Section: 7.1

Objective: Recognize different forms of quadratic functions, find the vertex, axis of symmetry, and y-intercept of quadratic graphs.

What does quadratic mean?

Forms of Quadratic Functions: an expression of degree 2.

Standard Form: $f(x)=ax^2+bx+c$, where $a\neq 0$. There are no parentheses.

$f(x)=ax^2+bx+c$ $y=ax^2+bx+c$

Example: $f(x)=-3x^2+2x-7$

$a=-3$ $b=2$ $c=-7$

Factored Form: $f(x)=a(x-p)(x-q)$, where $a\neq 0$. Written as a multiplication problem.

Example: $f(x)=(x-4)(x+5)$ $y=(x-4)(x+5)$

$a=1$ $p=4$ $q=-5$

Vertex Form: $f(x)=a(x-h)^2+k$, where $a\neq 0$. x only shows up once, as part of a perfect square.

Example: $f(x)=2(x+7)^2-1$

$y=2(x+7)^2-1$

$a=2$ $h=7$ $k=-1$

Examples: State whether each quadratic function is in standard, factored, or vertex form. Identify the values of a, b, and c for standard form; a, p, and q for factored form; or a, h, and k for vertex form.

a) $f(x)=2(x+3)(x-5)$
$a=2$ $p=-3$ $q=5$
factored $a(x-p)(x-q)$

b) $f(x)=-(x+4)^2-5$
$a=-1$ $h=-4$ $k=-5$
vertex

c) $f(x)=x^2+2x+4$
$a=1$ $b=2$ $c=4$
standard

d) $f(x)=-x^2+5x+0$
$a=-1$ $b=5$ $c=0$
standard

e) $f(x)=3x(x-2)$ $3(x-0)(x-2)$
$a=3$ $p=0$ $q=2$
factored

f) $f(x)=2(x+1)^2-3$
$a=2$ $h=-1$ $k=-3$
vertex

g) $f(x)=-(x+5)^2+0$
$a=-1$ $h=-5$ $k=0$
vertex

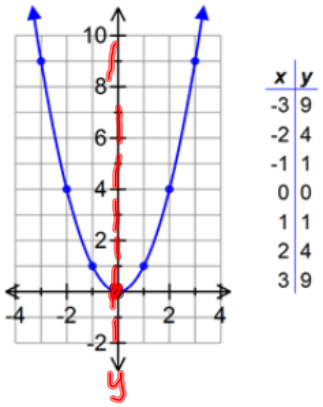
h) $f(x)=-3x^2+4$ $-3x^2+0x+4$
$a=-3$ $b=0$ $c=4$

i) $f(x)=5x^2$ $f(x)=5x^2$
standard
$y=5x^2+0x+0$
$a=5$ $b=0$ $c=0$

vertex
$y=5(x-0)^2+0$
$a=5$ $h=0$ $k=0$

factored
$y=5x^2$
$y=5(x-0)(x-0)$
$a=5$ $p=0$ $q=0$

The graph of $y = x^2$:



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Vertex: (0,0)

Axis of Symmetry: $x = 0$ (x coord of vertex)

Direction of Opening: up

***y*-intercept:** (0, 0)

A) Parabola: The shape of the graph of a quadratic function.

B) *Axis of Symmetry:* A line that cuts a parabola in half. If you were to fold a parabola along its axis of symmetry, the two sides would overlap. The equation of the axis of symmetry looks like $x = \#$.

C *Vertex:* The "tip" of the parabola – the point at which it changes direction.

- If the **parabola opens up** ($a > 0$), the vertex is the lowest point on the graph, or the ***minimum point.***
- If the **parabola opens down** ($a < 0$), the vertex is the highest point on the graph, or the ***maximum point.***

D) *y-intercept:* The point where the graph crosses or touches the y-axis written as an ordered pair (0,y)

Finding the vertex in each form.

1) Vertex Form of a Quadratic Function: $y = a(x-h)^2 + k$

a is the number in front of the parentheses.
(If there isn't a number written in front, $a = 1$. If there's just a negative sign in front, $a = -1$.)

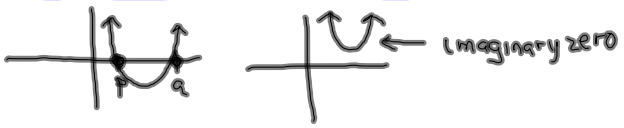
h is the *opposite* of the number with x in the parentheses.
k is the number at the end.

Vertex: (h,k)

2) Standard Form: $f(x) = ax^2 + bx + c$ *or* $y = ax^2 + bx + c$

- Just like with the other forms, the graph opens up if a is positive and opens down if a is negative.
- Vertex:
  - The x-coordinate of the vertex is $\frac{-b}{2a}$. (The opposite of b divided by 2 times a)
  - To find the y-coordinate, plug the x-coordinate into the original equation.

3) Factored Form: $f(x) = a(x-p)(x-q)$ *where* $a \neq 0$
Factored form shows the zeros or x-intercepts of the quadratic.
Zeros of a Function: The values of x that make $f(x)$ or y equal zero. If the zeros are real, they tell you the places where the graph crosses the x-axis, or the ***x-intercepts*** of the graph.
Other words for zeros are *solutions to* $f(x) = 0$, *roots*, *x-intercepts*.



Finding the axis of symmetry, direction of opening, and *y*-intercept is the same in all forms.

Axis of Symmetry: Find the x-coordinate of the vertex and write the equation of the vertical line.

In standard form it is $x = \frac{-b}{2a}$. In vertex form it is $x = h$. In factored form its $x = \frac{(p+q)}{2}$

$X = \frac{(p+q)}{2}$

Direction of Opening:

- Opens up if a is **positive**.
- Opens down if a is **negative**.

Finding the *y*-intercept:

1. Plug in 0 for x. Let x=0
2. Simplify. **Don't forget to use order of operations.**

Write the form each quadratic equation is in. Find the vertex and the direction of the opening of the graph for each of the following quadratic equations. Find the *y*-intercept and axis of symmetry.

$a(x-h)^2+k$

a) $y = (x-7)^2 + 9$

$h =$ 7 , $k =$ 9 $a = 1$

vertex (h,k)

Let x=0

$y = (0-7)^2+9$

58

Form: vertex

Vertex: (7,9)

Axis of Symmetry: x=7

Direction of opening: ↑ up

y-intercept: (0,58)

b) $y = 3x^2 - 12x - 10$

$a =$ 3 , $b =$ -12 $c = -10$

positive ↑

$\frac{-b}{2a} = \frac{12}{2 \cdot 3} = \frac{12}{6} = 2$

$y = 3(2)^2 - 12(2) - 10$

y coord of vertex

Form: Standard

Vertex: (2,-22)

Axis of Symmetry: x=2

Direction of opening: up

y-intercept: (0,-10)

(0, c)

c) $y = -(x+4)(x-6)$

$p =$ −4 , $q =$ 6 a: −1

$\frac{(p+q)}{2}$ $\frac{-4+6}{2}$ $\frac{2}{2}=1$

Form: factored

Vertex: (1, 25)

Axis of Symmetry: X=1

Direction of opening: down

y-intercept: a is neg (0,24)

$y = -(1+4)(1-6)$

$= -(5)(-5)$

$y = 25$

yint $-(0+4)(0-6)$

24

d) $y = -x^2 + 4x - 10$

$a =$ −1 , $b =$ 4

$\frac{-b}{2a}$ $\frac{-4}{2(-1)}$ $\frac{-4}{-2}=2$

use calculator

Table x=2 y=−6

Form: standard

Vertex: (2,−6)

Axis of Symmetry: x=2

Direction of opening: down

y-intercept: a is neg (0,−10)

e) $y = -3(x+2)^2 - 1$

$h =$ −2 , $k =$ −1

$-3(0+2)^2-1$ calculator

$-3(4)-1$

-13

a: −3

Form: vertex

Vertex: (−2,−1)

Axis of Symmetry: x=−2

Direction of opening: down

y-intercept: (0,−13)

f) $y = \frac{1}{2}(x-3)(x-7)$

$p =$ 3 , $q =$ 7

$x = \frac{(3+7)}{2} = \frac{10}{2} = 5$

$\frac{1}{2}(5-3)(5-7)$

a: $\frac{1}{2}$

Form: factored

Vertex: (5, −2)

Axis of Symmetry: x=5

Direction of opening: up

y-intercept: (0, 10.5)

$\frac{1}{2}(0-3)(0-7) = 10.5$

g) $y = -5x^2 - 10x + 12$

$a =$ -5 , $b =$ -10

$\frac{-b}{2a} \quad \frac{10}{2(-5)} = \frac{10}{-10} = -1$

$y = -5(-1)^2 - 10(-1) + 12$

Form: Standard

Vertex: (-1, 17)

Axis of Symmetry: x = -1

Direction of opening: down

y-intercept: (0, 12)

(0, c)

$a = -5$

h) $y = \frac{2}{3}x^2 - 4$

$a =$ $\frac{2}{3}$, $b =$ 0 Standard

$h =$ 0 , $k =$ -4 ⇐ vertex

$a = \frac{2}{3}$

Form: standard or vertex

Vertex: (0, -4)

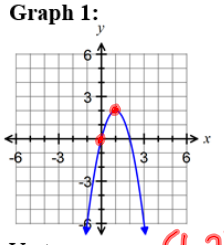
Axis of Symmetry: x = 0

Direction of opening: up

y-intercept: (0, -4)

For each of the following graphs, find the vertex, axis of symmetry, and y-intercept.

Graph 1:



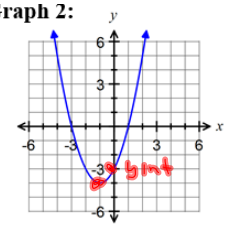
Vertex: (1, 2)

Axis of Symmetry: x = 1

y-intercept: (0, 0)

Is the value of "a" positive or negative? down

Graph 2:



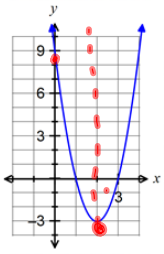
Vertex: (-1, -4)

Axis of Symmetry: x = -1

y-intercept: (0, -3)

Is the value of "a" positive or negative? positive

Graph 3:



Vertex: (2, -3)

Axis of Symmetry: x = 2

y-intercept: (0, 9)

Is the value of "a" positive or negative? positive

up