Section 3.1 Objective: Exponent rules

The following properties are true for all real numbers a and b and all integers m and n, provided that no denominators are 0 and that 0^0 is not considered.

1 as an exponent: $a^1 = a$

$7^1 = 7,\ \pi^1 = \pi,\ (-10)^1 = -10$

$8^1 = 8$

0 as an exponent: $a^0 = 1$

$2^0 = 1,\ 27^0 = 1,\ \left(-\frac{5}{8}\right)^0 = 1$

$(7x^2y^3)^0 = 1$

$7(14x^3y^2z^{15})^0$

$7 \cdot 1 = 7$

The Product Rule: $a^m \cdot a^n = a^{m+n}$

e.g.) $x^2 \cdot x^5 = x^{2+5} = x^7$

$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

$4^6 \cdot 4^3 = 4^{6+3} = 4^9$

$(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$

The Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

e.g.) $\frac{x^5}{x^2} = x^{5-2} = x^3$

$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^3$

$\frac{7^3}{7^2} = \frac{7 \cdot 7 \cdot 7}{7 \cdot 7} = 7^1$

$7^{3-2} = 7^1$

The Power Rule: $(a^m)^n = a^{mn}$

MULTIPLY

e.g.) $(x^2)^5 = x^{(2)(5)} = x^{10}$ $(3^6)^4 = 3^{24}$

$x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2$

$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{10}$ $(8^4)^7 = 8^{28}$

Raising a product to a power: $(ab)^n = a^n b^n$

$(2k)^4 = 2^4 \cdot k^4 = 16k^4$ $(803w)^{13} = 803^{13} w^{13}$

$2k \cdot 2k \cdot 2k \cdot 2k$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot k \cdot k \cdot k \cdot k$

$16k^4$

$4(2y)^3$

$4 \cdot 2^3 y^3$

$4 \cdot 8 y^3$

$32y^3$

Raising a quotient to a power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$$\left(\frac{p}{q^2}\right)^3 = \frac{p^3}{(q^2)^3} = \frac{p^3}{q^6}$$

Example $\left(\frac{2}{y}\right)^3 = \frac{2^3}{y^3} = \boxed{\frac{8}{y^3}}$

Negative exponents: $a^{-n} = \frac{1}{a^n}$

$2^{-3} = \frac{1}{2^3}$, $7x^3y^{-4} = \frac{7x^3}{y^4}$

$\frac{1}{a^{-n}} = a^n$ $\frac{1}{x^{-9}} = x^9$, $\frac{b}{c^{-3}d} = \frac{bc^3}{d}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$ $\left(\frac{2}{v}\right)^{-3} = \left(\frac{v}{2}\right)^{3} = \frac{v^3}{2^3} = \frac{v^3}{8}$

$\frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$ $\frac{2^{-3}}{V^{-3}}$ $\frac{V^3}{2^3}$

To *simplify* an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Examples: Simplify the following expressions.

a) $m^5 \cdot m^7$

$m^{5+7} = m^{12}$

product rule

b) $(5a^2b^3)(3a^4b^5)$

$5 \cdot 3\, a^2 \cdot a^4 b^3 \cdot b^5$

$15\, a^{2+4} \cdot b^{3+5}$

$15a^6b^8$

c) $\frac{r^9}{r^3} = r^6$

r^{9-3}

Quotient rule

d) $\frac{p^3}{p^7}$

$p^{3-7} = p^{-4}$ $\frac{1}{p^4}$

$\frac{1}{p^4}$

e) $\frac{10x^{11}y^5}{2x^4y^7}$

$\frac{10}{2} \cdot \frac{x^{11}}{x^4} \cdot \frac{y^5}{y^7}$

$\frac{10}{2}=5$ ⇓

$5 \cdot x^{11-4}\ y^{5-7}$

$5x^7y^{-2} = \boxed{\frac{5x^7}{y^2}}$

f) $\frac{4x^3y^2}{6x^7y}$

$\frac{4}{6} \cdot \frac{x^3}{x^7} \cdot \frac{y^2}{y^1}$

$\frac{2}{3} \cdot x^{3-7}\ y^{2-1}$

$\frac{2}{3}x^{-4}y^1$

$\boxed{\frac{2y^1}{3x^4}}$

g) $(-2)^4$

16

h) -2^4

$(-)\ 2^4$

-16

i) $5x^{-4}y^3 \cdot x^2y^{-1}$

$5\ x^{-4} \cdot x^2 \cdot y^3 \cdot y^{-1}$

$5\ x^{-4+2}\ y^{3+-1}$

$5\ x^{-2}\ y^2$

$\boxed{\frac{5y^2}{x^2}}$

j) $\frac{1}{6^{-2}}$

$\frac{6^2}{1} = \boxed{36}$

k) $9^{-3} \cdot 9^8$

9^{-3+8}

$\boxed{9^5}$

l) $\frac{3x^2}{15x^{-3}y^{-4}}$

$\frac{3x^2x^3y^4}{15}$

$\frac{3}{15}x^{2+3}y^4$

$\boxed{\frac{1x^5y^4}{5}}$

m) $(3^5)^4$

$3^{5\cdot4}$

3^{20}

n) $\frac{y^{-5}}{y^{-4}}$

$\frac{y^4}{y^5}$ $\frac{yyyy}{yyyyy}$

$\boxed{\frac{1}{y}}$

o) $(y^{-5})^7$

$y^{-5\cdot7}$

$y^{-35} = \boxed{\frac{1}{y^{35}}}$

p) $(a^{-3})^{-7}$

$a^{-3\cdot-7}$

$\boxed{a^{21}}$

q) $(-2x)^3$

$(-2)^3x^3$

$\boxed{-8x^3}$

r) $\left(\frac{x^2}{2}\right)^4$

$\frac{(x^2)^4}{2^4} = \boxed{\frac{x^8}{16}}$

s) $(3x^5y^{-1}z^{-2})^{-2}$

$3^{-2}x^{5\cdot-2}y^{-1\cdot-2}$

$3^{-2}x^{-10}y^2$

$\boxed{\frac{y^2}{3^2x^{10}}}$ or $\boxed{\frac{y^2}{9x^{10}}}$

t) $\left(\frac{y^2z^3}{5}\right)^{-3}$

$\frac{y^{2\cdot-3}z^{3\cdot-3}}{5^{-3}}$

$\frac{y^{-6}z^{-9}}{5^{-3}}$

$\boxed{\frac{5^3}{y^6z^9}}$