

Date:

Section: 1.6

Objective: Deal with equations and graphs involving multiple transformations.

**Order of Transformations**

When you graph from an equation, you need to apply transformations to the parent graph in a specific order:

**1. Reflections (A negative sign in front of the equation):**

a. Vertical Reflection – graph is reflected over the x-axis

**2. Stretches/Compressions (The number in front of the equation, called a):**

a. Vertical Stretch – the y-coordinates are multiplied by a number greater than 1.

b. Vertical Compression – the y-coordinates are multiplied by a number between 0 and 1.

**3. Translations:**

a. **Horizontal Translations:** graph is shifted to the left or right  
(The **OPPOSITE** of the number being added to or subtracted from x, called h.) *For x-3, h=3*

b. **Vertical Translations:** graph is shifted up or down  
(The number that is added or subtracted at the end of the equation, called k. It has the **SAME** sign as the equation!)

Answer the following questions using the equations:

$y = a\sqrt{x-h} + k$ ,  $y = a|x-h| + k$ ,  $y = a(x-h)^2 + k$

*missing a=1  
missing h or k=0*

Given the following equations find a, h, and k.

*y = 1√x-6+8*

A.  $y = \sqrt{x-6} + 8$   
*invisible 1*  
a = 1

B.  $y = 3(x+4)^2 - 2$   
a = 3

*y = -1|x-0|+7*  
C.  $y = -|x| + 7$   
*invisible 1*  
a = -1

*opp* h = 6

*opp* h = -4

h = 0

*same* k = 8

*same* k = -2

k = 7

For each function, identify the parent graph ( $y = \sqrt{x}$ ,  $y = x^2$ , or  $y = |x|$ ), then list the transformations needed to get from the parent graph to the final graph. Make sure to list the transformations in the order in which they should be applied.

A.  $y = -4(x+2)^2$   
Parent:  $y = x^2$

- Transformations:
1. reflection over x-axis
  2. vertical stretch
  3. left 2

B.  $y = \frac{1}{4}|x-3| - 6$   
Parent:  $y = |x|$

- Transformations:
1. vertical compression
  2. right 3
  3. down 6

C.  $y = -2\sqrt{x-1}$   
Parent:  $y = \sqrt{x}$

- Transformations:
1. reflection over x-axis
  2. vertical stretch
  3. down 1

## General Strategy:

- Figure out what shape the **parent graph** is – does the equation have  $|$ ,  $^2$ , or  $\sqrt{\quad}$ ? Make an  $x, y$  table with the key points of the parent graph.
- Make a second  $x, y$  table to apply any **reflections** and/or **stretches/compressions**:
  - Multiply the  $y$ -coordinates of the parent graph by  $a$  (the number *out in front*, including any negatives.)
- Make a third  $x, y$  table to apply any **translations** to the *new* points:
  - Add  $h$  to the  $x$ -coordinates to move the graph right or left. Remember,  $h$  is the opposite of the number inside with the  $x$ . Do the opposite of the sign in the equation!
  - Add  $k$  to the  $y$ -coordinates to move the graph up or down. Remember,  $k$  is the number added or subtracted at the end of the equation.  $k$  has the same sign as the equation!

**Example:**  $y = -2(x+3)^2 + 1$

$a = \underline{-2}$  ← reflection & stretch (opp same)  
 $h = \underline{-3}$  ← left 3 (opp)  
 $k = \underline{1}$  ← up 1 (same)

**Parent Table:**

$y = x^2$

$x$	$y$
-2	4
-1	1
0	0
1	1
2	4

**Table for stretch and reflection:**

Multiply the  $y$ -value of the parent table by  $a$

$y = -3x^2$

$x$	$y$ ( $ay = -3y$ )
-2	-12
-1	-3
0	0
1	-3
2	-12

multiply  $y$ 's by -3

reflection & vertical stretch

**Table for translation or shift:**

Add  $h$  to  $x$ -value of the previous table  
 Add  $k$  to  $y$ -value of the previous table

$y = -2(x+3)^2 + 1$

$x$ ( $x+h = x-3$ )	$y$ ( $ay+k = -3y+1$ )
-5	-11
-4	-2
-3	1
-2	-2
-1	-11

subtract 3 from  $x$ 's  
 add 1 to  $y$ 's

left 3 and up 1

**For each graph, do the following:**

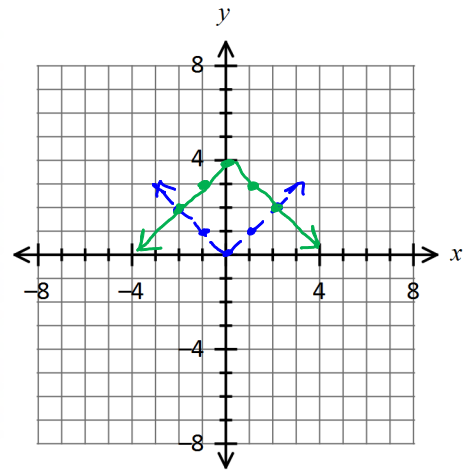
- Identify the parent graph ( $y = |x|$ ,  $y = x^2$ , or  $y = \sqrt{x}$ ).
- Fill in the  $x, y$  table for the parent graph.
- Draw the graph of the parent graph with a dashed line.
- List the transformations in the correct order.
- Make a second  $x, y$  table to apply the reflections and stretches/compressions (by multiplying the  $y$ -coordinates by the number in front or multiply by  $a$ ).
- Make a third and final  $x, y$  table to apply the translations. (Add  $h$  and  $k$  to the  $x$ 's and  $y$ 's from the previous table to move the graph in the correct directions.)
- Draw the final graph with a solid line.
- State the vertex or endpoint, domain, and range of the final graph.

13.  $y = -|x| + 4$   $a = \underline{-1}$   $h = \underline{0}$   $k = \underline{4}$

Parent Graph:  $y = |x|$

Transformations:

reflection over x-axis  
up 4



$y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

reflect over x-axis  
multiply y's by  $a = -1$

x	y
-2	-2
-1	-1
0	0
1	-1
2	-2

up 4  
 $y + 4$

x	y
-2	2
-1	3
0	4
1	3
2	2

Vertex:  $(0, 4) \leftarrow (h, k)$

Domain:  $(-\infty, \infty)$

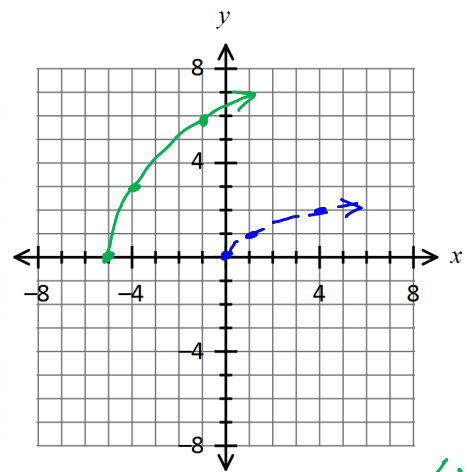
Range:  $(-\infty, 4]$

14.  $y = 3\sqrt{x+5}$   $a = \underline{3}$   $h = \underline{-5}$   $k = \underline{0}$

Parent Graph:  $y = \sqrt{x}$

Transformations:

vertical stretch (by 3)  
left 5



$y = \sqrt{x}$

x	y
0	0
1	1
4	2

vertical stretch  
multiply y's by  $a = 3$

x	y
0	0
1	3
4	6

left 5  
 $x - 5$

x	y
-5	0
-4	3
-1	6

Endpoint:  $(-5, 0) \leftarrow (h, k)$

Domain:  $[-5, \infty)$

Range:  $[0, \infty)$

$$y = -(x-5)^2 + 3$$

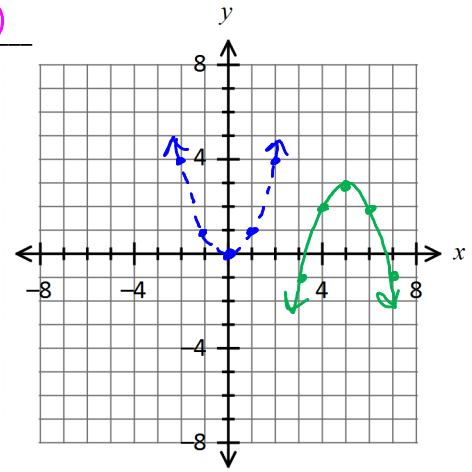
*opp same*

15.  $y = -(x-5)^2 + 3$     $a = \underline{-1}$     $h = \underline{5}$     $k = \underline{3}$

Parent Graph:  $y = x^2$

Transformations:

reflect over x-axis  
right 5  
up 3



Vertex:  $\underline{(5, 3)}$  ←  $(h, k)$

Domain:  $\underline{(-\infty, \infty)}$

Range:  $\underline{(-\infty, 3]}$

$$y = x^2$$

x	y
-2	4
-1	1
0	0
1	1
2	4

reflect over x-axis  
multiply y's by a = -1

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

x's + 5  
y's + 3

x	y
3	-1
4	2
5	3
6	2
7	-1

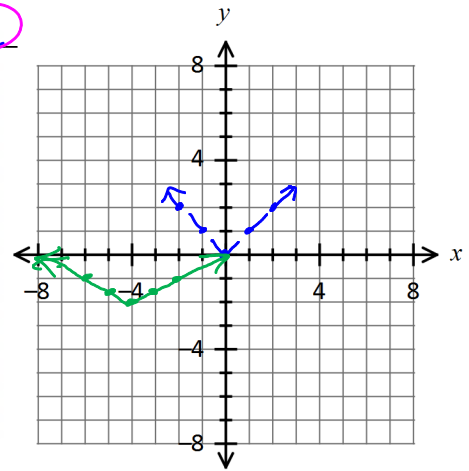
16.  $y = \frac{1}{2}|x+4|-2$     $a = \underline{\frac{1}{2}}$     $h = \underline{-4}$     $k = \underline{-2}$

*opp same*

Parent Graph:  $y = |x|$

Transformations:

vertical compression (by  $\frac{1}{2}$ )  
left 4  
down 2



Vertex:  $\underline{(-4, -2)}$  ←  $(h, k)$

Domain:  $\underline{(-\infty, \infty)}$

Range:  $\underline{[-2, \infty)}$

x	y
-2	2
-1	1
0	0
1	1
2	2

Vertical compr.  
multiply y's by a = 1/2

x	y
-2	1
-1	1/2
0	0
1	1/2
2	1

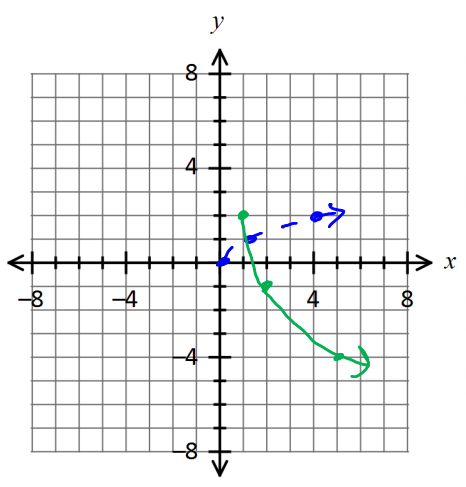
x's - 4  
y's - 2

x	y
-6	-1
-5	-1/2
-4	-2
-3	-1/2
-2	-1

18.  $y = -3\sqrt{x-1} + 2$   $a = \underline{-3}$   $h = \underline{1}$   $k = \underline{2}$

Parent Graph:  $y = \sqrt{x}$

Transformations:  
 reflection over x-axis  
 vertical stretch (by 3)  
 right 1  
 up 2



x	y	reflect and stretch multiply y's by a = -3	x	y	x's + 1 y's + 2	x	y
0	0		0	0		1	2
1	1		1	-3		2	-1
4	2		4	-6		5	-4

Endpoint:  $(1, 2) \leftarrow (h, k)$   
 Domain:  $[1, \infty)$   
 Range:  $(-\infty, 2]$

Write an equation for each transformation of the parent function  $y = \sqrt{x}$ .

- a) 5 units up:  $y = \sqrt{x} + 5$
- b) 8 units right:  $y = \sqrt{x-8}$

Write an equation for each transformation of the parent function  $y = x^2$ .

- a) 5 units left, 12 units down:  $y = (x+5)^2 - 12$
- b) Vertical compression by a factor of  $\frac{1}{2}$ , 9 units right:  $y = \frac{1}{2}(x-9)^2$

Write an equation for each transformation of the parent function  $y = |x|$

- a) Vertical stretch by a factor of 4, reflect over the x-axis, 10 units up:  $y = -4x^2 + 10$