

## Section 4.5 notes

**Objective: Combining Functions with Arithmetic**

It can be useful to combine two functions to make a new function. For instance, you may have a function describing the revenue from a product and a function describing the costs of producing the product. By subtracting the two functions, you can create a function describing the profit made from the product.

**Tips:**

- Use parentheses carefully. Write each function in parentheses.
- Be careful with negatives. If there is a negative outside parentheses, it changes the sign of *everything* inside the parentheses.
- When adding or subtracting, combine like terms.
- When multiplying, distribute or FOIL and make sure to use exponent rules correctly.

**Examples:** Let  $f(x) = 3x - 5$  and  $g(x) = x^2 + 5x - 2$ .

Perform the indicated operations.

a)  $h(x) = f(x) + g(x)$

$$\begin{array}{r} \underline{3x-5} + \underline{x^2+5x-2} \\ \hline x^2 + 8x - 7 \end{array}$$

b)  $h(x) = f(x) - g(x)$

$$\begin{array}{r} 3x-5 - (x^2+5x-2) \\ \underline{3x-5} - x^2 - \underline{5x} + \underline{2} \\ -2x - 3 - x^2 \\ \hline -x^2 - 2x - 3 \end{array}$$

c)  $h(x) = g(x) - f(x)$

$$\begin{array}{r} x^2 + 5x - 2 - (3x - 5) \\ \underline{x^2 + 5x - 2} - \underline{3x} + \underline{5} \\ \hline x^2 + 2x + 3 \end{array}$$

d)  $h(x) = 2f(x) + 3g(x)$

$$\begin{array}{r} 2(3x-5) + 3(x^2+5x-2) \\ \underline{6x-10} + \underline{3x^2+15x-6} \\ 21x - 16 + 3x^2 \\ \hline 3x^2 + 21x - 16 \end{array}$$

**Examples:** Let  $f(x) = 3x - 5$  and  $g(x) = x^2 + 5x - 2$ .

e)  $h(x) = -f(x) + 4g(x)$

$$-(3x-5) + 4(x^2+5x-2)$$

$$\underline{-3x+5} + 4x^2 + \underline{20x-8}$$

$$\boxed{17x - 3 + 4x^2}$$

f)  $h(x) = f(x) - 5f(x)$

$$3x-5 - 5(3x-5)$$

$$\underline{3x-5} - \underline{15x+25}$$

$$\boxed{-12x+20}$$

g)  $h(x) = f(x) \cdot g(x)$

$$(3x-5)(x^2+5x-2)$$

	$3x$	$-5$
$x^2$	$3x^3$	$-5x^2$
$+5x$	$15x^2$	$-25x$
$-2$	$-6x$	$10$

$$\boxed{3x^3 + 10x^2 - 31x + 10}$$

h)  $h(x) = f(x) \cdot f(x)$

	$3x$	$-5$
$3x$	$9x^2$	$-15x$
$-5$	$-15x$	$25$

$$\boxed{9x^2 - 30x + 25}$$

**Evaluating Combined Functions**

To evaluate a combined function for certain values of  $x$ , replace  $x$  with the specified number in each function, then add, subtract, multiply or divide. Make sure to follow order of operations!

**Examples:** Let  $f(x) = 2x - 7$ , and let  $g(x) = -x^2 + 3$ .

Evaluate the following.

a)  $f(2) + g(1)$

$$\begin{array}{l} f(x) \\ 2x-7 \\ 2(2)-7 \\ -3 \end{array} \quad \begin{array}{l} g(x) \\ -x^2+3 \\ -(1)^2+3 \\ -1+3 \\ 2 \end{array}$$

$$\underline{-3} + \underline{2}$$

$$\boxed{-1}$$

b)  $f(0) - g(-3)$

$$\begin{array}{l} f(x) \\ 2x-7 \\ 2(0)-7 \\ -7 \end{array} \quad \begin{array}{l} g(x) \\ -x^2+3 \\ -(-3)^2+3 \\ -9+3 \\ -6 \end{array}$$

$$\underline{-7} - \underline{-6}$$

$$\boxed{-1}$$

c)  $f(-2) \cdot 3g(2)$

$$\begin{array}{l} f(x) \\ 2x-7 \\ 2(-2)-7 \\ -11 \end{array} \quad \begin{array}{l} g(x) \\ -x^2+3 \\ -(2)^2+3 \\ -4+3 \\ -1 \end{array}$$

$$-11 \quad 3(-1)$$

$$-11 \cdot -3$$

$$\boxed{33}$$

**Examples:** Let  $f(x) = 3x - 5$  and  $g(x) = (x + 3)(x - 1)$   
 Perform the indicated operations and state the domain of the new function.

a)  $r(x) = \frac{g(x)}{f(x)}$

$$\frac{(x+3)(x-1)}{(3x-5)}$$

$3x - 5 \neq 0$   
 $+5 +5$   
 $8x \neq \frac{5}{3}$

Domain:  $X \neq \frac{5}{3}$

b)  $r(x) = \frac{f(x)}{g(x)}$

$$\frac{(3x-5)}{(x+3)(x-1)}$$

Domain:

$x+3 \neq 0$     $x-1 \neq 0$   
 $X \neq -3$     $X \neq 1$

c)  $r(x) = \frac{2f(x)}{f(x)}$

$$\frac{2(3x-5)}{(3x-5)} = 2$$

Domain:

$X \neq \frac{5}{3}$

d)  $r(x) = \frac{g(x)}{-3g(x)}$

$$\frac{1}{-3}$$

Domain:

$X \neq -3$     $X \neq 1$

**Examples:** Let  $f(x) = 3x - 5$  and  $g(x) = (x + 3)(x - 1)$

Evaluate the following functions with the given values and functions.

a)  $\frac{f(2)}{g(-2)}$

$$\frac{3x-5}{(x+3)(x-1)} \quad \frac{3(2)-5}{(-2+3)(-2-1)} = \frac{1}{-3}$$

b)  $\frac{-2f(5)}{g(-1)} = \frac{-20}{-4} = 5$

$$f(5) = 3x - 5$$

$$3(5) - 5$$

$$-2(10)$$

$$g(-1)$$

$$(x+3)(x-1)$$

$$(-1+3)(-1-1)$$

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## Story Problems Involving Combined Functions

a) A company estimates that its cost and revenue can be modeled by the functions  $C(x) = 0.6x^2 + 49x + 150$  and  $R(x) = 100x + 75$ , where  $x$  is the number of items produced. The company's profit,  $P$ , can be modeled by  $P(x) = R(x) - C(x)$ . Find the profit equation and determine the profit when 60 items are produced.

$$C(x) = 0.6x^2 + 49x + 150$$

$$R(x) = 100x + 75$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 100x + 75 - (0.6x^2 + 49x + 150)$$

$$P(x) = 100x + 75 - 0.6x^2 - 49x - 150$$

$$P(x) = -0.6x^2 + 51x - 75$$

$$P(60) = -0.6(60)^2 + 51(60) - 75$$

$$P(60) = \$825$$

b) A service committee is organizing a fundraising dinner. The cost of renting a facility is \$250 plus \$3 per person, or  $C(x) = 3x + 250$ , where  $x$  represents the number of people attending the fundraiser. The committee wants to charge attendees \$20 each or  $R(x) = 20x$ . How many people must attend the fundraiser for the event to raise \$500?

$$C(x) = 3x + 250 \quad R(x) = 20x$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 20x - (3x + 250)$$

$$20x - 3x - 250$$

$$P(x) = 17x - 250$$

$$\textcircled{b} \quad 500 = 17x - 250$$

$$+250 \quad +250$$

$$\frac{750}{17} = \frac{17x}{17}$$

$$44.117 = x$$

$$x = 45 \text{ people}$$