Section 7.5

Objective: more quadratic story notes

Steps for solving stories:

1. READ the story, write down the information needed and define a variable
2. Write an equation
3. Solve for variable
4. Check

Tips for solving story problems:

- Identify what you know.
- What are you trying to find out?
- Draw a picture or diagram to help you visualize the situation.
- Carefully define your variables.
- Translate the words into symbols.
- Use appropriate units.
- Make sure your answer makes sense.

Hints:

- ***Sum:*** + ***Difference:*** – ***Product:*** × ***Quotient:*** ÷

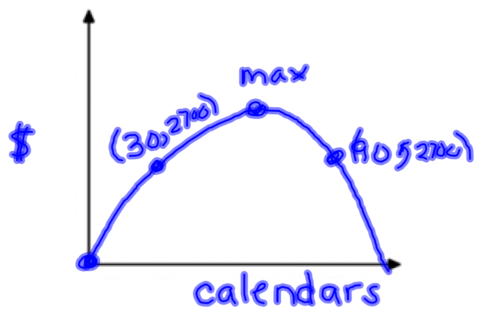
★ ***Words that tell you to look for the vertex:*** maximum, minimum, highest, lowest, biggest, littlest, largest, smallest, maximize, minimize.

EXAMPLES:

1. A ski club sells calendars to raise money. The profit, P, in dollars, from selling x calendars is given by the equation $P(x) = 120x - x^2$.

   Define your variables: $x =$ Calendars, $P(x) = y =$ Profit \$

   Sketch a graph of the situation. Label the axes clearly.



$P(x) = -x^2 + 120x$

$a = -1 \quad b = 120 \quad c = 0$ yint

How much profit will the club make from selling 50 calendars?

$P(50) = -(50)^2 + 120(50)$

\$3500

How many calendars must be sold for the club to make \$2700?

$2700 = -x^2 + 120x$

$+x^2 - 120x \quad +x^2 - 120x$

$x^2 - 120x + 2700 = 0$

$x = \dfrac{+120 \pm \sqrt{(-120)^2 - 4(1)(2700)}}{2(1)}$

$\dfrac{120 \pm \sqrt{3600}}{2(1)}$

$\dfrac{(120 \pm 60)}{2} \to \dfrac{(120+60)}{2} = 90$ calendars

$\dfrac{(120 \pm 60)}{2} \to \dfrac{(120-60)}{2} \quad \dfrac{60}{2} = 30$

How many calendars must be sold to maximize profit?

$\dfrac{-b}{2a} = \dfrac{-120}{2(-1)} = \dfrac{-120}{-2} = 60$ calendars

vertex

What is the maximum profit?

Find y-value plug in 60 for x

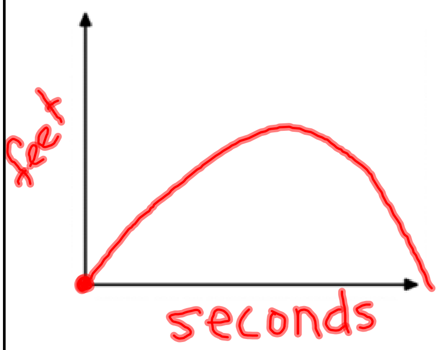
$y = -(60)^2 + 120(60)$

$\boxed{\$3600}$

2. A rock is thrown upward from the ground by the wheel of a truck. Its height in feet above the ground after t seconds is given by the function $h(t) = -16t^2 + 20t$.

Define your variables: $x = t =$ seconds, $h(t) = y =$ height in feet

Draw a sketch of the graph representing the path of the height of the rock. Label your axes.



$-16t^2 + 20t$

$a = -16 \quad b = 20 \quad c = 0$

How long does it take the rock to reach its maximum height?

max vertex

$X = \frac{-b}{2a} \quad \frac{-20}{2(-16)} = \frac{-20}{-32}$

.625 sec

What is the maximum height of the rock?

$-16(.625)^2 + 20(.625)$

$\boxed{6.25 \text{ ft}}$

How long will it take for the rock to return to the ground?

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad 0 = -16t^2 + 20t$

$\frac{-20 \pm \sqrt{(20)^2 - 4(-16)(0)}}{2(-16)}$

$\frac{-20 \pm \sqrt{400}}{-32}$

$\frac{-20 \pm 20}{-32}$

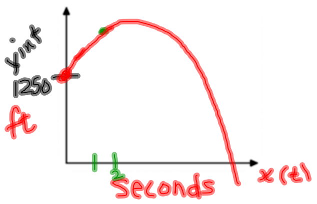
$\frac{(-20+20)}{-32} = \frac{0}{-32} = 0$

$\frac{(-20-20)}{-32} = \boxed{1.25 \text{ sec}}$

3. A penny is thrown upward from the observation deck on the 102nd floor of the Empire State Building. It's height, h, in feet, after t seconds is given by the equation $h(t) = -16t^2 + 92t + 1250$

Define your variables: $x = t =$ seconds, $h(t) = y =$ height in feet

Draw a sketch of the graph representing the path of the height of the penny. Label your axes.



$h(t) = -16t^2 + 92t + 1250$

$a = -16$ $b = 92$ $c = 1250$

yint

What is the height of the observation deck? (In other words, how high is the penny at $t = 0$?)

y value

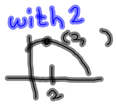
1250 ft

How high is the penny after 2 seconds?

replace t with 2

$-16(2)^2 + 92(2) + 1250$

$\boxed{1370 \text{ ft}}$



The Empire State Building has a lightning rod with a tip that is 1454 ft above the ground. Will the penny reach the top of the lightning rod? (Hint: Find the maximum height and see if it's larger or smaller than 1454 ft.)

vertex

$\frac{-b}{2a}$ $\frac{-92}{2(-16)}$ $\frac{-92}{-32} = 2.875$

$\boxed{\text{NO}}$

$1382.25 < 1454$

$-16(2.875)^2 + 92(2.875) + 1250$

1382.25 ft

When will the penny be 1110 feet above the ground?

$1110 = -16x^2 + 92x + 1250$

$y_1 = -16x^2 + 92x + 1250$

$y_2 = 1110$

Change window

Find intersections 2nd trace #5 $\boxed{x = 7 \text{ sec}}$

By hand

$1110 = -16x^2 + 92x + 1250$
$-1110 \qquad\qquad -1110$

$0 = -16x^2 + 92x + 140$

$x = \frac{-92 \pm \sqrt{(92)^2 - 4(-16)(140)}}{2(-16)}$

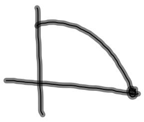
$\frac{-92 \pm \sqrt{17424}}{-32}$

$\frac{-92 \pm 132}{-32}$

$\frac{(-92 + 132)}{-32} = -1.25$

$\frac{(-92 - 132)}{-32} = \boxed{7}$

How long will it take for the penny to hit the ground?



$0 = -16x^2 + 92x + 1250$

$\frac{-92 \pm \sqrt{92^2 - 4(-16)(1250)}}{2(-16)}$

$\frac{-92 \pm \sqrt{88464}}{-32}$

$\frac{-92 \pm 297.48}{-32}$

$\frac{(-92 + 297.48)}{-32}$ $\frac{(-92 - 297.48)}{-32}$

$\cancel{-6.42}$ no neg time

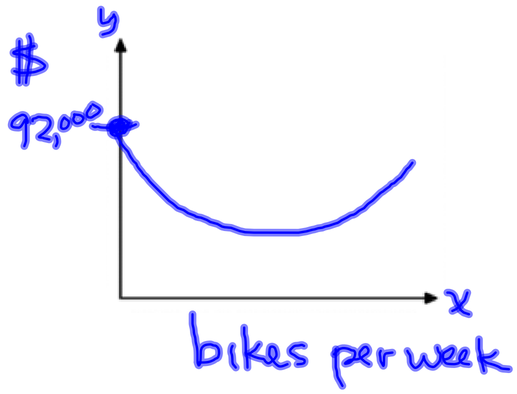
$\approx \boxed{12.2 \text{ sec}}$

4. The cost C in dollars, of manufacturing x bikes per week at a production plant is given by the function $C(x) = 2x^2 - 800x + 92{,}000$.

cost in dollars

Define your variables: $x =$ bikes per week, $C(x) = y =$ dollars

Sketch a rough graph of the cost equation. Be sure to label your axes. Use the y-intercept and the direction of opening to help draw the graph.



$C(x) = 2x^2 - 800x + 92{,}000$

$a = 2 \quad b = -800 \quad c = 92{,}000$

y int

How much does it cost to manufacture 50 bikes per week? Show your work.

replace x with 50

$C(50) = 2(50)^2 - 800(50) + 92{,}000$

$\boxed{\$57{,}000}$

Find the number of bikes that must be manufactured each week to minimize the cost. Show your work.

x vertex

$\frac{-b}{2a} \quad \frac{800}{2(2)} \quad \frac{800}{4} = \boxed{200 \text{ bikes}}$ per week

Find the minimum cost. Show your work.

y value

$C(200) = 2(200)^2 - 800(200) + 92{,}000$

$\$12{,}000$