Section 6.1   **Objective: Solving quadratics**

Quadratic Equation: Any equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Zero Product Property: If the product of several factors is equal to zero, then at least one of the factors is equal to zero.

- The only way to end up with zero when you multiply is if one of the numbers being multiplied is zero.

- If a and b are real numbers and $a \cdot b = 0$, then $a = 0$ or $b = 0$ or both. $(\underline{\quad})(\underline{\quad}) = 0$

   ★ **This is only true if <u>one side of the equation is zero.</u>**

If $a \cdot b = 1$, it ***does not mean*** that $a = 1$ or $b = 1$.

$(2)\left(\frac{1}{2}\right) = 1, \quad \left(\frac{3}{4}\right)\left(\frac{4}{3}\right) = 1$, etc.

DON'T split up $(x+5)(x-3) = 1$ into $x + 5 = 1$ and $x - 3 = 1$.
<u>That's</u> **wrong!**

Solving Quadratic Equations by Factoring:
1. Get a zero on one side of the equation.
2. Factor completely.
3. Set each factor *containing a variable* equal to 0.
4. Solve the resulting equations.

Examples: Solve each equation by factoring.

a) $(x-3)(x+5)=0$

$$x-3=0 \qquad x+5=0$$
$$+3 \; +3 \qquad\quad -5 \; -5$$
$$\boxed{x=3} \qquad \boxed{x=-5}$$

b) $(3x)(x+4)=0$

$$\frac{3x}{3}=\frac{0}{3} \qquad x+4=0$$
$$\qquad\qquad -4 \; -4$$
$$\boxed{x=0} \qquad \boxed{x=-4}$$

c) $2(x+5)(3x-4)=0$

$\cancel{2\neq 0}$ $x+5=0$ $3x\cancel{-4}=0$
         $\underline{-5\ -5}$ $\cancel{+4}\ +4$

$\boxed{x=-5}$ $\dfrac{\cancel{3}x}{\cancel{3}}=\dfrac{4}{3}$

$\boxed{x=\dfrac{4}{3}}$

d) $(x+7)^2=0$

$(x+7)(x+7)=0$

$x\cancel{+7}=0$
     -7

$\boxed{x=-7}$

e) $3x^2=0$

$3\cdot x\cdot x=0$

$\cancel{3\neq 0}$ $\boxed{x=0}$ $x=0$

f) $x^2-8x=0$

factor x^2-8x
GCF $x\cancel{x}$ $8\cancel{x}$

$(x)(x-8)=0$

$\boxed{x=0}$ $x-8=0$
              $+8\ +8$

$\boxed{x=8}$

g) $x^2 + 7x + 6 = 0$

Add
$$\begin{array}{c|c} 6 & 7 \\ \hline 1\cdot 6 & 1+6 \end{array}$$

$$\begin{array}{c|c|c} & x & 1 \\ \hline x & x^2 & 1x \\ \hline 6 & 6x & 6 \end{array}$$

$(x+1)(x+6) = 0$

$x + 1 = 0$ $x + 6 = 0$
$\quad -1\ -1$ $\quad -6\ -6$

$\boxed{x = -1}$ $\boxed{x = -6}$

h) $x^2 - 4x = 12$

$\qquad -12$

$$\begin{array}{c|c} -12 & -4 \\ \hline 2 \cdot -6 & \end{array}$$

$x^2 - 4x - 12 = 0$

$$\begin{array}{c|c|c} & x & 2 \\ \hline x & x^2 & 2x \\ \hline -6 & -6x & -12 \end{array}$$

$(x+2)(x-6) = 0$

$x + 2 = 0$ $x - 6 = 0$
$\ -2\ -2$ $\quad +6\ +b$

$\boxed{x = -2}$ $\boxed{x = 6}$

i) $\qquad 4x^2 = 9$

$\sqrt{4}\ \ 4x^2 - 9 = 0$

Difference of 2 perfect squares
$(2x + 3)(2x - 3) = 0$

$2x + 3 = 0$ $2x - 3 = 0$
$\ -3\ -3$ $\quad +3\ +3$

$\dfrac{2x}{2} = -\dfrac{3}{2}$ $\dfrac{2x}{2} = \dfrac{3}{2}$

$\boxed{x = -\dfrac{3}{2}}$ $\boxed{x = \dfrac{3}{2}}$

j) $-x^2 - 10x = 25$

$0 = x^2 + 10x + 25$

$$\begin{array}{c|c} 25 & 10 \\ \hline 5\cdot 5 & \end{array}$$

$$\begin{array}{c|c|c} & x & 5 \\ \hline x & x^2 & 5x \\ \hline 5 & 5x & 25 \end{array}$$

$(x+5)(x+5) = 0$

$x + 5 = 0$ $x + 5 = 0$
$\ -5\ -5$ $\ -5\ -5$

$x = -5$ $\boxed{x = -5}$

4

k) GCF $3x^2 + 15x + 18 = 0$

$3(x^2 + 5x + 6) = 0$

factor

$\begin{array}{c|c|c} & x & 2 \\ \hline x & x^2 & 2x \\ \hline 3 & 3x & 6 \end{array}$

$\begin{array}{c|c} 6 & 5 \\ \hline 1 \cdot 6 & 1+6 \\ 2 \cdot 3 & 2+3 \end{array}$

$(x+2)(x+3) = 0$

$\begin{array}{ccc} x+2=0 & & x+3=0 \\ -2-2 & & -3-3 \end{array}$

$\boxed{x = -2}$ $\boxed{x = -3}$

l) $2x^2 = x$

$2x^2 - x = 0$

GCF

$x(2x - 1) = 0$

$\boxed{x = 0}$ $2x - 1 = 0$
                        $+1 \quad +1$

$\dfrac{2x}{2} = \dfrac{1}{2}$

$\boxed{x = \dfrac{1}{2}}$

m)   box 1   box 4
      $4x^2 + 5x - 6 = 0$

$ac = -24$ $b = 5$

$\begin{array}{c|c|c} & 4x & -3 \\ \hline 1x & 4x^2 & -3x \\ \hline 2 & 8x & -6 \end{array}$

$\begin{array}{c|c} -24 & 5 \quad \text{add} \\ \hline -1 \cdot 24 & 23 \\ -2 \cdot 12 & 10 \\ \hline -3 \cdot 8 & 5 \end{array}$

$(4x - 3)(x + 2) = 0$

$\begin{array}{cc} 4x - 3 = 0 & 1x + 2 = 0 \\ +3 \; +3 & -2 \; -2 \end{array}$

$\dfrac{4x}{4} = \dfrac{3}{4}$ $\boxed{x = -2}$

$\boxed{x = \dfrac{3}{4}}$

n) $2x^2 - 21x = 11$

$2x^2 - 21x - 11 = 0$

no GCF

$ac = 2 \cdot -11 = -22$
$b = -21$

$\begin{array}{c|c} -22 & -21 \quad \text{add} \\ \hline 1 \cdot 22 & -22+1 \end{array}$

$\begin{array}{c|c|c} & x & -11 \\ \hline 2x & 2x^2 & -22x \\ \hline 1 & 1x & -11 \end{array}$

$(x - 11)(2x + 1) = 0$

$\begin{array}{cc} x - 11 = 0 & 2x + 1 = 0 \\ +11 \; +11 & -1 \; -1 \end{array}$

$\boxed{x = 11}$ $\dfrac{2x}{2} = \dfrac{-1}{2}$

$\boxed{x = \dfrac{-1}{2}}$

5

o)　　　$3x^2 - 15 = 4x$

$3x^2 - 15 - 4x = 0$

$3x^2 - 4x - 15 = 0$

$ac = 3 \cdot -15 = -45$

$b = -4$

MULT	ADD
-45	-4
1 · 45	1 + -45
3 · 15	3 + -15
5 · -9	5 + -9

$$\boxed{3x \quad 5}$$

	3x	5
X	$3x^2$	$5x$
-3	$-9x$	-15

$(x - 3)(3x + 5) = 0$

$x - 3 = 0$　　　$3x + 5 = 0$
　$+3 \quad +3$　　　　　$-5 \quad -5$

$\boxed{X = 3}$　　　$3x = -5$

　　　　　　　$\boxed{X = \frac{-5}{3}}$

p)　　$11x = -5x^2 - 2$

$5x^2 + 11x + 2 = 0$

$ac = 5 \cdot 2 = 10$

$b = 11$

MULT	ADD
10	11
10 · 1	10 + 1

	x	2
5x	$5x^2$	$10x$
1	$1x$	2

$(x + 2)(5x + 1) = 0$

$x + 2 = 0$　　　$5x + 1 = 0$
　$-2 \quad -2$　　　　$-1 \quad -1$

$\boxed{X = -2}$　　$\dfrac{5X}{5} = \dfrac{-1}{5}$

　　　　　　　$\boxed{X = \frac{-1}{5}}$