Section: 7.7 Objective: Notes on Quadratic Inequalities

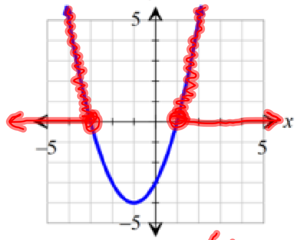
Review Example: Solve $f(x) = x^2 + 2x - 3$. $(x+3)(x-1)$

1 · 3

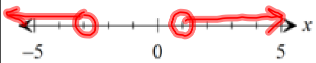
Notice that each of these inequalities below involves the value of $x^2 + 2x - 3$, which is represented by the y-coordinate of the graph. In each case, we are trying to figure out what x-values (x-coordinates) make the inequality true. When trying to find where $x^2 + 2x - 3 > 0$, we are trying to figure out what x-coordinates have a y-coordinate that is bigger than zero—in other words, *where is the graph above the x-axis?*

$(x+3)(x-1)=0$

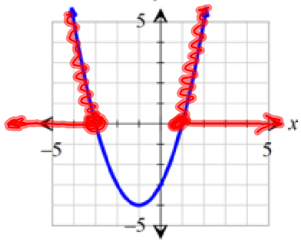
a) $x^2 + 2x - 3 > 0$ > above



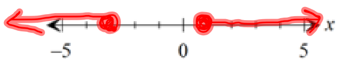
$(-\infty, -3) \cup (1, \infty)$



b) $x^2 + 2x - 3 \geq 0$ above zeros

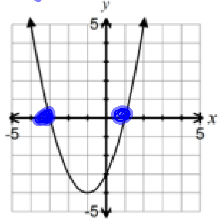


$(-\infty, -3] \cup [1, \infty)$



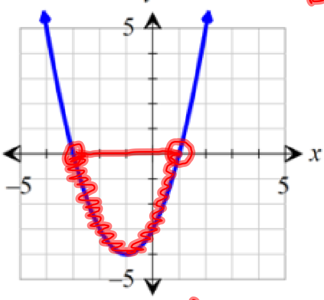
$f(x) = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

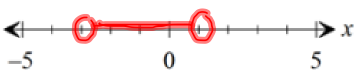


$x+3=0$, $x=-3$ $x-1=0$, $x=1$

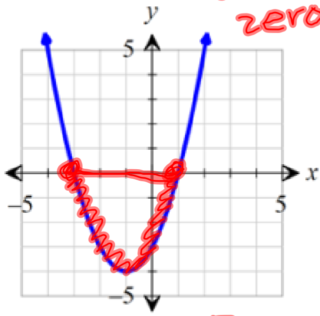
c) $x^2 + 2x - 3 < 0$ below



$(-3, 1)$



d) $x^2 + 2x - 3 \leq 0$ below zeros



$[-3, 1]$

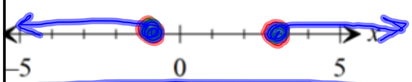
Solving a Quadratic Inequality Using the Graph:

1. Write the inequality in standard form. Replace the inequality sign with an equal sign and solve the equation $ax^2 + bx + c = 0$ by factoring, completing the square or using the quadratic formula. This gives you the x-intercepts of the graph of $y = ax^2 + bx + c$
2. Graph $y = ax^2 + bx + c$. The graph does not have to be very detailed. A rough sketch of a parabola opening in the correct direction with the correct x-intercepts is all you need.
3. The solutions of $ax^2 + bx + c > 0$ are the x-values for which the graph is **above** the x-axis.
   The solutions of $ax^2 + bx + c \geq 0$ are the x-values for which the graph is **on or above** the x-axis.
   The solutions of $ax^2 + bx + c < 0$ are the x-values for which the graph is **below** the x-axis.
   The solutions of $ax^2 + bx + c \leq 0$ are the x-values for which the graph is **on or below** the x-axis.
4. If the inequality involves $\leq$ or $\geq$, the x-intercepts **are included** in the solution set (use brackets).
   If the inequality involves $<$ or $>$, the x-intercepts **are not included** in the solution set (use parentheses).

Examples: Solve each inequality and graph the solution set on a number line. Write answer in interval notation

a) $(x-3)(x+1) \geq 0$ above zeros

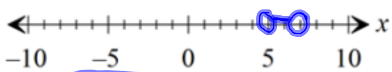
$x - 3 = 0$ $\quad$ $x + 1 = 0$
$+3 \; +3$ $\quad$ $-1 \; -1$
$x = 3$ $\quad$ $x = -1$



$\boxed{(-\infty,-1] \cup [3,\infty)}$

b) $(x-7)(x-5) < 0$ below

$x - 7 = 0$ $\quad$ $x - 5 = 0$
$+7 \; +7$ $\quad$ $+5 \; +5$
$x = 7$ $\quad$ $x = 5$

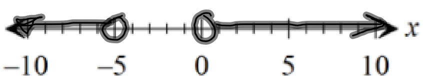


$\boxed{(5,7)}$

c) $x^2 + 5x > 0$ above

$x(x+5) = 0$

$\boxed{x=0}$ $\quad$ $x + 5 = 0$
$-5 \; -5$
$\boxed{x=-5}$



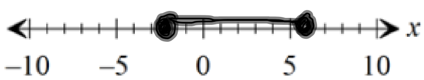
$\boxed{(-\infty,-5) \cup (0,\infty)}$

d) $x^2 - 4x - 12 \leq 0$ below zeros

-12	-4
1 -12	1 -12
2 6	2 -6
3 4	3 -4

$(x+2)(x-6) = 0$

$x + 2 = 0$ $\quad$ $x - 6 = 0$
$-2 \; -2$ $\quad$ $+6 \; +6$
$x = -2$ $\quad$ $x = 6$



$[-2, 6]$

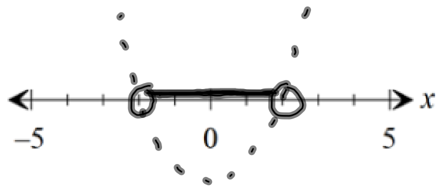
e) $x^2 - 4 < 0$ before

$(x+2)(x-2)=0$

$x+2=0$ $\quad$ $x-2=0$

$-2 \ -2$ $\quad$ $+2 \ +2$

$x=-2$ $\quad$ $x=2$



$(-2, 2)$

f) $x^2 + 10x \geq -9$ above zeros

9 | 10
1.9 |

$x^2+10x+9 \geq 0$

$(x+1)(x+9)$

$x+1=0$ $\quad$ $x+9=0$

$-1 \ -1$ $\quad$ $-9 \ -9$

$x=-1$ $\quad$ $x=-9$

$(-\infty, -9] \cup [-1, \infty)$

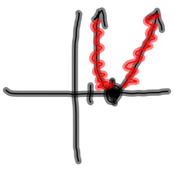
g) $x^2 - 4x + 4 > 0$ above

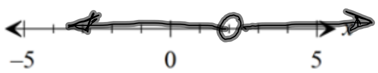
$(x-2)(x-2)=0$

$x-2=0$ $\quad$ $x-2=0$

$+2 \ +2$ $\quad$ $+2 \ +2$

$x=2$ $\quad$ $x=2$





$(-\infty, 2) \cup (2, \infty)$

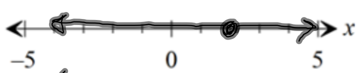
h) $x^2 - 4x + 4 \geq 0$ above zeros

$(x-2)(x-2)$

$x-2=0$ $\quad$ $x-2=0$

$+2$ $\quad$ $+2$

$x=2$ $\quad$ $x=2$



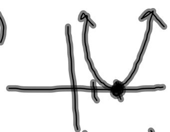
$(-\infty, \infty)$

$(-\infty, 2] \cup [2, \infty)$

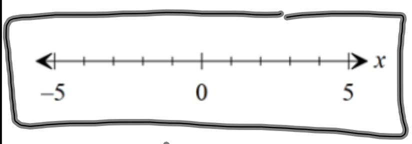
i) $x^2 - 4x + 4 < 0$ below

$(x-2)(x-2)$

$x=2$



nothing below



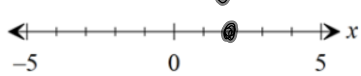
$\varnothing$

no solution

j) $x^2 - 4x + 4 \leq 0$ below zeros

$(x-2)(x-2)$

$x=2$



$[2]$