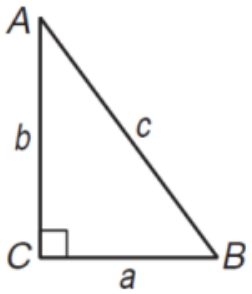
Section 11.5 Objective: Solving Triangles Notes

Solving a Triangle: Figuring out the lengths of all three sides and the measures of all three angles of a triangle. DRAW AND LABEL A TRIANGLE (Label the pieces you know with numbers and the pieces you don't know with variables)! Decide which of these choices your triangle is like. Then follow the instructions. Show work!

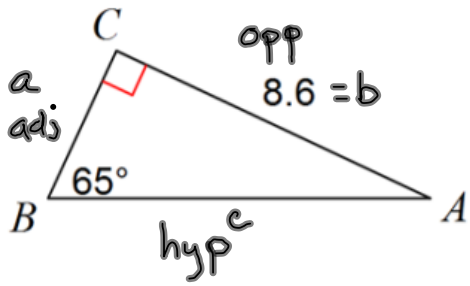


- **If you know the measure of one angle and the length of one side,**
  - To find the measure of the other angle:
    Use the fact that the angles in a triangle add to 180° to find the measure of the other angle.
  - To find the lengths of the other sides: If you know the measure of one angle and the length of one side, use sin, cos, or tan to figure out the lengths of the other sides.

- If you know the lengths of two of the sides,
  - To find the length of the other side: use the Pythagorean Theorem to find the length of the third side.
  - To find the measures of the angles: Use inverse functions ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$). Then use the fact that the angles in a triangle add to 180° to find the measure of the third angle.

Examples: Solve $\triangle ABC$. Round answers to the nearest tenth. Show all your work.

a)



$m\angle A = 25°$ $m\angle B = 65$ $m\angle C = 90$

$a = 4.0$ $b = 8.6$ $c = 9.5$

To find $\angle A$ $180° - 65° - 90° = 25°$

Use $\sin\theta$ to find "c"

$$\frac{\sin 65}{1} = \frac{8.6}{c}$$

$c \sin 65° = 8.6$

$c = 8.6 \div \sin 65$

$c = 9.5$

Use $\tan\theta$ to find a

$$\frac{\tan 65°}{1} = \frac{8.6}{a}$$

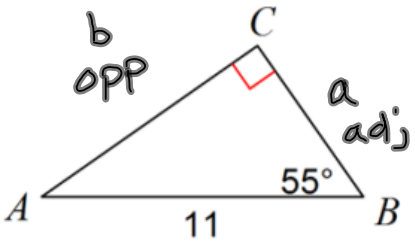
$a \tan 65° = 8.6$

$a = 8.6 \div \tan 65°$

$a = 4.0$

* Once you have found 2 sides of triangle you could use $a^2 + b^2 = c^2$ to find 3rd side.

b)



hyp

$m\angle A = 35°$ $m\angle B = 55°$ $m\angle C = 90°$

$a = 6.3$ $b = 9.0$ $c = 11$

$\angle A = 180° - 90° - 55° = 35°$

$$\frac{\cos 55}{1} = \frac{a}{11}$$

$11 \cos 55 = a$

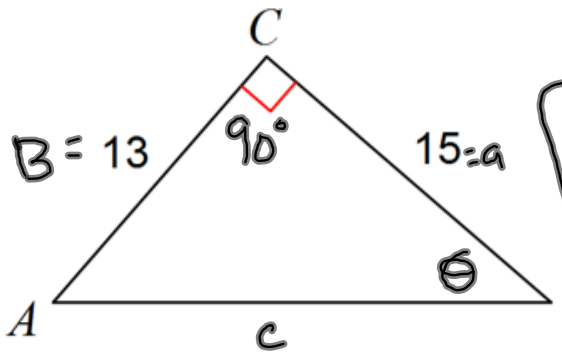
$6.3 = a$

$$\frac{\sin 55}{1} = \frac{b}{11}$$

$b = 11 \sin 55°$

$b = 9.0$

c)



$m\angle A = 49.1$

$m\angle B = 40.9$

$m\angle C = 90°$

$a = 15$

$b = 13$

$c = \sqrt{394} \approx 19.8$

Choose an angle "θ"

$\tan B = \frac{13}{15}$

$\tan^{-1}\left(\frac{13}{15}\right) \approx 40.9° = m\angle B$

$m\angle A = 180 - 90 - 40.9 = 49.1$

$a^2 + b^2 = c^2$

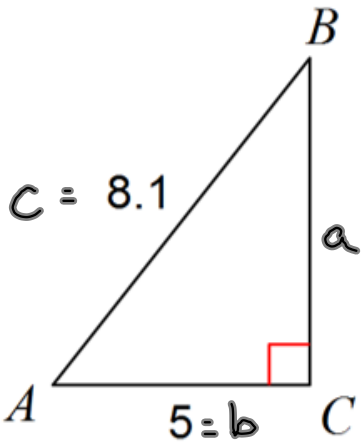
$15^2 + 13^2 = c^2$

$394 = c^2$

$\sqrt{394} = c$

$c = \sqrt{394}$ or ≈ 19.8

d)



$m\angle A = 51.9°$

$m\angle B = 38.1°$

$m\angle C = 90°$

$a = 6.37$ or 6.4

$b = 5$

$c = 8.1$

use $a^2 + b^2 = c^2$ to find a

$a^2 + 5^2 = 8.1^2$

$a^2 = 8.1^2 - 5^2$

$a = \sqrt{8.1^2 - 5^2}$

$a = \sqrt{40.61}$

$a \approx 6.4$

choose to find $m\angle A$

$\cos A = \frac{adj}{hyp}$

$\cos A = \frac{5}{8.1}$

$\cos^{-1}\left(\frac{5}{8.1}\right) = m\angle A$

$m\angle A \approx 51.9°$

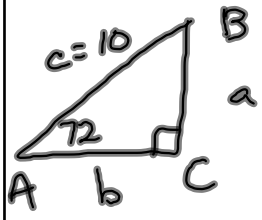
$m\angle B = 180° - 90° - 51.9°$

$m\angle B = 38.1°$

$m\angle C = 90°$

Draw and label the triangle with the given measurements. Then solve the triangle.

a) $m\angle A = 72°,\ c = 10$



$m\angle A = 72°$

$m\angle B = 18°$

$m\angle C = 90°$

$a = 9.5$

$b = 3.1$

$c = 10$

$180 - 72 - 90 = m\angle B$

$m\angle B = 18°$

$\sin A = \frac{a}{c}$

$\frac{\sin 72}{1} = \frac{a}{10}$

$10 \sin 72 = a$

$a = 9.5$

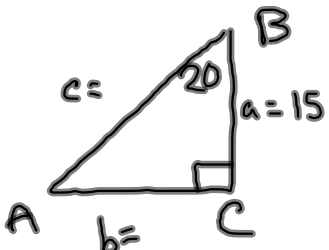
$\cos A = \frac{b}{c}$

$\frac{\cos 72}{1} = \frac{b}{10}$

$10 \cos 72 = b$

$3.1 = b$

b) $m\angle B = 20°,\ a = 15$



$m\angle A = 70°$

$m\angle B = 20$

$m\angle C = 90$

$m\angle A = 180 - 90 - 20$

$m\angle A = 70°$

$a = 15$

$b = 5.5$

$c = 15.96$ or 16.0

$\tan B = \frac{b}{a}$

$\frac{\tan 20}{1} = \frac{b}{15}$

$15 \tan 20 = b$

$5.5 = b$

$\frac{\cos 20}{1} = \frac{15}{c}$

$c \cdot \cos 20 = 15$

$c = 15 \div \cos 20$

$c = 15.96$

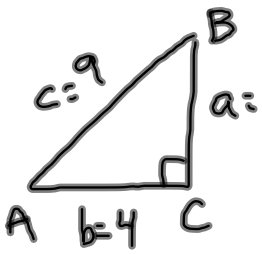
OR $a^2 + b^2 = c^2$

$15^2 + 5.5^2 = c^2$

$\sqrt{15^2 + 5.5^2} = c$

$15.96 \approx c$

c) $b = 4,\ c = 9$



$m\angle A = 63.6°$ $a \approx 8.1$

$m\angle B = 26.4°$ $b = 4$

$m\angle C = 90°$ $c = 9$

$a^2 + b^2 = c^2$

$a^2 + 4^2 = 9^2$

$a^2 = 9^2 - 4^2$

$a = \sqrt{9^2 - 4^2}$

$a = \sqrt{65} \approx 8.1$

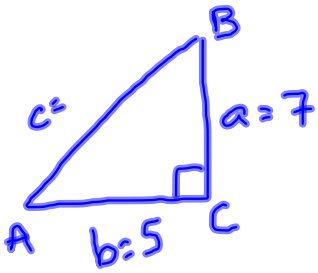
$\cos A = \frac{b}{c}$

$\cos A = \frac{4}{9}$

$\cos^{-1}\left(\frac{4}{9}\right) = 63.6°$

$m\angle B = 180 - 90 - 63.6°$

d) $a = 7,\ b = 5$



$m\angle A = 54.5°$ $a = 7$

$m\angle B = 35.5°$ $b = 5$

$m\angle C = 90°$ $c \approx 8.6$

$a^2 + b^2 = c^2$

$7^2 + 5^2 = c^2$

$\sqrt{7^2 + 5^2} = c$

$8.6 \approx c$

$m\angle A$ use $\tan A$

$\tan A = \frac{opp}{adj}$

$\tan A = \frac{7}{5}$

$\tan^{-1}\left(\frac{7}{5}\right) \approx 54.5°$

$m\angle B = 180 - 90 - 54.5$

$m\angle B \approx 35.5°$