Objective: Exponent rules

The following properties are true for all real numbers a and b and all integers m and n, provided that no denominators are 0 and that 0^0 is not considered.

1 as an exponent:

$$a^1 = a$$

e.g.)
$$7^1 = 7$$
, $\pi^1 = \pi$, $(-10)^1 = -10$

0 as an exponent:

$$a^{0} = 1$$

e.g.)
$$2^0 = 1$$
, $27^0 = 1$, $\left(-\frac{5}{8}\right)^0 = 1$

The Product Rule:

$$a^m \cdot a^n = a^{m+n}$$

e.g.)
$$x^2 \cdot x^5 = x^{2+5} = x^7$$

The Quotient Rule:

$$\frac{a^m}{a^n} = a^{m-n}$$

e.g.)
$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

The Power Rule:

$$\left(a^{m}\right)^{n}=a^{mn}$$

e.g.)
$$(x^2)^5 = x^{(2)(5)} = x^{10}$$

Raising a product to a power:

$$(ab)^n = a^n b^n$$

e.g.)
$$(2k)^4 = 2^4 \cdot k^4 = 16k^4$$

Raising a quotient to a power:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

e.g.)
$$\left(\frac{p}{q^2}\right)^3 = \frac{p^3}{\left(q^2\right)^3} = \frac{p^3}{q^6}$$

Negative exponents:

$$a^{-n} = \frac{1}{a^n}$$

e.g.)
$$2^{-3} = \frac{1}{2^3}$$
, $7x^3y^{-4} = \frac{7x^3}{y^4}$

$$\frac{1}{a^{-n}} = a^n$$

e.g.)
$$\frac{1}{x^{-9}} = x^9$$
, $\frac{b}{c^{-3}d} = \frac{bc^3}{d}$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^{n}}{a^{n}}$$
 e.g.) $\left(\frac{2}{v}\right)^{-3} = \left(\frac{v}{2}\right)^{3} = \frac{v^{3}}{2^{3}} = \frac{v^{3}}{8}$

e.g.)
$$\left(\frac{2}{v}\right)^{-3} = \left(\frac{v}{2}\right)^3 = \frac{v^3}{2^3} = \frac{v^3}{8}$$

To simplify an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Examples: Simplify the following expressions.

a)
$$m^5 \cdot m^7$$

b)
$$(5a^2b^3)(3a^4b^5)$$
 c) $\frac{r^9}{r^3}$

c)
$$\frac{r^9}{r^3}$$

d)
$$\frac{p^3}{p^7}$$

e)
$$\frac{10x^{11}y^5}{2x^4y^7}$$

$$f) \frac{4x^3y^2}{6x^7y}$$

g)
$$(-2)^4$$

i)
$$5x^{-4}y^3 \cdot x^2y^{-1}$$

j)
$$\frac{1}{6^{-2}}$$

k)
$$9^{-3} \cdot 9^{8}$$

$$1) \ \frac{3x^2}{15x^{-3}y^{-4}}$$

m)
$$(3^5)^4$$

n)
$$\frac{y^{-5}}{y^{-4}}$$

o)
$$(y^{-5})^7$$

p)
$$(a^{-3})^{-7}$$

q)
$$(-2x)^3$$

r)
$$\left(\frac{x^2}{2}\right)^4$$

s)
$$(3x^5y^{-1})^{-2}$$

$$t) \left(\frac{y^2 z^3}{5}\right)^{-3}$$