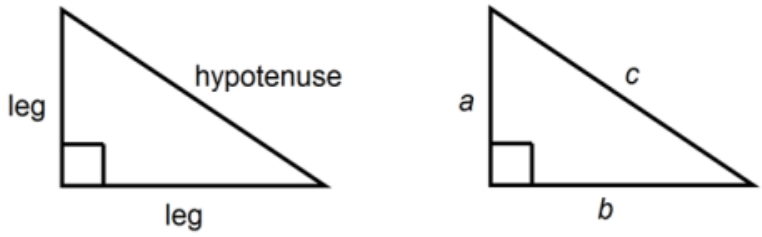
Section 11.1

Objective: Pythagorean Theorem and Distance Notes

The Pythagorean Theorem: In a right triangle, the sum of the squares of the legs equals the square of the hypotenuse.

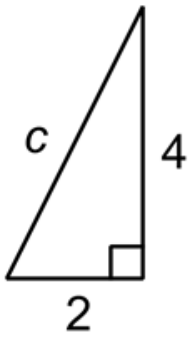
The Pythagorean Theorem: In a right triangle, $a^2 + b^2 = c^2$, or $\textbf{leg}^2 + \textbf{leg}^2 = \textbf{hypotenuse}^2$.



★ **The hypotenuse (the longest side – the one across from the right angle) should always be by itself on one side of the equation.**

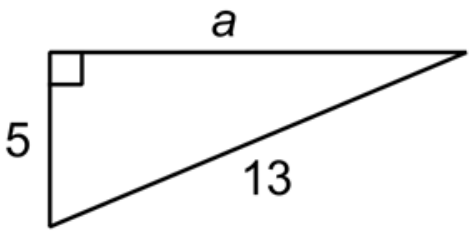
★ *****It does not matter which leg is a or b.**

To find the length of the hypotenuse:



$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$c^2 = 20$$
$$c = \sqrt{20}$$
$$c = 2\sqrt{5} \approx 4.47$$

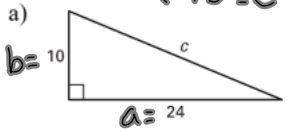
To find the length of a leg:



$$a^2 + 5^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 = 169 - 25$$
$$a^2 = 144$$
$$a = \sqrt{144}$$
$$a = 12$$

Examples: Find the length of the missing side of each triangle. Write answer as **exact** and **rounded** to the nearest hundredth.

a)



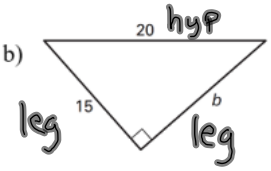
$a^2+b^2=c^2$

$hyp = \sqrt{leg^2+leg^2}$

$c = \sqrt{10^2+24^2}$

$c = 26$

b)



$a^2+b^2=c^2$

$15^2+b^2=20^2$

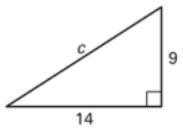
$b^2=20^2-15^2$

$b=\sqrt{20^2-15^2}$

$b=\sqrt{175}$ or $5\sqrt{7}$

≈ 13.23

c)



$a^2+b^2=c^2$

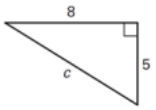
$9^2+14^2=c^2$

$\sqrt{9^2+14^2}=c$

$\sqrt{277}=c$

≈ 16.64

d)



$a^2+b^2=c^2$

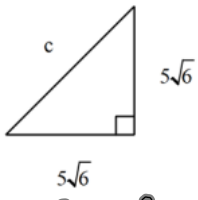
$5^2+8^2=c^2$

$\sqrt{5^2+8^2}=c$

$\sqrt{89}=c$

≈ 9.43

e)



$a^2+b^2=c^2$

$(5\sqrt{6})^2+(5\sqrt{6})^2=c^2$

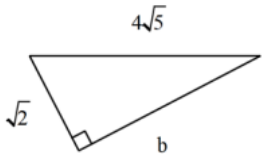
$150+150=c^2$

$\sqrt{300}=\sqrt{c^2}$

$10\sqrt{3}=c$

≈ 17.32

f)



$a^2+b^2=c^2$

$(\sqrt{2})^2+b^2=(4\sqrt{5})^2$

$2+b^2=4\sqrt{5}\cdot 4\sqrt{5}$

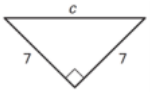
$2+b^2=80$

$b^2=78$

$b=\sqrt{78}$

≈ 8.83

g)



$a^2+b^2=c^2$

$7^2+7^2=c^2$

$98=c^2$

$\sqrt{98}=c$

or $7\sqrt{2}=c$

≈ 9.90

h)

$a^2+b^2=c^2$

$a^2+8.4^2=11.9^2$

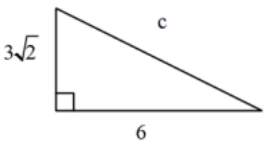
$a^2=11.9^2-8.4^2$

$a^2=71.05$

$a=\sqrt{71.05}$

≈ 8.43

i)



$a^2+b^2=c^2$

$(3\sqrt{2})^2+6^2=c^2$

$(3\sqrt{2})(3\sqrt{2})+36=c^2$

$18+36=c^2$

$54=c^2$

$\sqrt{54}=c$

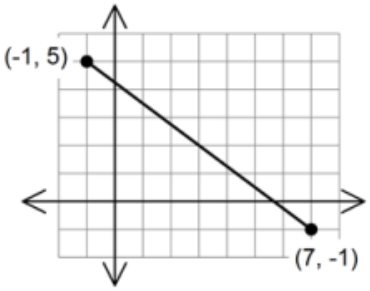
or $3\sqrt{6}=c$

≈ 7.35

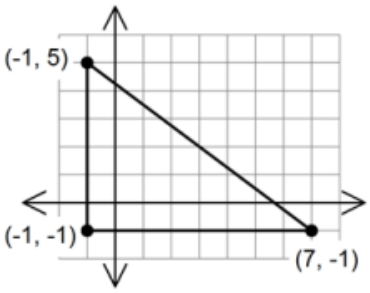
Distance Formula:

Example: Find the distance between $(-1,5)$ and $(7,-1)$.

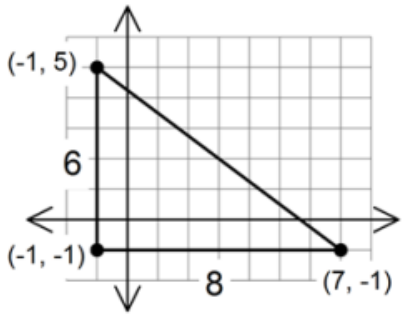
1. Plot the two points on a graph and connect them with a segment.



2. Draw a right triangle with your segment as the hypotenuse.



3. Figure out the lengths of the legs.



4. Plug into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = \sqrt{100} = 10$$

-or-

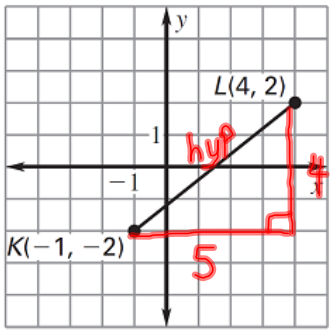
Use the distance formula (but be careful with your negatives!)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7-(-1))^2 + ((-1)-5)^2}$$
$$= \sqrt{(8)^2 + (-6)^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100} = 10$$

Examples: Find the distance between each set of points.

a)

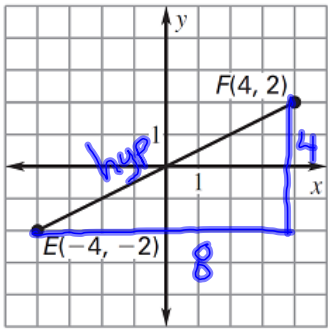


$c^2 = 5^2 + 4^2$

$c^2 = 41$

$c = \sqrt{41} \approx 6.4$

b)



$c^2 = \sqrt{8^2 + 4^2}$

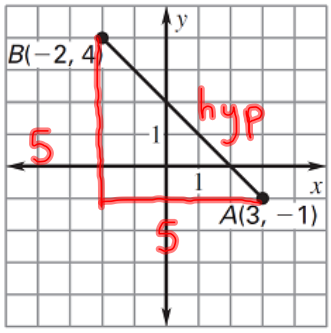
$c^2 = \sqrt{64+16}$

$c = \sqrt{80}$

$4\sqrt{5}$

≈ 8.94

c)



$c^2 = 5^2 + 5^2$

$c^2 = 50$

$c = \sqrt{50}$ or $5\sqrt{2}$

≈ 7.07

d) $(5,6)$ and $(-1,-2)$

$x_1\ y_1 \quad x_2\ y_2$

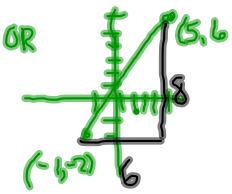
$d=\sqrt{(-1-5)^2+(-2-6)^2}$

$d=\sqrt{(-6)^2+(-8)^2}$

$d=\sqrt{100}$

$\boxed{d=10}$

OR



$a^2+b^2=c^2$

$6^2+8^2=c^2$

$\sqrt{6^2+8^2}=c$

$\sqrt{100}=c$

$10=c$

e) $(4,7)$ and $(9,-3)$

$x_1, y_1 \quad x_2\ y_2$

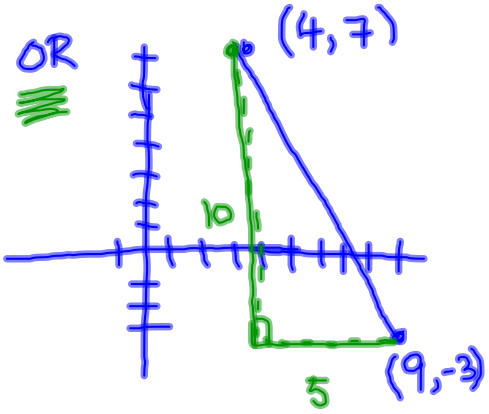
$d=\sqrt{(9-4)^2+(-3-7)^2}$

$d=\sqrt{5^2+(-10)^2}$

$d=\sqrt{25+100}$

$\boxed{d=\sqrt{125} \text{ or } 5\sqrt{5} \approx 11.18}$

OR



$a^2+b^2=c^2$

$5^2+10^2=c^2$

$125=c^2$

$\sqrt{125}=c$

$5\sqrt{5} \approx 11.18$

f) $(-2,3)$ and $(-5,7)$

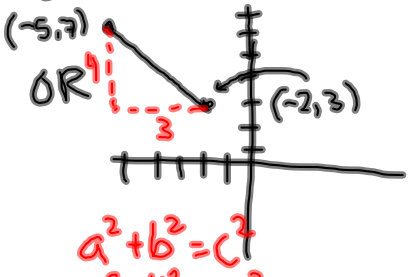
$(x_1,y_1) \quad (x_2,y_2)$

$d=\sqrt{(-5--2)^2+(7-3)^2}$

$d=\sqrt{(-3)^2+(4)^2}$

$d=\sqrt{9+16}$

$\boxed{d=\sqrt{25} \text{ or } 5}$



$a^2+b^2=c^2$

$3^2+4^2=c^2$

$9+16=c^2$

$\sqrt{25}=c$

$\boxed{5=c}$