Objective: Rational Exponents

If n is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number then $a^{1/n} = \sqrt[n]{a}$.

★ **The denominator of the exponent tells you what type of root to take.**

Examples: Write an equivalent expression using radical notation and, if possible, simplify.

a) $25^{1/2}$

b) $64^{1/3}$

c) $\left(xy^2 z\right)^{1/6}$

d) $\left(36x^{10}\right)^{1/2}$

e) $2x^{1/4}$

f) $\left(2x\right)^{1/4}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[7]{2xy}$

b) $\sqrt[4]{\dfrac{ab^3}{7}}$

c) $\sqrt{3z}$

d) $3\sqrt{z}$

e) $\sqrt[5]{xy^2 z}$

Positive Rational Exponents

If m and n are positive integers (where $n \neq 1$) and $\sqrt[n]{a}$ exists, then $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

e.g.) $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$　　or　　$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Examples: Write an equivalent expression using radical notation and simplify.

a) $t^{5/6}$

b) $9^{3/2}$

c) $64^{2/3}$

d) $\left(2x\right)^{3/4}$

e) $2x^{3/4}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[3]{x^5}$

b) $\sqrt[7]{9^2}$

c) $\left(\sqrt[5]{6n}\right)^3$

d) $6\sqrt[5]{n^3}$

e) $\left(\sqrt[4]{2m}\right)^2$

Negative Rational Exponents

For any rational number m/n, and any nonzero real number $a^{m/n}$, $a^{-m/n} = \dfrac{1}{a^{m/n}}$.

★ **The sign of the base is not affected by the sign of the exponent.**

Examples: Write an equivalent expression using positive exponents and, if possible, simplify.

a) $49^{-1/2}$

b) $(3mn)^{-2/5}$

c) $7x^{-2/3}$

Laws of Exponents: The laws of exponents apply to rational exponents as well as integer exponents.

Examples: Use the laws of exponents to simplify.

a) $2^{2/5} \cdot 2^{1/5}$

b) $\dfrac{x^{7/3}}{x^{4/3}}$

c) $\left(19^{2/5}\right)^{5/3}$

d) $x^{1/2} \cdot x^{2/3}$

e) $y^{-4/7} \cdot y^{6/7}$

f) $\dfrac{z^{3/4}}{z^{2/5}}$

g) $\dfrac{x^{3/4} \cdot x^{1/6} \cdot y}{y^{1/2}}$

h) $\dfrac{\left(2x^{2/5} y^{-1/3}\right)^5}{x^2 y}$

To Simplify Radical Expressions:
1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

Examples: Use rational exponents to simplify. Do not use exponents that are fractions in the final answer.

a) $\sqrt[8]{z^4}$

b) $\left(\sqrt[3]{a^2bc^4}\right)^9$

c) $\sqrt{x} \cdot \sqrt[4]{x}$

d) $\sqrt[6]{y^2} \cdot \sqrt[9]{y}$

e) $\dfrac{\sqrt[3]{k}}{\sqrt[7]{k^2}}$

f) $\dfrac{\sqrt[8]{m^4}}{\sqrt[6]{m}}$

g) $\sqrt[4]{\sqrt[5]{x}}$

h) $\sqrt[3]{2} \cdot \sqrt[5]{3}$