Section 3.4 **Objective: Rational Exponents**

If n is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number then $a^{1/n} = \sqrt[n]{a}$.

The denominator of the exponent tells you what type of root to take.

Examples: Write an equivalent expression using radical notation and, if possible, simplify.

numerator is power

a) $25^{1/2}$ ← denominator is index

$\sqrt[2]{25^1}$

$5 \cdot 5$

$\boxed{5}$

power

index

b) $64^{1/3}$

$\sqrt[3]{64^1}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2$

$\boxed{4}$

power

c) $(xy^2z)^{1/6}$ index

$\sqrt[6]{(xy^2z)^1}$

power

radicand

d) $(36x^{10})^{1/2}$ index

$\sqrt[2]{36x^{10}}$

$\boxed{6x^5}$

e) $2x^{1/4}$

$2\sqrt[4]{x^1}$

f) $(2x)^{1/4}$

$\sqrt[4]{(2x)^1}$

Examples: Write an equivalent expression using exponential notation.

denominator

a) $\sqrt[7]{(2xy)^1}$

$(2xy)^{\frac{1}{7}}$

denominator

b) $\sqrt[4]{\left(\frac{ab^3}{7}\right)^1}$ power of radicand

$\left(\frac{ab^3}{7}\right)^{\frac{1}{4}}$

index is 2

c) $\sqrt{3z}$ power of entire radicand is 1.

$(3z)^{\frac{1}{2}}$

• no index number index is 2 denominator

d) $3\sqrt{z^1}$ power

$3z^{\frac{1}{2}}$

denominator

numerator

e) $\sqrt[5]{(xy^2z)^1}$

$(xy^2z)^{\frac{1}{5}}$

Positive Rational Exponents

If m and n are positive integers (where $n \neq 1$)

and $\sqrt[n]{a}$ exists, then $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

e.g.) $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ $\qquad$ $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

or $8 \boxed{\wedge} (2 \div 3)$

Examples: Write an equivalent expression using radical notation and simplify.

a) $t^{5/6}$ power index

$\sqrt[6]{t^5}$

b) $9^{3/2}$ power index

9 ^ (3÷2)

$\sqrt[2]{9^3}$

$\boxed{27}$

c) $64^{2/3}$

$\sqrt[3]{64^2}$

calc 64 ^ (2÷3)

$\boxed{16}$

d) $(2x)^{3/4}$ power index

$\sqrt[4]{(2x)^3}$

e) $2x^{3/4}$ power index

$\boxed{2\sqrt[4]{x^3}}$

only x has fraction
2 times $x^{\frac{3}{4}}$

Examples: Write an equivalent expression using exponential notation.

denominator

a) $\sqrt[3]{x^5}$ ← numerator

base

$\boxed{x^{\frac{5}{3}}}$

b) $\sqrt[7]{9^2}$

$9^{\frac{2}{7}}$

c) $\left(\sqrt[5]{6n}\right)^3$

$(6n)^{\frac{3}{5}}$

d) $6\sqrt[5]{n^3}$

$6n^{\frac{3}{5}}$

e) $\left(\sqrt[4]{2m}\right)^2$

$\boxed{(2m)^{\frac{2}{4}}}$ or $(2m)^{\frac{1}{2}}$

Negative Rational Exponents

For any rational number m/n, and any nonzero real number $a^{m/n}$,

$$a^{-m/n} = \frac{1}{a^{m/n}}.$$

★ **The sign of the base is not affected by the sign of the exponent.**

Examples: Write an equivalent expression using positive exponents and, if possible, simplify.

a) $49^{-1/2}$

$\frac{1}{49^{\frac{1}{2}}}$

$\frac{1}{\sqrt{49^1}} = \boxed{\frac{1}{7}}$

b) $(3mn)^{-2/5}$

$\boxed{\frac{1}{(3mn)^{\frac{2}{5}}}}$ OR

$\frac{1}{\sqrt[5]{(3mn)^2}}$

c) $7x^{-2/3}$

$7(x)^{-\frac{2}{3}}$

$\boxed{\frac{7}{x^{\frac{2}{3}}}}$

or $\frac{7}{\sqrt[3]{x^2}}$

Laws of Exponents: The laws of exponents apply to rational exponents as well as integer exponents.

Examples: Use the laws of exponents to simplify.

a) $2^{2/5} \cdot 2^{1/5}$

Add powers

$2^{\frac{2}{5}+\frac{1}{5}} = \boxed{2^{\frac{3}{5}}}$

b) $\dfrac{x^{7/3}}{x^{4/3}}$

subtract powers

$x^{\frac{7}{3}-\frac{4}{3}}$

$x^{\frac{3}{3}} = \boxed{x^1}$

c) $\left(19^{2/5}\right)^{5/3}$

$19^{\frac{2}{5}\cdot\frac{5}{3}} = \boxed{19^{\frac{2}{3}}}$

d) $x^{1/2} \cdot x^{2/3}$

$x^{\frac{1}{2}+\frac{2}{3}} = x^{\frac{7}{6}}$

e) $y^{-4/7} \cdot y^{6/7}$

$y^{-\frac{4}{7}+\frac{6}{7}} = \boxed{y^{\frac{2}{7}}}$

f) $\dfrac{z^{3/4}}{z^{2/5}}$

$z^{\frac{3}{4}-\frac{2}{5}} = \boxed{z^{\frac{7}{20}}}$

g) $\dfrac{x^{3/4} \cdot x^{1/6} \cdot y^1}{y^{1/2}}$

$x^{\frac{3}{4}+\frac{1}{6}} \; y^{1-\frac{1}{2}}$

$\boxed{x^{\frac{11}{12}} \; y^{\frac{1}{2}}}$

h) $\dfrac{\left(2x^{2/5}y^{-1/3}\right)^5}{x^2 y}$

$$\frac{2^5 \cdot x^{\frac{2}{5}\cdot\frac{5}{1}} \cdot y^{-\frac{1}{3}\cdot 5}}{x^2 y^1}$$

$$\frac{2^5 \, x^2 \cdot y^{-\frac{5}{3}}}{x^2 \, y^1}$$

$$2^5 x^{2-2} \; y^{-\frac{5}{3}-1}$$

$$32\, x^0 \, y^{-\frac{8}{3}}$$

$x^0 = 1$

$$\boxed{\frac{32}{y^{\frac{8}{3}}}}$$

To Simplify Radical Expressions:

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

Examples: Use rational exponents to simplify. Do not use exponents that are fractions in the final answer.

a) $\sqrt[8]{z^4}$

$z^{\frac{4}{8}} = z^{\frac{1}{2}}$

$\boxed{\sqrt{z}}$

b) $\left(\sqrt[3]{a^2bc^4}\right)^9$

$(a^2bc^4)^{\frac{1}{3}\cdot 9}$

$(a^2bc^4)^{\frac{9}{3}}$

$(a^2bc^4)^3$

$\boxed{a^6b^3c^{12}}$

c) $\sqrt{x}\cdot\sqrt[4]{x}$

$x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}$

$x^{\frac{1}{2}+\frac{1}{4}}$

$x^{\frac{3}{4}} = \boxed{\sqrt[4]{x^3}}$

d) $\sqrt[6]{y^2}\cdot\sqrt[9]{y}$

$y^{\frac{2}{6}}\cdot y^{\frac{1}{9}}$

$y^{\frac{2}{6}+\frac{1}{9}}$

$y^{\frac{4}{9}} = \boxed{\sqrt[9]{y^4}}$

e) $\dfrac{\sqrt[3]{k}}{\sqrt[7]{k^2}}$

$\dfrac{k^{\frac{1}{3}}}{k^{\frac{2}{7}}}$

$k^{\frac{1}{3}-\frac{2}{7}} = k^{\frac{1}{21}}$

$\boxed{\sqrt[21]{k}}$

f) $\dfrac{\sqrt[8]{m^4}}{\sqrt[6]{m}}$

$\dfrac{m^{\frac{4}{8}}}{m^{\frac{1}{6}}}$

$m^{\frac{4}{8}-\frac{1}{6}} = m^{\frac{1}{3}}$

$\boxed{\sqrt[3]{m}}$

g) $\sqrt[4]{\sqrt[5]{x}}$

$\sqrt[4]{x^{\frac{1}{5}}}$

$\left(x^{\frac{1}{5}}\right)^{\frac{1}{4}}$

$x^{\frac{1}{20}}$

$\boxed{\sqrt[20]{x^1}}$

h) $\sqrt[3]{2} \cdot \sqrt[5]{3}$

$2^{\frac{1}{3}} \cdot 3^{\frac{1}{5}}$

get a common denominator

make equal fractions

$2^{\frac{5}{15}} \cdot 3^{\frac{3}{15}}$

$\sqrt[15]{2^5 \cdot 3^3}$

or $\sqrt[15]{864}$