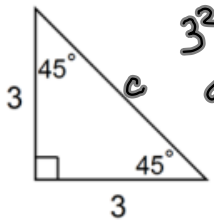
Section 11.2 Objective: Special Right Triangles Notes

How do I find a missing side of a right triangle?

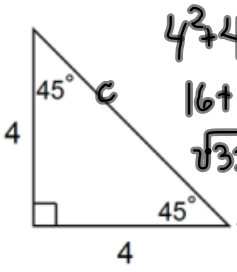
Use the Pythagorean theorem **to find the length of the hypotenuse for each right triangle. Express your answers in simplest radical form.**

a)



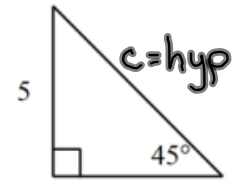
$3^2+3^2=c^2$
$9+9=c^2$
$18=c^2$
$\sqrt{18}=c$
$3\sqrt{2}=c$

b)



$4^2+4^2=c^2$
$16+16=c^2$
$\sqrt{32}=c$
$4\sqrt{2}=c$

c)

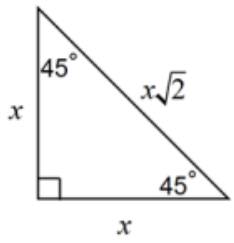


$5^2+5^2=c^2$
$25+25=c^2$
$50=c^2$
$\sqrt{50}=c$
$5\sqrt{2}=c$

There are 2 triangles that when you do the Pythagorean Theorem with them you will always get the same ratio.

45°-45°-90° Right Triangles:

- **Legs are** x (the same)
- **Hypotenuse =** $x\sqrt{2}$ / leg times $\sqrt{2}$



If given hypotenuse:

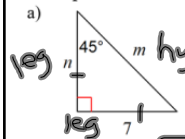
hyp = leg $\sqrt{2}$

to find leg divide hypotenuse by $\sqrt{2}$

SO: $\text{leg} = \frac{\text{hypotenuse}}{\sqrt{2}}$

Examples: Find the value of each variable.

a)



leg hyp leg

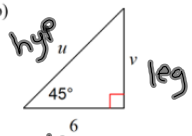
leg = 7 $\boxed{n = 7}$

m is hypotenuse

$m = \text{leg}\sqrt{2}$

$\boxed{m = 7\sqrt{2}}$

b)



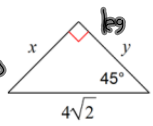
leg leg

$\boxed{v = 6}$

$u = \text{hyp} = \text{leg}\sqrt{2}$

$\boxed{u = 6\sqrt{2}}$

c)



leg leg hyp

leg = leg $x = y$

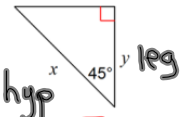
$\text{leg} = \frac{\text{hyp}}{\sqrt{2}}$

$\text{leg} = \frac{4\sqrt{2}}{\sqrt{2}}$

leg = 4

$\boxed{x = y = 4}$

d)



leg leg hyp

$\boxed{y = 2\sqrt{5}}$

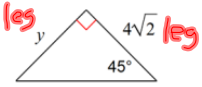
$\text{hyp} = \text{leg}\sqrt{2}$

$x = \text{hyp} = (2\sqrt{5})(\sqrt{2})$

$= 2\sqrt{5 \cdot 2}$

$\boxed{x = 2\sqrt{10}}$

e)



hyp

$y = 4\sqrt{2}$

$\text{hyp} = \text{leg}\sqrt{2}$

$x = (4\sqrt{2})(\sqrt{2})$

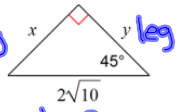
$x = 4\sqrt{2} \cdot \sqrt{2}$

$x = 4\sqrt{2 \cdot 2}$

$x = 4\sqrt{4} = 4 \cdot 2$

$\boxed{x = 8}$

f)



hyp

$x = y$

$\text{leg} = \frac{\text{hyp}}{\sqrt{2}}$

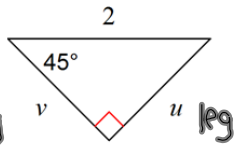
$\boxed{\text{leg} = \frac{2\sqrt{10}}{\sqrt{2}}}$

$\text{leg} = 2\sqrt{10} \div \sqrt{2}$

$\boxed{2\sqrt{5}}$

$x = y = 2\sqrt{5}$

g)



$u = v$

$\text{leg} = \frac{\text{hyp}}{\sqrt{2}}$

$\text{leg} = \frac{2}{\sqrt{2}}$

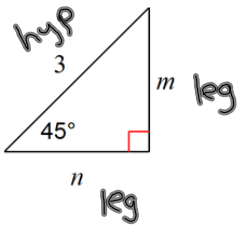
$\boxed{u = v = \frac{2}{\sqrt{2}}}$ or

if you use calculator you will get $\sqrt{2}$

here is why

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

h)



$\text{leg} = \text{leg}$ $m = n$

$\text{leg} = \frac{\text{hyp}}{\sqrt{2}}$

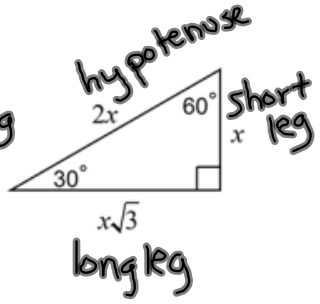
$\boxed{\text{leg} = \frac{3}{\sqrt{2}}}$

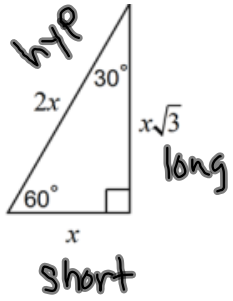
calculator will give a decimal or

$\frac{3\sqrt{2}}{2}$

30°-60°-90° Right Triangles:

- **Hypotenuse =** 2 times short leg
- **Long Leg =** short leg times $\sqrt{3}$





given hypotenuse find short leg first

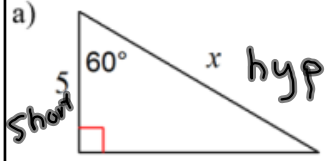
short leg = hypotenuse ÷ 2

given long leg find short leg first

$$\text{short leg} = \frac{\text{long leg}}{\sqrt{3}}$$

Examples: Find the value of each variable.

a)



hyp = short leg (2)

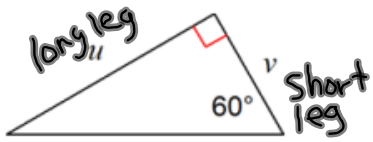
$x = 5(2) = 10$ $\boxed{x = 10}$

y = long leg

long leg = short $\sqrt{3}$

$\boxed{y = 5\sqrt{3}}$

b)



Short leg = hyp ÷ 2

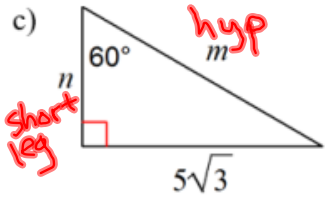
$v = 6 \div 2$

$\boxed{v = 3}$

long leg = (short) $\sqrt{3}$

$\boxed{u = 3\sqrt{3}}$

c)



Short = $\frac{\text{long}}{\sqrt{3}}$

$n = \frac{5\sqrt{3}}{\sqrt{3}}$

$\boxed{n = 5}$

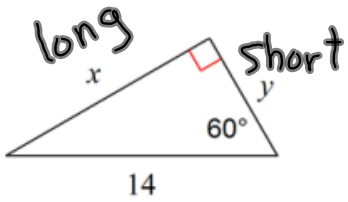
m = hyp

hyp = (short)(2)

$m = 5(2)$

$\boxed{m = 10}$

d)



hyp

Short = hyp ÷ 2

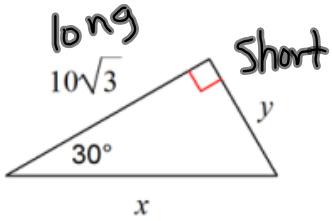
$y = 14 \div 2$

$\boxed{y = 7}$

X = short ($\sqrt{3}$)

$\boxed{X = 7\sqrt{3}}$

e)



hyp

Short = $\frac{\text{long leg}}{\sqrt{3}}$

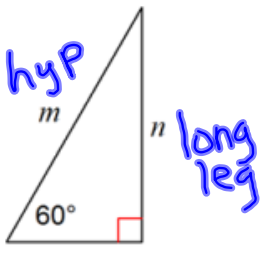
$y = \frac{10\sqrt{3}}{\sqrt{3}}$

$\boxed{y = 10}$

$x = 10(2)$

$\boxed{x = 20}$

f)

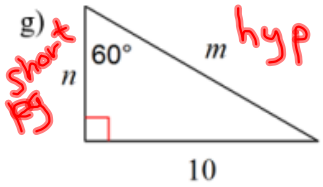


Short leg

$\boxed{n = 8\sqrt{3}}$

$m = 8(2)$

$\boxed{m = 16}$

g) 

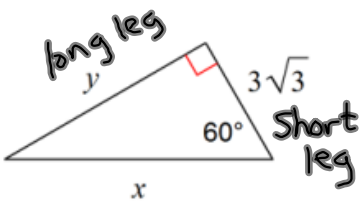
short leg, hyp, long leg

Short = long / $\sqrt{3}$

short = $n = \frac{10}{\sqrt{3}}$

hyp = $m = 2\left(\frac{10}{\sqrt{3}}\right)$

$m = \frac{20}{\sqrt{3}}$

h) 

long leg, short leg, hyp

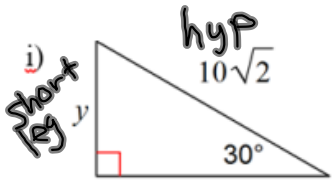
$y = (3\sqrt{3})(\sqrt{3})$

$y = 3\sqrt{9}$

$y = 3 \cdot 3 = 9$

$x = (3\sqrt{3})(2)$

$x = 6\sqrt{3}$

i) 

short leg, hyp, long leg

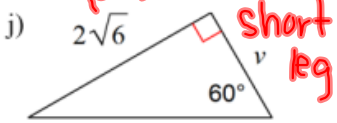
Short = hyp ÷ 2

$y = \frac{10\sqrt{2}}{2}$

$y = 5\sqrt{2}$

$x = (5\sqrt{2})(\sqrt{3})$

$x = 5\sqrt{6}$

j) 

long leg, short leg, hyp

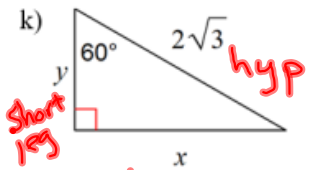
Short leg = $\frac{\text{long leg}}{\sqrt{3}}$

$v = \frac{2\sqrt{6}}{\sqrt{3}}$ or $2\sqrt{2}$

hyp = (short leg)(2)

$u = \left(\frac{2\sqrt{6}}{\sqrt{3}}\right)(2)$ or $u = (2\sqrt{2})(2)$

$u = \frac{4\sqrt{6}}{\sqrt{3}}$ or $u = 4\sqrt{2}$

k) 

short leg, hyp, long leg

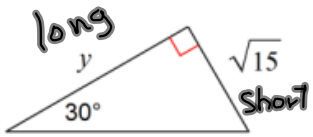
short = hyp ÷ 2

$y = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$y = \sqrt{3}$

$x = (\sqrt{3})(\sqrt{3})$

$x = 3$

l) 

long, short, hyp

$\text{hyp} = (\sqrt{15})(2)$

$x = 2\sqrt{15}$

$\text{long} = (\sqrt{15})(\sqrt{3})$

$\text{long} = \sqrt{45}$

or $3\sqrt{5}$

$y = \sqrt{45}$ or $3\sqrt{5}$

Now let's mix the two types together. First you need to decide which type of special right triangle it is. Then you can use the pattern to find the missing sides.

Examples: Find the value of each variable.

a)

45-45-90

$y=5 \quad x=5\sqrt{2}$

b)

30-60-90

$n=6\sqrt{3} \quad m=12$

c)

45-45-90

hyp = $9\sqrt{2}$

$x=y=\frac{9\sqrt{2}}{\sqrt{2}}$

$x=y=9$

d)

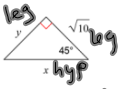
30-60-90

$y=28\div 2$

$y=14$

$x=14\sqrt{3}$

e)



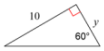
45-45-90

$y=\sqrt{10}$

$x=\sqrt{10}(\sqrt{2})$

$x=\sqrt{20}$ or $2\sqrt{5}$

f)



30-60-90

x = hyp

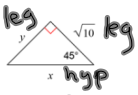
y = short leg

10 = long leg

$y=\frac{10}{\sqrt{3}}$

$x=\frac{20}{\sqrt{3}}$

g)



45-45-90

$y=\sqrt{10}$

$x=\sqrt{10}\cdot\sqrt{2}$

$x=\sqrt{20}$

or $x=2\sqrt{5}$

h)

30-60-90

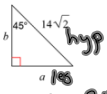
$n=\left(\frac{\sqrt{3}}{2}\right)\sqrt{3}$

$n=\frac{3}{2}$

$m=\left(\frac{\sqrt{3}}{2}\right)(2)$

$m=\sqrt{3}$

i)



45-45-90

$14\sqrt{2}$ = hyp

a = b

$\text{leg}=\frac{\text{hyp}}{\sqrt{2}}$

$a=b=\frac{14\sqrt{2}}{\sqrt{2}}$

$a=b=14$

j)

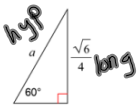
30-60-90

$\text{Short}=\frac{\text{long}}{\sqrt{3}}$

$y=\frac{8\sqrt{3}}{\sqrt{3}}=8$

$x=8(2)=16$

k)



30-60-90

short = long ÷ $\sqrt{3}$

$b=\left(\frac{\sqrt{6}}{4}\right)\div\sqrt{3}$

$\frac{\sqrt{6}}{4}\div\frac{\sqrt{3}}{1}$

$\frac{\sqrt{6}}{4}\cdot\frac{1}{\sqrt{3}}$

$\frac{\sqrt{6}}{4\sqrt{3}}$ or $\frac{\sqrt{2}}{4}$

$a=\frac{2\sqrt{6}}{4\sqrt{3}}$ or $\frac{2\sqrt{2}}{4}$

or $\frac{\sqrt{6}}{2\sqrt{3}}$ or $\frac{\sqrt{2}}{2}$

l)

45-45-90

$v=7\sqrt{6}$

$u=(7\sqrt{6})(\sqrt{2})$

$u=7\sqrt{12}$ or

$7\cdot 2\sqrt{3}$

or $14\sqrt{3}$