

**The Product Rule for Radicals:** For any real numbers  $\sqrt[n]{a}$  and  $\sqrt[n]{b}$ ,  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ .

**Caution:** The product rule doesn't work if you are trying to multiply the even roots of negative numbers, because those roots are not real numbers. For example,  $\sqrt{-2} \cdot \sqrt{-8} \neq \sqrt{16}$ .

**Caution:** The product only applies when the radicals have the same index:  $\sqrt[3]{5} \cdot \sqrt[4]{6} \neq \sqrt[12]{30}$ .

**Examples:** Multiply.

a)  $\sqrt{7} \cdot \sqrt{5}$

b)  $5\sqrt{2} \cdot \sqrt{8}$

c)  $2\sqrt{5} \cdot 7\sqrt{15}$

d)  $\sqrt{3} \cdot \sqrt{3}$

e)  $(\sqrt{8})^2$

f)  $(3\sqrt{11})^2$

g)  $\sqrt[3]{3} \cdot \sqrt[3]{9}$

h)  $2\sqrt[3]{10} \cdot 6\sqrt[3]{25}$

**Question:** Can you add and subtract radicals the same way you multiply and divide them?

e.g.) Since  $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ , does  $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ ? **NO!!!!!!!!!!!!**

**Don't make the following mistakes:**

- $\sqrt{2} + \sqrt{5} \neq \sqrt{7}$
- $\sqrt{9+16} \neq 3+4$
- $\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$
- $\sqrt{x^2 - 4} \neq x - 2$
- $(\sqrt{x} + \sqrt{y})^2 \neq x + y$

**Like Radicals:** Radicals with the same index *and* the same radicand.

**Examples:** Determine whether the following are like radicals. If they are not, explain why not.

a)  $\sqrt{3}$  and  $\sqrt{2}$

b)  $4\sqrt{5}$  and  $-3\sqrt{5}$

c)  $2\sqrt{x}$  and  $\sqrt[3]{x}$

**Steps for Adding and Subtracting Radicals:**

1. Simplify each radical completely.
2. Combine like radicals. When you add or subtract radicals, you can *only* combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.

**Examples:**

a)  $5\sqrt{3x} - 7\sqrt{3x}$

b)  $4\sqrt{11} + 8\sqrt{11}$

c)  $10\sqrt{6} + 3\sqrt{2} - 8\sqrt{6}$

d)  $\sqrt{20} - \sqrt{50} + \sqrt{45}$

e)  $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$

f)  $\sqrt[3]{54} - 5\sqrt[3]{16} + \sqrt[3]{2}$

**Multiplying Radical Expressions:** Use the Product Property. Use the Distributive Property and FOIL to multiply radical expressions with more than one term.

**Examples:** Multiply.

a)  $\sqrt{3}(5 + \sqrt{30})$

b)  $\sqrt{2}(\sqrt{6} - 3\sqrt{2})$

c)  $(\sqrt{5} - \sqrt{6})(\sqrt{7} + 1)$

d)  $(5\sqrt{3} - 4\sqrt{2})(\sqrt{3} + \sqrt{2})$

e)  $(4\sqrt{3} - 1)^2$

f)  $(\sqrt{2} + 5)(\sqrt{2} - 5)$