SM 2

Date: **Section:** 1.2

Objective: Use interval notation to describe inequalities. Determine whether or not a relation is a function and find its domain and range based on a graph.

Domain: The set of all inputs (the x-values) of a relation.

- If a relation is represented by a graph, the domain is the set of all x-coordinates of points on the graph. You can think of it as the graph's shadow on the x-axis.

Range: The set of all outputs (the y-values) of a relation.

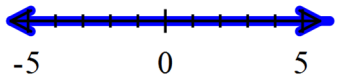
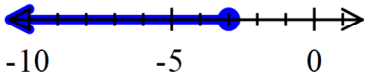
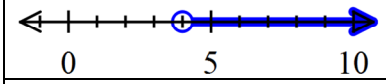
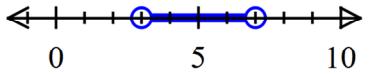
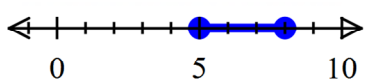
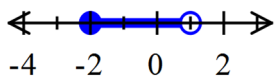
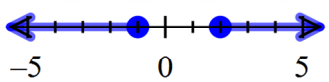
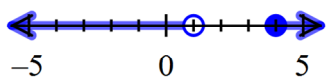
- If a relation is represented by a graph, the range is the set of all y-coordinates of points on the graph. You can think of it as the graph's shadow on the y-axis.

If the graph is a set of unconnected points, the domain and range are just lists of the x and y coordinates, respectively. However, if the graphs contain connected points, they contain an infinite number of points, so we can't list the coordinates. One way we solve this is to use ***interval notation.***

Interval Notation

Domain and range are often written in ***interval notation.***

- The numbers in the parentheses are the endpoints – the points where the interval starts and stops.
- If an endpoint is *included* in the interval, put it in a bracket [or].
- If an endpoint is *not included* in the interval, put it in a parenthesis (or).
- If the interval goes on forever, use $-\infty$ or ∞. Infinity symbols always get put in parentheses (or).
- Multiple intervals are connected with the union sign $\cup$, which is the math symbol for "or".

Graph	Interval Notation	Inequality Notation	Meaning
	$(-\infty,\infty)$	$\mathbb{R}$	All real numbers
	$(-\infty,-3]$	$x \le -3$	Everything less than or equal to -3
	$(4,\infty)$	$x > 4$	Everything greater than 4
	$(3,7)$	$3 < x < 7$ $x > 3$ and $x < 7$	Everything between 3 and 7, not including either 3 or 7
	$[5,8]$	$5 \le x \le 8$ $x \ge 5$ and $x \le 8$	Everything between 5 and 8, including both 5 and 8
	$[-2,1)$	$-2 \le x < 1$ $x \ge -2$ and $x < 1$	Everything between -2 and 1, including -2, but not including 1
	$(-\infty,-1] \cup [2,\infty)$	$x \le -1$ or $x \ge 2$	Everything that is *either* less than *or* equal to -1 or greater than or equal to 2
	$(-\infty,1) \cup [4,\infty)$	$x < 1$ or $x \ge 4$	Everything that is *either* less than 1 *or* greater than or equal to 4

Tips for domain and range

- ***Read the domain from left to right and look at the numbers on the x-axis.***
  Read the range from down to up and look at the numbers on the y-axis.
  - Write the lower value or $-\infty$ first and the higher value or ∞ last.

- If the graph *goes on forever without a boundary* in some direction, the domain or range will involve $-\infty$ or ∞.
  - Always use parentheses around $-\infty$ or ∞.

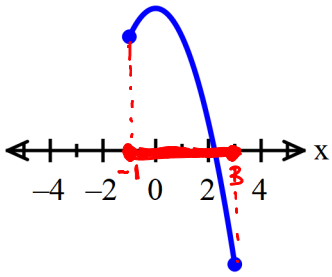
arrow on the end, no dotted line to indicate upper or lower limit

- Use a bracket [or] if the domain or range starts or ends at a number – there is a point at that number on the graph with that x- or y-coordinate.
  - This happens if the graph *has a* ***vertex*** (a point where the graph changes direction) or an ***endpoint*** (a point where the graph starts or stops – represented by a filled-in circle).

- Use a parenthesis (or) if a number *is not included* in the domain or range (the graph gets really, really close to that x- or y-coordinate, but never actually gets there).
  - This happens when there is an ***asymptote*** (a line that the graph gets really close to, but never actually touches). It can also happen if there are holes or gaps in the graph.

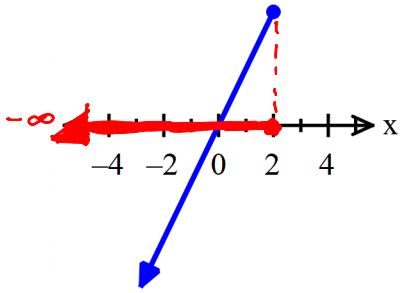
Example: State the domain in interval notation.

a)



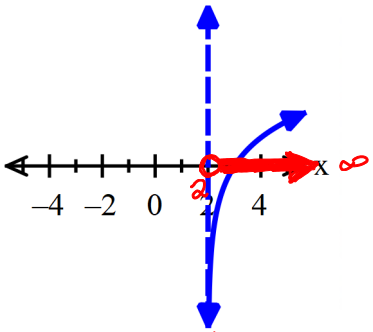
Domain: $[-1, 3]$

b)



Domain: $(-\infty, 2]$

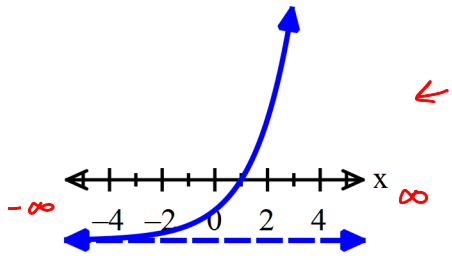
c)



Domain: $(2, \infty)$

Graph never actually gets to this line. There isn't a point on the graph that actually has an x-coordinate of 2, so 2 isn't part of the domain. That's why we use a (instead of a [.

d)



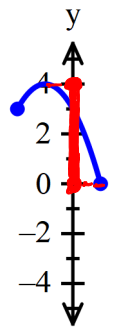
← no boundary on right

Goes on forever to right, just very steeply

Domain: $(-\infty, \infty)$

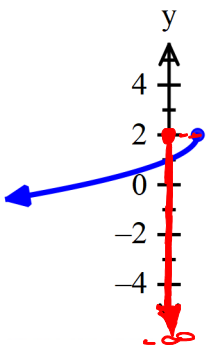
Example: State the range in interval notation.

a)



Range: [0, 4]

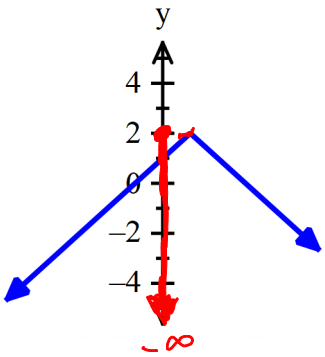
b)



no lower limit

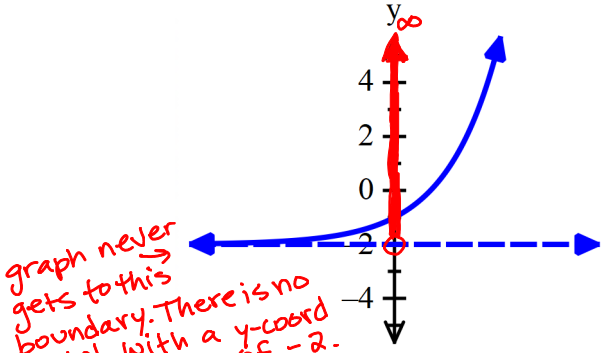
Range: (-∞, 2]

c)



Range: (-∞, 2]

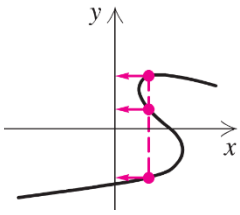
d)



graph never gets to this boundary. There is no point with a y-coord of -2.

Range: (-2, ∞)

Vertical Line Test

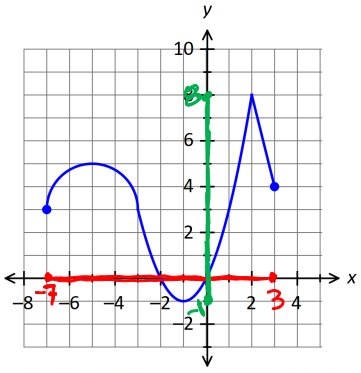


If it is possible for a vertical line to cross a graph more than once, then the graph is not the graph of a function.

The graph at left is not a function because one x-value has 3 different y-values.

Examples: Determine whether each graph is the graph of a function. Then state the domain and range.

a)

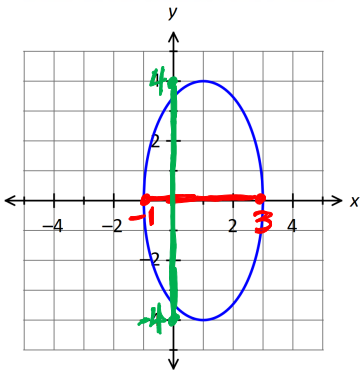


Function? (Yes) / No

Domain? $[-7, 3]$

Range? $[-1, 8]$

b)

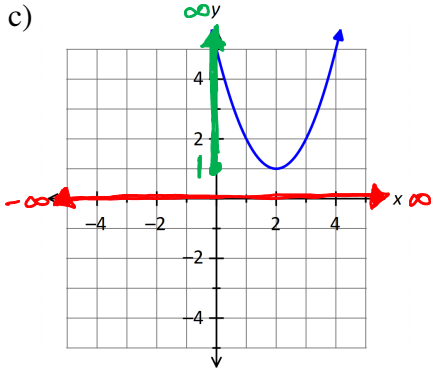


Function? Yes / (No)

Domain? $[-1, 3]$

Range? $[-4, 4]$

c)

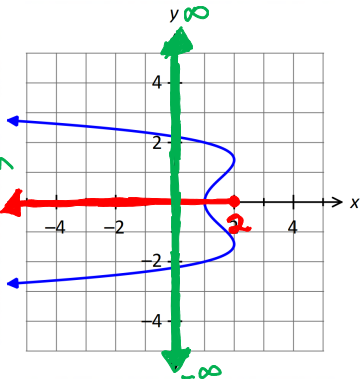


Function? (Yes) / No

Domain? $(-\infty, \infty)$

Range? $[1, \infty)$

d)



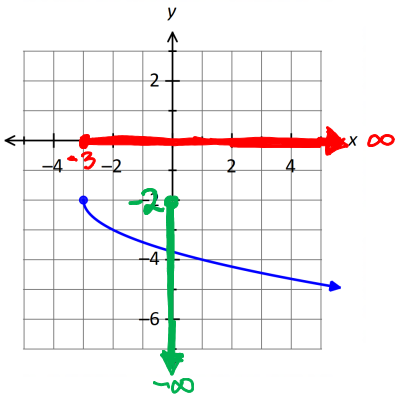
flat, but no boundaries on top & bottom goes on forever up & down

Function? Yes / (No)

Domain? $(-\infty, 2]$

Range? $(-\infty, \infty)$

e)

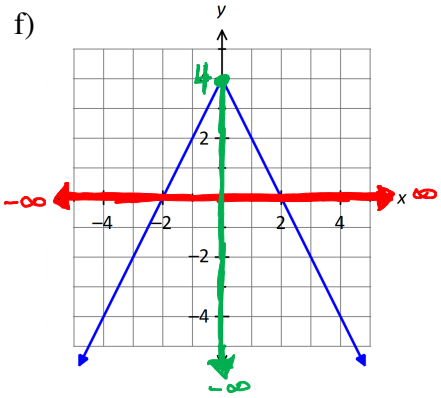


Function? (Yes) / No

Domain? $[-3, \infty)$

Range? $(-\infty, -2]$

f)

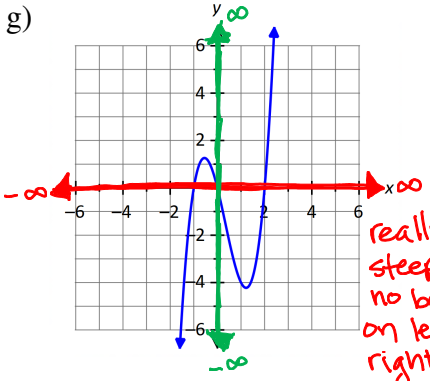


Function? (Yes) / No

Domain? $(-\infty, \infty)$

Range? $(-\infty, 4]$

g)



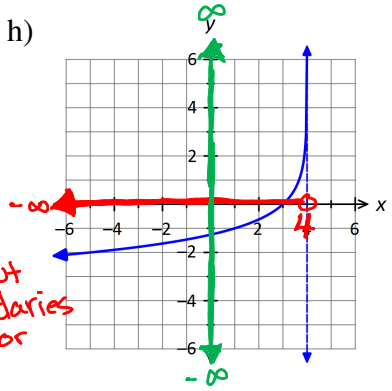
really steep, but no boundaries on left or right

Function? (Yes) / No

Domain? $(-\infty, \infty)$

Range? $(-\infty, \infty)$

h)

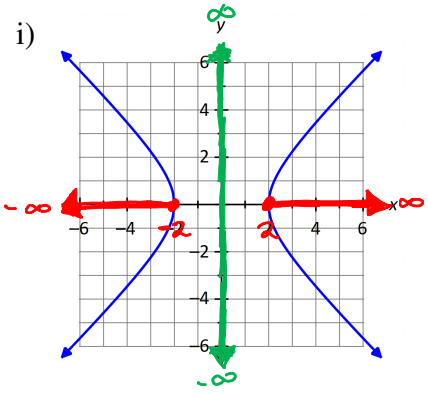


Function? (Yes) / No

Domain? $(-\infty, 4)$ ← Graph never gets to $x = 4$, so use) instead of] on the 2

Range? $(-\infty, \infty)$

i)



Function? Yes / (No)

Domain? $(-\infty, -2] \cup [2, \infty)$

Range? $(-\infty, \infty)$