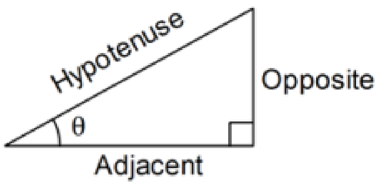
Section 11.3 **Objective: Trigonometric Functions, Missing Angles Notes**

Trigonometry: The study of the relationships among the sides and angles of right triangles

Trigonometric Ratio: A ratio of the lengths of two sides of a right triangle. The three main trigonometric ratios are sine (sin), cosine (cos), and tangent (tan). If θ (theta – the angle) is an acute angle of a right triangle, "adj" is the length of the leg adjacent (next to) θ, "opp" is the length of the leg opposite θ, and "hyp" is the length of the hypotenuse, then:

The 3 main or most common trigonometric ratios:



1) $\sin\theta = \dfrac{\text{opp side}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

2) $\cos\theta = \dfrac{\text{adj side}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

3) $\tan\theta = \dfrac{\text{opp side}}{\text{adj side}}$ $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

- ★ ***Adjacent*** means " next to θ " (θ is the angle you are focusing on)
- ★ ***Opposite*** means " across from θ " (θ is the angle you are focusing on)
- ★ ***Hypotenuse*** is the side across from the right angle

A common way to remember this is: SOH – CAH - TOA

Steps to find the trigonometric ratios

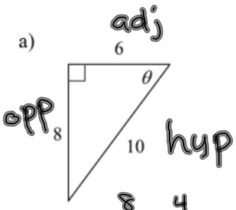
1) Label the sides of the triangle as hyp, opp, adj

2) If a side is missing, use the Pythagorean Theorem to find the missing side

3) Write the fraction using SOH – CAH - TOA

4) Remember: Use exact answers and simplify the fraction (unless it says to find the decimal approximation)

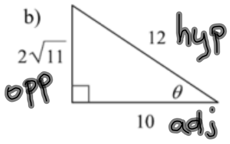
Examples: Label the sides as opposite, adjacent, and hypotenuse. Find the lengths of any missing sides. Then find the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a)


$\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

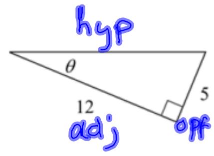
$\tan\theta = \frac{8}{6} = \frac{4}{3}$

b)


$\sin\theta = \frac{2\sqrt{11}}{12}$ or $\frac{\sqrt{11}}{6}$

$\cos\theta = \frac{10}{12} = \frac{5}{6}$

$\tan\theta = \frac{2\sqrt{11}}{10}$ or $\frac{\sqrt{11}}{5}$

c)


find hypotenuse

$a^2+b^2=c^2$

$12^2+5^2=c^2$

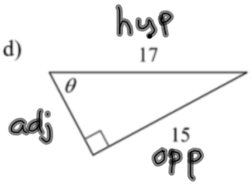
$\sqrt{144+25}=c$

$13=c$

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

d)


find adj side

$a^2+b^2=c^2$

$a^2+15^2=17^2$

$a^2=17^2-15^2$

$a^2=64$

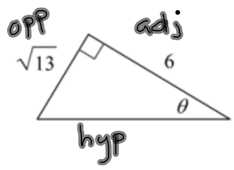
$a=\sqrt{64}$

$a=8$

$\sin\theta = \frac{15}{17}$

$\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{15}{8}$

e)


find hypotenuse

$a^2+b^2=c^2$

$6^2+(\sqrt{13})^2=c^2$

$36+13=c^2$

$49=c^2$

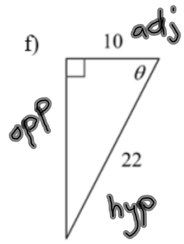
$\sqrt{49}=c$

$7=c$

$\sin\theta = \frac{\sqrt{13}}{7}$

$\cos\theta = \frac{6}{7}$

$\tan\theta = \frac{\sqrt{13}}{6}$

f)


find opposite

$a^2+b^2=c^2$

$10^2+b^2=22^2$

$b^2=22^2-10^2$

$b=\sqrt{384}$

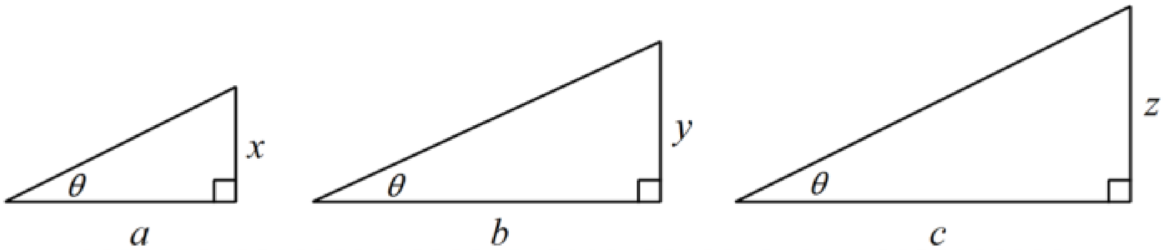
or $8\sqrt{6}$

$\sin\theta = \frac{\sqrt{384}}{22}$ or $\frac{4\sqrt{6}}{11}$ simplified

$\cos\theta = \frac{10}{22}$ or $\frac{5}{11}$

$\tan\theta = \frac{\sqrt{384}}{10}$ or $\frac{4\sqrt{6}}{5}$ simplified

No matter how big the triangle is, the values of the trigonometric functions for a certain size angle will remain the same. For example, the triangle below shows three similar right triangles, which means that θ is the same size angle in all three triangles and that $\tan\theta = \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$. The value of the tangent is the same in all three triangles even though they are different sizes. The same is true for the sine and cosine.



We can use this totally awesome fact to use the sides of a triangle to figure out how big the angles in the triangle are.

In the diagram on the previous page, measure the labeled sides of the diagrams, in millimeters. Record the measurements below.

$x =$ 20 $a =$ 40 $y =$ 60 $b =$ 30 $z =$ 33 $c =$ 66

Fill in the blanks:

$\tan\theta = \frac{x}{a} \approx$ .5 $\tan\theta = \frac{y}{b} \approx$ .5 $\tan\theta = \frac{z}{c} \approx$ .5

★ **Make sure your calculator is in DEGREE mode!**

Now we are going to use the *inverse tangent* function to get three estimates for the size of θ. If you measured correctly, they should be very close to the same number. The only reason they are different at all is because it is hard to measure the side lengths exactly.

Make sure you are in degree mode.

In the calculator, type $\tan^{-1}\left(\frac{opp}{adj}\right)$ with each of your three fractions. Record your three estimates of θ below:

Estimate 1: 26.6° Estimate 2: 26.6° Estimate 3: 26.6°

Inverse Functions:

- The inverse sine of x ($\sin^{-1} x$) is the angle whose sine is x. If $\sin\theta = x$, then $\theta = \sin^{-1} x$.
- The inverse cosine of x ($\cos^{-1} x$) is the angle whose cosine is x. If $\cos\theta = x$, then $\theta = \cos^{-1} x$.
- The inverse tangent of x ($\tan^{-1} x$) is the angle whose tangent is x. If $\tan\theta = x$, then $\theta = \tan^{-1} x$.

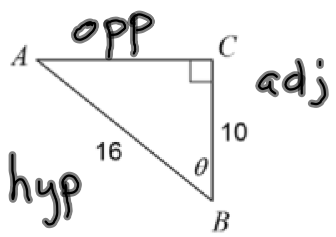
When do you use inverse functions? fraction or decimal

Use inverse functions when you know the sine, cosine, or tangent of an angle and want to know how big the angle is.

degree of angle

Examples: Label the sides as opposite, adjacent, or hypotenuse. Write an equation involving sine, cosine or tangent. Then find the measure of θ to the nearest tenth of a degree.

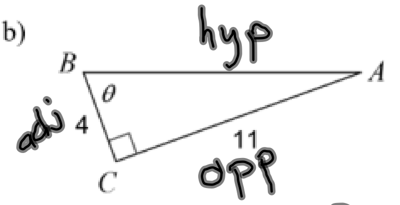
a)



$\cos\theta = \frac{adj}{hyp} = \frac{10}{16}$

$\cos^{-1}\left(\frac{10}{16}\right) \approx 51.3°$

b)

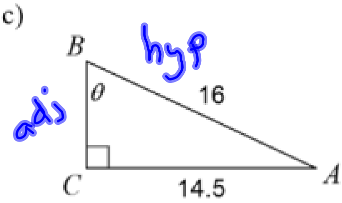


use $\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{11}{4}$

$\tan^{-1}\left(\frac{11}{4}\right) \approx 70.0°$

c)

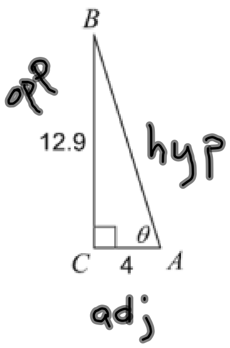


use $\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{14.5}{16}$

$\sin^{-1}\left(\frac{14.5}{16}\right) = 65.0°$

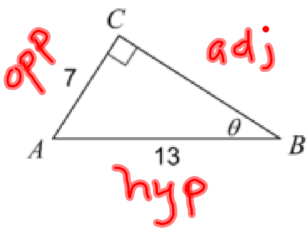
d)



use $\tan\theta = \frac{opp}{adj}$

$\tan^{-1}\left(\frac{12.9}{4}\right) \approx 72.8°$

e)



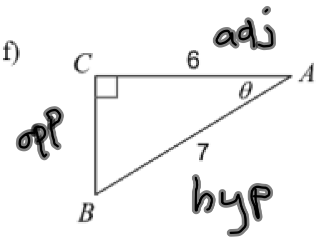
use $\sin\theta$

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{7}{13}$

$\sin^{-1}\left(\frac{7}{13}\right) \approx 32.6°$

f)



Use $\cos\theta$

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{6}{7}$

$\cos^{-1}\left(\frac{6}{7}\right) \approx 31.0°$