Section 6.3

Objective: solving quadratic equations using the Quadratic Formula

We've learned how to solve quadratic equations by factoring, but what do we do if we have an equation with something that can't be factored, like $x^2 + 5x + 2 = 0$?

The Quadratic Formula:

A quadratic equation written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, has the solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving a Quadratic Equation Using the Quadratic Formula:

1. Write the equation in standard form: $ax^2 + bx + c = 0$.
2. Identify a, b, and c. Plug them into the equation. Be careful with parentheses.
3. Simplify. Be careful to follow order of operations and deal with negatives correctly.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples: Solve each equation using the quadratic formula.

a) $1x^2 + 4x + 7 = 0$ (a = 1, b = 4, c = 7)

number in front of x^2

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(7)}}{2(1)}$$

$(4)^2 - 4(1)(7)$

$$X = \frac{-4 \pm \sqrt{-12}}{2}$$

$$\frac{-4 \pm i\sqrt{12}}{2}$$ simplify $\sqrt{12}$

$$\frac{-4 \pm i2\sqrt{3}}{2}$$

$$\frac{2(-2 \pm i\sqrt{3})}{2}$$

$$-2 \pm i\sqrt{3}$$

b) $3m^2+16m+5=0$

a = 3, b = 16, c = 5

$$x=\frac{-16\pm\sqrt{(16)^2-4(3)(5)}}{2(3)}$$

$$\frac{-16\pm\sqrt{196}}{6}$$

$$\frac{-16\pm14}{6}$$

$\frac{(-16+14)}{6}=\boxed{\frac{-1}{3}}$

$\frac{(-16-14)}{6}=\boxed{-5}$

c) $2w^2-4w=3$

$2w^2-4w-3=0$

a = 2, b = -4, c = -3

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-3)}}{2(2)}$$

$$\frac{4\pm\sqrt{40}}{4}$$

$$\frac{4\pm2\sqrt{10}}{4}$$

$$\frac{2(2\pm\sqrt{10})}{4\ 2}=\boxed{\frac{2\pm\sqrt{10}}{2}}$$

d) $-n^2+4n-4=0$

a = -1, b = 4, c = -4

$$x=\frac{-4\pm\sqrt{(4)^2-4(-1)(-4)}}{2(-1)}$$

$$\frac{-4\pm\sqrt{0}}{-2}\qquad\frac{-4\pm0}{-2}=\frac{-4}{-2}=\boxed{2}$$

e) $r^2+9=0$ no "r" so "b" is 0

a = 1, b = 0, c = 9

number in front of r^2 | number in front of r

$$x=\frac{-0\pm\sqrt{(0)^2-4(1)(9)}}{2(1)}$$

$$\frac{0\pm\sqrt{-36}}{2}$$

$$\frac{\pm i\sqrt{36}}{2}\Rightarrow\frac{\pm6i}{2}\Rightarrow\boxed{\pm3i}$$

f) $6u^2 - 2u = 0$

a = 6, b = -2, c = 0

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(0)}}{2(6)}$$

$$\frac{2 \pm \sqrt{4}}{12}$$

$$\frac{2 \pm 2}{12} \rightarrow \frac{(2+2)}{12} = \frac{4}{12} = \frac{1}{3}, \quad \frac{(2-2)}{12} = \frac{0}{12} = 0$$

g) $z = -3z^2 - 3$ $\quad ax^2+bx+c=0$

$3z^2+1z+3 = 0$

a = 3, b = 1, c = 3

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(3)}}{2(3)}$$

$$\frac{-1 \pm \sqrt{-35}}{6}$$

$$\frac{-1 \pm i\sqrt{35}}{6}$$

h) $\frac{1}{4}y^2 - y + \frac{1}{2} = 0$

a = $\frac{1}{4}$, b = -1, c = $\frac{1}{2}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)}}{2\left(\frac{1}{4}\right)}$$

$$\frac{1 \pm \sqrt{\frac{1}{2}}}{\frac{1}{2}}$$