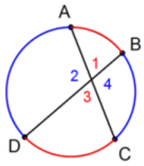
Section 12.2 **Objective: Tangent and Chord Theorems Notes**

Theorem:

- If two chords intersect inside a circle, then the measure of each angle formed is the average of the measures of the arcs intercepted by the angle and its vertical angle.



$m\angle 1 = m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$

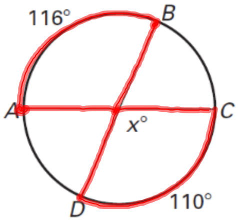
$m\angle 2 = m\angle 4 = \frac{1}{2}\left(m\overset{\frown}{BC} + m\overset{\frown}{AD}\right)$

Remember this crosses inside circle
Add arcs

Angle $= \frac{1}{2}$(big arc + little arc)

Examples: Find the value of x.

a)

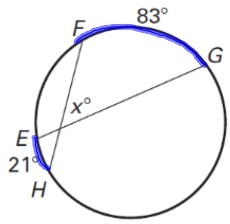


$x = \frac{1}{2}(116 + 110)$

$x = \frac{1}{2}(226)$

$x = 113$

b)

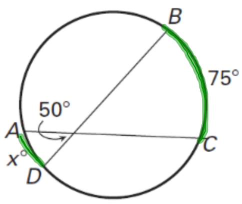


$x = \frac{1}{2}(83 + 21)$

$x = \frac{1}{2}(104)$

$x = 52°$

c)



This time we need an arc.

Set up equation

$50 = \frac{1}{2}(75 + x)$

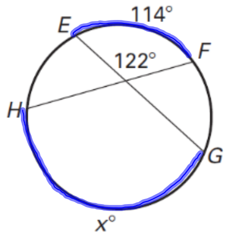
To get rid of $\frac{1}{2}$
multiply both sides by 2

$2(50) = \cancel{2 \cdot \frac{1}{2}}(75 + x)$

$100 = 75 + x$
$-75 \quad -75$ Subtract 75 from both sides

$\boxed{25° = x}$

d)



angle $= \frac{1}{2}$(big arc + little arc)

$122 = \frac{1}{2}(x + 114)$

$2(122) = 2 \cdot \frac{1}{2}(x + 114)$

$244 = x + 114$
$-114 \quad -114$

$130 = x$

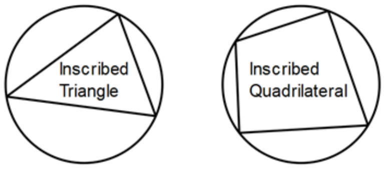
Another way to remember:

If you have an angle and an arc and need to find the other arc

Multiply angle by 2 = big arc + little arc

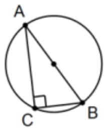
2(angle) = big arc + little arc

Inscribed Polygon: A polygon whose vertices all lie on a circle.



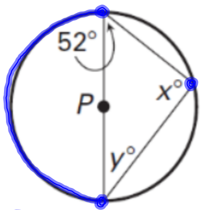
Theorems:

- If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.
- If $\triangle ABC$ is a right triangle with hypotenuse $\overline{AB}$, then $\overline{AB}$ is a diameter of the circle.
- If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.
- If $\overline{AB}$ is a diameter of the circle, then $\triangle ABC$ is a right triangle with $\overline{AB}$ as hypotenuse.



Examples: Find the values of x and y in $\odot P$.

a)



$52 + x + y = 180$

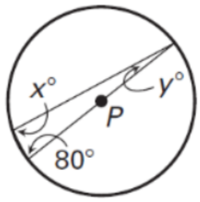
Since x is opposite hypotenuse and hypotenuse is diameter

$\boxed{x = 90°}$

$y = 180 - 90 - 52$

$\boxed{y = 38°}$

b)



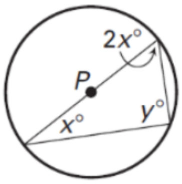
x is opposite diameter so

$\boxed{x = 90°}$

$y = 180 - 90 - 80$

$\boxed{y = 10°}$

c)



y is opposite diameter

So $\boxed{y = 90°}$

Sum of angles in a triangle is 180°

so $x + 2x + y = 180$

$y = 90$ $\quad 3x + y = 180$

$3x + 90 = 180$

$-90 \quad -90$

$3x = 90$

$\frac{3x}{3} = \frac{90}{3}$

$\boxed{x = 30}$

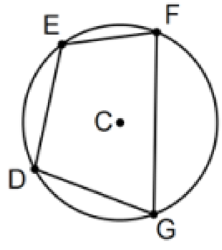
$2x = ?$

$2(30) = 60$

$\boxed{2x = 60}$

Theorem:

- If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.



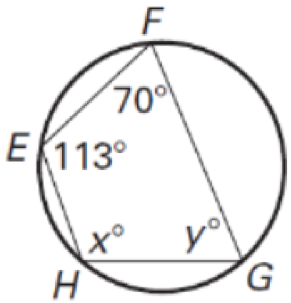
$m\angle D + m\angle F = 180°$

$m\angle E + m\angle G = 180°$

opposite angles add to 180°

Examples: Find the values of x and y.

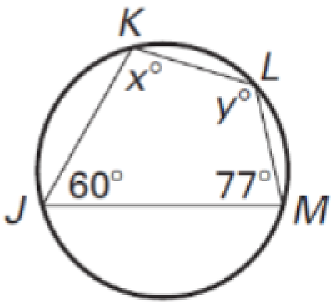
a)



$x + 70 = 180$
$-70 \quad -70$
$x = 110°$

$y + 113 = 180$
$-113 \quad -113$
$\boxed{y = 67°}$

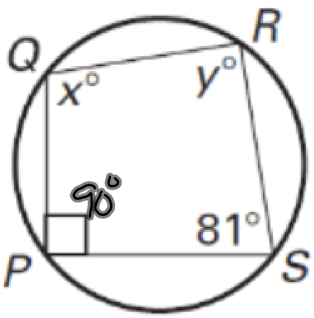
b)



$x + 77 = 180$
$-77 \quad -77$
$\boxed{x = 103°}$

$y + 60 = 180$
$-60 \quad -60$
$\boxed{y = 120°}$

c)

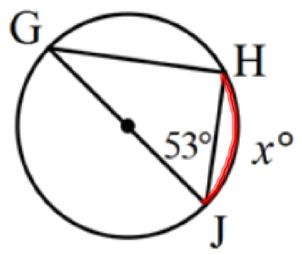


$x + 81 = 180$
$-81 \quad -81$
$\boxed{x = 99°}$

$y + 90 = 180$
$-90 \quad -90$
$\boxed{y = 90}$

Find the measure of the arc or angle indicated.

a)



$\overline{GJ}$ is a diameter

so $\widehat{GH} + \widehat{HJ} = 180°$

$\angle GJH = 53$

$2(53) = \widehat{GH}$

$116 = \widehat{GH}$

$180 - 116 = x$

$\boxed{74° = x}$

OR · first find $m\angle G$

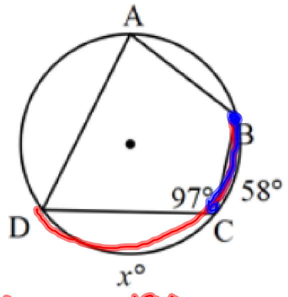
$\angle H = 90$

$180 - 90 - 53 = 37$

$\angle G = 37$

$2(37) = 74°$

b)



$\angle A + \angle C = 180$

$\angle A + 97 = 180$

$\angle A = 180 - 97$

$\angle A = 83$

$m\,\widehat{DCB} = 2(83)$

$m\,\widehat{DCB} = 166$

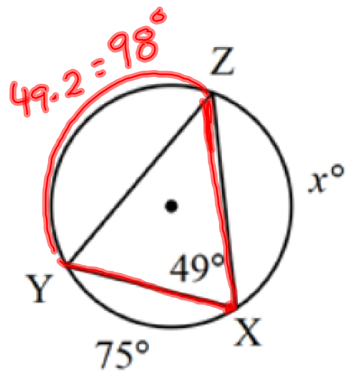
$\widehat{DCB} = \widehat{DC} + \widehat{CB}$

$166 = x + 58$

$166 - 58 = x$

$\boxed{108° = x}$

c)



YZ is not a diameter

$\angle X = 49$

$2(\angle X) = \widehat{YZ}$

$2(49) = 98$

An entire circle $= 360°$

so $x + 98 + 75 = 360$

$x + 173 = 360$

$\underline{-173 \quad -173}$

$\boxed{x = 187°}$

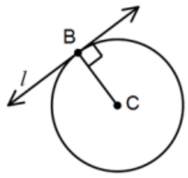
Theorems about Tangents:

- If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency.

  If line l is tangent to $\odot C$ at B, then $l \perp \overline{CB}$.

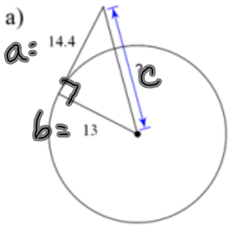
- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

  If $l \perp \overline{CB}$, then line l is tangent to $\odot C$ at B.



Examples: Find the length of the missing segment. Assume that segments which appear to be tangent to the circle are tangent to the circle.

a)



use $a^2+b^2=c^2$

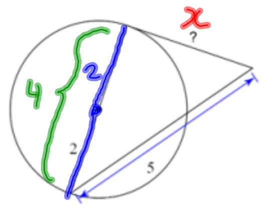
$14.4^2+13^2=c^2$

$376.36=c^2$

$\sqrt{376.36}=c$

$\boxed{19.4 \approx c}$

b)



radius = 2 so
diameter = 4

$x^2+4^2=5^2$

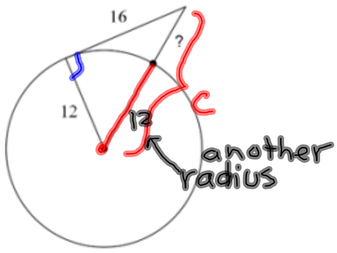
$x^2+16=25$

$x^2=25-16$

$x^2=9$

$\boxed{x=3}$

c)



$12^2+16^2=c^2$

$400 = \text{hypotenuse}^2$

$\sqrt{400} = \text{hypotenuse}$

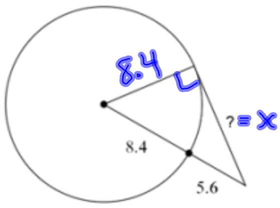
$20 = \text{hypotenuse}$

$20 = 12 + ?$

$20 - 12 = ?$

$\boxed{8 = ?}$

d)



radius = 8.4

hypotenuse = 8.4+5.6

$x^2+8.4^2=(8.4+5.6)^2$

$x^2+8.4^2=(14)^2$

$x^2=14^2-8.4^2$

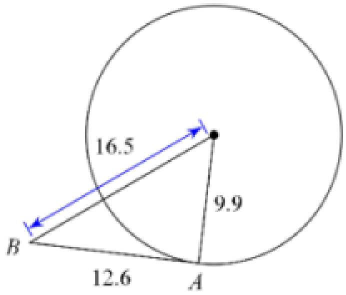
$x^2=125.44$

$x=\sqrt{125.44}$

$\boxed{x \approx 11.2}$

Examples: Determine whether $\overline{AB}$ is tangent to the circle. Explain your reasoning.

a)



Does $a^2+b^2=c^2$

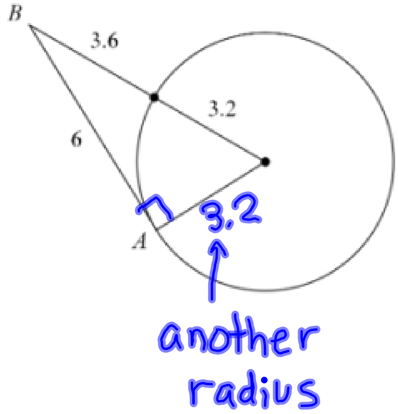
if yes it's tangent
if no it is not tangent

$9.9^2+12.6^2 \stackrel{?}{=} 16.5^2$

$\boxed{256.77 \neq 272.25}$

$\boxed{\text{Not tangent}}$

b)



$a^2+b^2 \stackrel{?}{=} c^2$

$3.2^2+6^2 \stackrel{?}{=} (3.6+3.2)^2$

$46.24 \stackrel{?}{=} (6.8)^2$

$\boxed{46.24 = 46.24 \text{ yes tangent}}$