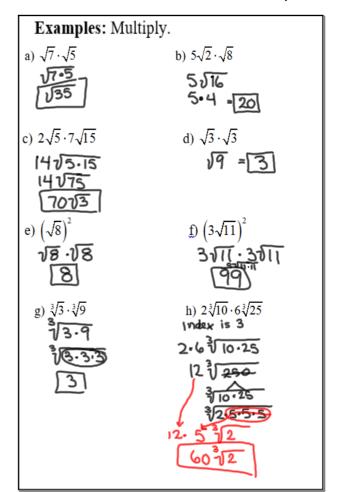---

Section 3.3   **Objective: Radical operations**

The Product Rule for Radicals:

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$.

Caution: The product rule doesn't work if you are trying to multiply the even roots of <u>negative</u> numbers, because those roots are not real numbers. For example, $\sqrt{-2} \cdot \sqrt{-8} \neq \sqrt{16}$.

Caution: The product only applies when the radicals have the same index: $\sqrt[3]{5} \cdot \sqrt[4]{6} \neq \sqrt[12]{30}$.

$$\sqrt[3]{5} \cdot \sqrt[4]{6} \neq \sqrt[12]{5 \cdot 6}$$

index not same

Examples: Multiply.

a) $\sqrt{7} \cdot \sqrt{5}$

$\sqrt{7 \cdot 5}$

$\boxed{\sqrt{35}}$

b) $5\sqrt{2} \cdot \sqrt{8}$

$5\sqrt{16}$

$5 \cdot 4 = \boxed{20}$

c) $2\sqrt{5} \cdot 7\sqrt{15}$

$14\sqrt{5 \cdot 15}$

$14\sqrt{75}$

$\boxed{70\sqrt{3}}$

d) $\sqrt{3} \cdot \sqrt{3}$

$\sqrt{9} = \boxed{3}$

e) $\left(\sqrt{8}\right)^2$

$\sqrt{8} \cdot \sqrt{8}$

$\boxed{8}$

f) $\left(3\sqrt{11}\right)^2$

$3\sqrt{11} \cdot 3\sqrt{11}$

$\boxed{99}$

g) $\sqrt[3]{3} \cdot \sqrt[3]{9}$

$\sqrt[3]{3 \cdot 9}$

$\sqrt[3]{3 \cdot 3 \cdot 3}$

$\boxed{3}$

h) $2\sqrt[3]{10} \cdot 6\sqrt[3]{25}$

Index is 3

$2 \cdot 6 \sqrt[3]{10 \cdot 25}$

$12 \sqrt[3]{250}$

$\sqrt[3]{10 \cdot 25}$

$\sqrt[3]{2 \cdot 5 \cdot 5 \cdot 5}$

$12 \cdot 5 \sqrt[3]{2}$

$\boxed{60\sqrt[3]{2}}$

Question: Can you add and subtract radicals the same way you multiply and divide them?

e.g.) <u>Since</u> $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$, does $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$?   **NO!!!!!!!!!!**

Don't make the following mistakes:
- $\sqrt{2} + \sqrt{5} \neq \sqrt{7}$
- $\sqrt{9+16} \neq 3+4$
- $\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$
- $\sqrt{x^2 - 4} \neq x - 2$
- $\left(\sqrt{x} + \sqrt{y}\right)^2 \neq x + y$

Like Radicals: Radicals with the same index *and* the same radicand.

Examples: Determine whether the following are like radicals. If they are not, explain why not.

a) $\sqrt{3}$ and $\sqrt{2}$

No
one is a 3
the other is a 2

b) $4\sqrt{5}$ and $-3\sqrt{5}$

$\sqrt{5}$ $\sqrt{5}$

yes

$4\sqrt{5} + -3\sqrt{5}$

yes

$\boxed{1\sqrt{5}}$

c) $2\sqrt{x}$ and $\sqrt[3]{x}$

x is in $\sqrt{}$
are the index same? No
No the index are not same.

Steps for Adding and Subtracting Radicals:
1. Simplify each radical completely.
2. Combine like radicals. When you add or subtract radicals, you can *only* combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.

Examples:

a) $5\sqrt{3x} - 7\sqrt{3x}$ *Same index*
              Same prisoner 3x

$(5-7)\sqrt{3x}$

$\boxed{-2\sqrt{3x}}$

b) $4\sqrt{11} + 8\sqrt{11}$

$(4+8)\sqrt{11}$

$\boxed{12\sqrt{11}}$

c) $10\sqrt{6} + 3\sqrt{2} - 8\sqrt{6}$

$10\sqrt{6} - 8\sqrt{6} + 3\sqrt{2}$

$\boxed{2\sqrt{6} + 3\sqrt{2}}$

d) $\sqrt{20} - \sqrt{50} + \sqrt{45}$

$2\sqrt{5} - 5\sqrt{2} + 3\sqrt{5}$

$2\sqrt{5} + 3\sqrt{5} - 5\sqrt{2}$

$5\sqrt{5} - 5\sqrt{2}$

e) $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$

$2 \cdot 5\sqrt{2} + 4 \cdot 10\sqrt{5} - 6 \cdot 5\sqrt{5}$

$10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$

$10\sqrt{5} + 10\sqrt{2}$

f) $\sqrt[3]{54} - 5\sqrt[3]{16} + \sqrt[3]{2}$

$\sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} - 5\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2} + \sqrt[3]{2}$

$3\sqrt[3]{2} - 5 \cdot 2\sqrt[3]{2} + 1\sqrt[3]{2}$

$3\sqrt[3]{2} - 10\sqrt[3]{2} + 1\sqrt[3]{2}$

$\boxed{-6\sqrt[3]{2}}$

Multiplying Radical Expressions: Use the Product Property. Use the Distributive Property and FOIL to multiply radical expressions with more than one term.

Examples: Multiply.

a) $\sqrt{3}(5 + \sqrt{30})$

$5\sqrt{3} + \sqrt{90}$

$2\sqrt{3 \cdot 3 \cdot 5}$

$\boxed{5\sqrt{3} + 3\sqrt{10}}$

b) $\sqrt{2}(\sqrt{6} - 3\sqrt{2})$

$\sqrt{12} - 3\sqrt{4}$

$2\sqrt{3} - 3 \cdot 2$

$2\sqrt{3} - 6$

c) $(\sqrt{5} - \sqrt{6})(\sqrt{7} + 1)$

$\sqrt{5}(\sqrt{7} + 1) - \sqrt{6}(\sqrt{7} + 1)$

$\boxed{\sqrt{35} + 1\sqrt{5} - \sqrt{42} - \sqrt{6}}$

	$\sqrt{5}$	$-\sqrt{6}$
$\sqrt{7}$	$\sqrt{35}$	$-\sqrt{42}$
$+1$	$\sqrt{5}$	$-\sqrt{6}$

d) $\left(5\sqrt{3} - 4\sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right)$

$\sqrt{6} + 7$

e) $\left(4\sqrt{3} - 1\right)^2$

$(4\sqrt{3} - 1)(4\sqrt{3} - 1)$

	$4\sqrt{3}$	-1
$4\sqrt{3}$	$16\sqrt{9}$ (48)	$-4\sqrt{3}$
-1	$-4\sqrt{3}$	$+1$

$\boxed{-8\sqrt{3} + 49}$

f) $\left(\sqrt{2}+5\right)\left(\sqrt{2}-5\right)$

	$\sqrt{2}$	$+5$
$\sqrt{2}$	2	$5\sqrt{2}$
-5	$-5\sqrt{2}$	-25

$2 + 5\sqrt{2} - 5\sqrt{2} - 25$

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