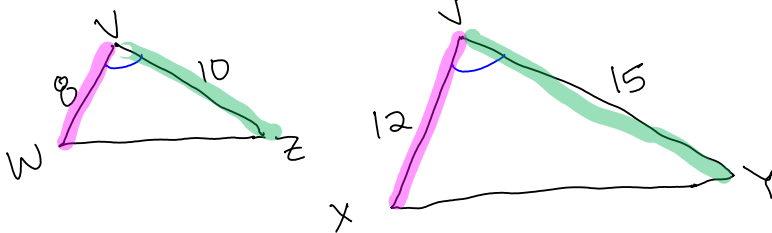
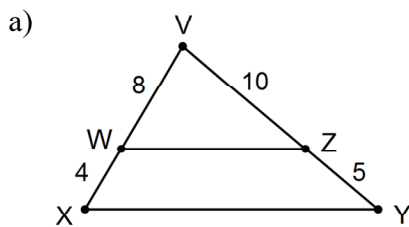
If $\frac{XZ}{MP} = \frac{XY}{MN}$ and $\angle X \cong \angle M$, then
 $\triangle ZXY \sim \triangle PMN$

Comparing corresponding sides are equal

AND the angles between the two sides are congruent, then the triangles are similar



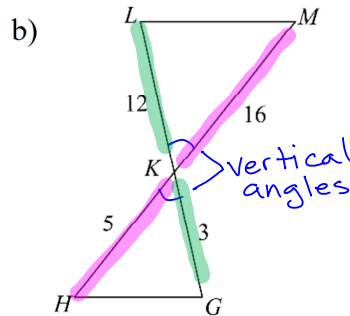
Examples: Determine whether the triangles are similar. If they are similar, write a similarity statement and determine the scale factor.



$\angle V \cong \angle X$ ← $\angle V$ is the same size in both triangles

$\frac{\text{short}}{\text{short}} = \frac{8}{12} = \frac{2}{3}$ $\frac{\text{long}}{\text{long}} = \frac{10}{15} = \frac{2}{3}$

Same
 $\triangle VWZ \sim \triangle XZY$ by SAS



$\angle LKM \cong \angle GKH$ (vertical angles)

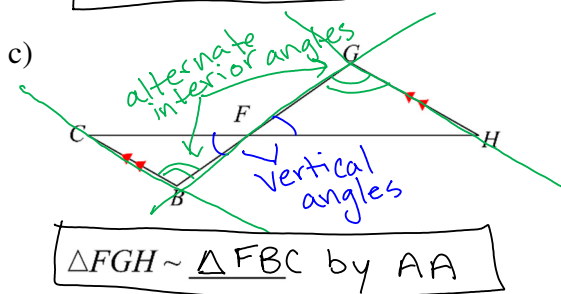
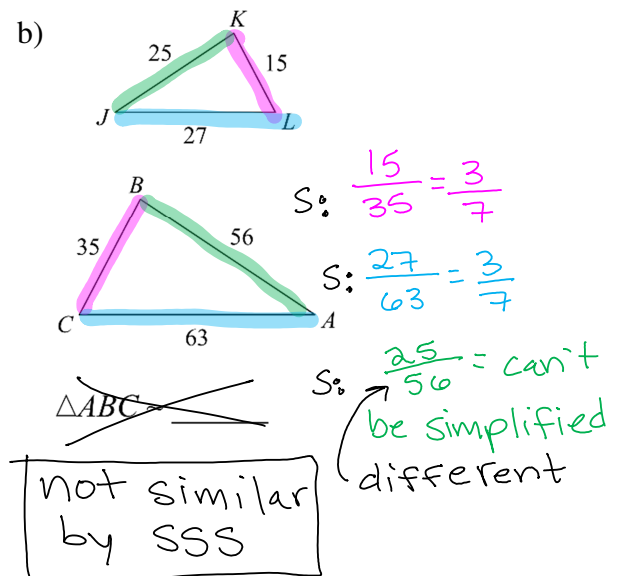
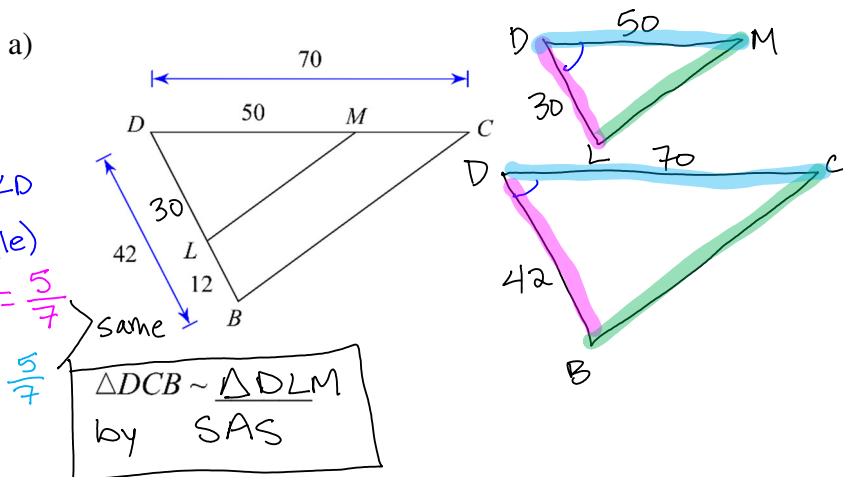
$\frac{\text{shortest}}{\text{shortest}} = \frac{12}{3} = 4$
 $\frac{\text{longest}}{\text{longest}} = \frac{16}{5}$ } different

not similar

Steps for triangle similarity problems:

- Look at the diagram and figure out what information it gives you – does it tell you about sides, angles, or both? (If you have overlapping triangles, draw them separately)!
- Figure out if there are any angles you know must be congruent because of other reasons. If you find any of them, mark them with congruency marks on the diagram. Possible reasons:
 - Overlapping angles – the same angle is part of both triangles.
 - Vertical angles (across from each other in an x).
 - If you are told two angle measures, you can find the third one by adding the two you know and subtracting from 180° . Show the work on this!
 - If there are parallel segments, look for corresponding angles or alternate interior angles.
- Write down congruency statements for any congruent angles. ($\angle P \cong \angle Q$ or $\angle ACB \cong \angle DCE$)
- Based on the information you have, decide what test to use:
 - If you have two sets of congruent angles, use AA.
 - If you have all three side lengths in both triangles, check for SSS.
 - If you know two side lengths in both triangles and know the angles between those two sides are congruent, check for SAS.
- If you have information about the sides, determine whether the “matching” sides are proportional. Make fractions out of the shortest/shortest, middle/middle, and longest/longest and simplify them. Write down all of these checks. If the fractions all simplify to the same thing, you have proportional sides. If not, the triangles aren't similar!
- If the triangles are similar, write a similarity statement ($\triangle ABC \sim \triangle DEF$). If they aren't similar, write “not similar”.

- Examples:**
- List any pairs of congruent angles and give a reason why each pair is congruent.
 - Determine whether corresponding sides are proportional by checking if their ratios are equal.
 - State which similarity test (AA, SSS, SAS) can be used to determine if the triangles are similar.
 - If the triangles are similar, complete the similarity statement. If they aren't, write “not similar”.



- A: $\angle CFB \cong \angle HFG$ (vertical \angle s)
 A: $\angle B \cong \angle G$ (alt. int \angle s)