Pre-Calculus Unit 4 review

Convert the angle given from degrees to radians.

Leave as a multiple of π. Show work!

1. $-36°$

$\frac{-36}{1} \cdot \frac{\pi}{180} = \frac{-36\pi}{180}$

$\boxed{\frac{-\pi}{5}}$

2. $20°$

$\frac{20}{1} \cdot \frac{\pi}{180}$

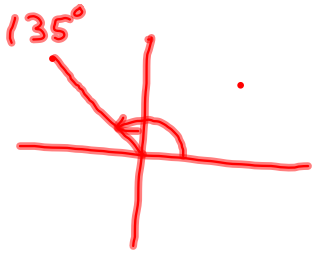
$\frac{20\pi}{180} = \boxed{\frac{\pi}{9}}$

Convert the angle given from radians to degrees. Round to the nearest tenth of a degree. Show work!

3. $\frac{3\pi}{4}$

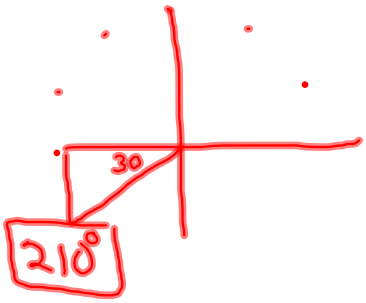
$\frac{3\pi}{4} \cdot \frac{180}{\pi}$

$\frac{540}{4} \Rightarrow \boxed{135°}$



4. $\frac{7\pi}{6}$

$\frac{7\pi}{6} \cdot \frac{30}{180} = \boxed{210°}$



Find the length of the arc intercepted by the given central angle α in a circle with radius r. Round answers to the nearest hundredth. Show work!

5. $\alpha = \frac{5\pi}{6}$, $r = 2.1\,cm$

(radians)(radius)

$S = ar$

$S = \frac{5\pi}{6} \cdot 2.1$

$= 5.497$

5.5 cm

$S = \frac{\text{degree}}{360} \times 2\pi r$

Find the area of a sector of a circle with the given central angle α and the given radius r of the circle. Round answers to the nearest hundredth. Show work!

6. $\alpha = 35°$, $r = 6\,m$

in degrees

$A_{sector} = \frac{1}{2} a r^2$ if in radians

$A_{sector} = \frac{\text{angle}}{360} \cdot \pi r^2$

$A = \frac{35}{360} \cdot \frac{\pi}{1} \cdot 6^2$

$A = 10.9955$

$11.00\ m^2$

Find the measure of two angles, one positive and one negative that are conterminal with the given angle.

7. $\frac{3\pi}{8}$

$\frac{3\pi}{8} + 2\pi$ $\quad$ $\frac{3\pi}{8} - 2\pi$

$\frac{3\pi}{8} + \frac{16\pi}{8}$ $\quad$ $\frac{3\pi}{8} - \frac{16\pi}{8}$

$\frac{19\pi}{8}$ $\quad$ $-\frac{13\pi}{8}$

8. $-62°$

$-62° + 360°$ $\quad$ $-62° - 360°$

$298°$ $\quad$ $-422°$

9. A wheel with a 13 inch diameter is turning at the rate of 22 revolutions per minute. Find the linear velocity of a point on the rim of the wheel in miles per hour. Round answer to nearest hundredth. (1 mile = 5280 feet)

$d = 13$ in $\quad r = 6.5$ in

22 rev per min

1 radian = 1 radius

1 rev = 2π radian

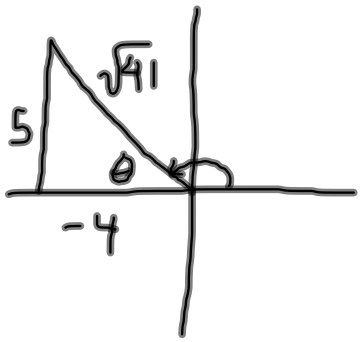
$$\frac{22 \text{ rev}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{6.5 \text{ in}}{1 \text{ rad}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}}$$

$$\frac{22(60)(2\pi)(6.5)}{(12)(5280)} = \frac{53909.72}{63360}$$

$\approx .85$ mph

Find the exact values of $\sin\alpha, \cos\alpha, \tan\alpha, \csc\alpha, \sec\alpha,$ *and* $\cot\alpha$ **where** α **is an angle in standard position whose terminal side contains the given point.**

10. $(-4, 5)$



$a^2+b^2=c^2$

$\sqrt{(-4)^2+5^2}=c^2$

$\sqrt{16+25}$

$\sqrt{41}=c$

$\sin a=\frac{y}{r}$ $\quad \sin a=\frac{5}{\sqrt{41}}$ $\quad \csc a=\frac{\sqrt{41}}{5}$

$\cos a=\frac{x}{r}$ $\quad \cos a=\frac{-4}{\sqrt{41}}$ $\quad \sec a=\frac{\sqrt{41}}{-4}$

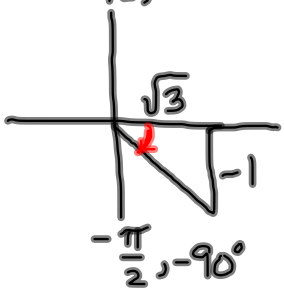
$\tan a=\frac{y}{x}$ $\quad \tan a=\frac{5}{-4}$ $\quad \cot a=\frac{-4}{5}$

Evaluate each expression without using a calculator. Give the result in degrees.

11. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ $\frac{y}{x}$

$\tan(\theta) = \frac{-1}{\sqrt{3}}$

$\pi/2, 90°$



$-\frac{\pi}{2}, -90°$

$-30°$

12. $\cos^{-1}\left(-\frac{1}{2}\right)$

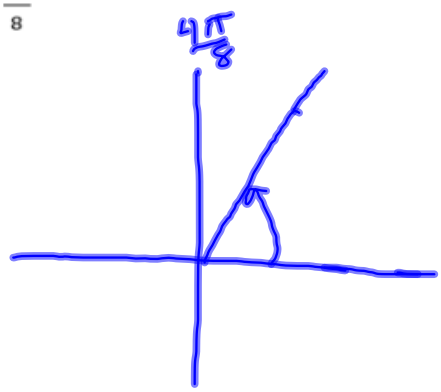
restricted $[0°, 180°]$

$\cos\theta = -\frac{1}{2}$

$120°$

Sketch the angle in standard position and name the quadrant in which the terminal side lies.

13. $\frac{3\pi}{8}$



$\frac{3\pi}{8} \cdot \frac{180}{\pi}$

or $\frac{4\pi}{8} = \frac{\pi}{2}$ so it must be in QI

$\frac{3\pi}{8}$ is smaller than $\frac{4\pi}{8} = \frac{\pi}{2}$

EXAMPLE 2.

$\frac{7\pi}{9}$ bigger than $\frac{\pi}{2}$ because $\frac{1}{2}$ of 9 is 4.5

$\frac{7\pi}{9} > \frac{4.5\pi}{9}$

$\frac{7\pi}{9} < \frac{9\pi}{9}$ II

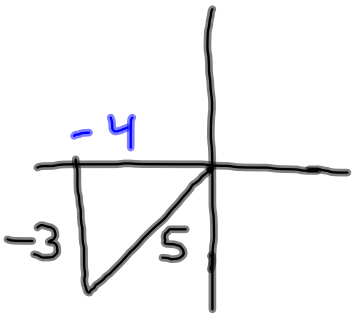
Find the exact value of the expression. Do not use a calculator.

14. $\cos a$, *if* $\sin a = -\frac{3}{5}$ *and* $\tan a > 0$

QIII or QIV

is positive

QI QIII



$\sin a = \frac{-3}{5} \quad \frac{y}{r}$

$x^2 + y^2 = r^2$

$x^2 + (-3)^2 = 5^2$

$x^2 = 25 - 9$

$x = \sqrt{16}$

$x = 4$

$\cos a = \frac{-4}{5}$

Find the equation for the curve in its final position. Pay attention to the order of the transformations!

15. The graph $y = \sin(x)$ is shifted a distance of $\frac{\pi}{4}$ to the left, reflected over the x-axis, stretched vertically by a factor of 3, and then translated 5 units upward.

$y = \sin(x)$

$y = \sin\left(x + \frac{\pi}{4}\right)$

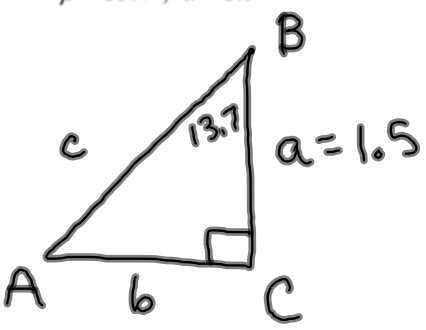
$y = -\sin\left(x + \frac{\pi}{4}\right)$

$y = -3\sin\left(x + \frac{\pi}{4}\right)$

$y = -3\sin\left(x + \frac{\pi}{4}\right) + 5$

Solve the right triangle using only the given sides and angles to find each missing part. Make a sketch. Round approximate answers to the nearest hundredth.

16. $\beta = 13.7°,\ a = 1.5$



$\angle A = 76.3°$

$\angle B = 13.7°$

$\angle C = 90°$

$a = 1.5$

$b = .37$

$c = 1.54$

$m\angle A = 90° - 13.7°$

$m\angle A = 180° - 90° - 13.7°$

$\tan(13.7) = \frac{b}{1.5}$

$1.5\tan(13.7) = b$

$.37 \approx b$

$a^2 + b^2 = c^2$

$1.5^2 + .37^2 = c^2$

$\sqrt{(1.5)^2 + (.37)^2}$

1.54

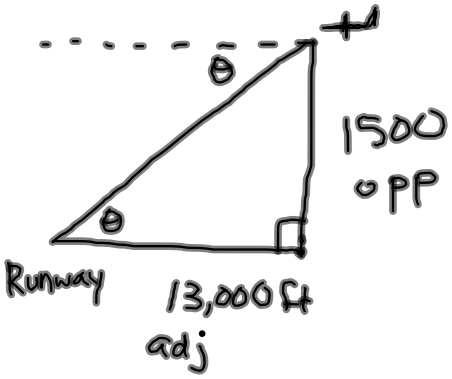
$\frac{\cos(13.7)}{1} = \frac{1.5}{c}$

$c = 1.5 \div \cos 13.7$

$c = 1.54$

Solve the following problem.

17. At an altitude of 1500 feet, the engine on a small plane fails. What angle of depression is needed to reach an airport runway that is 13000 feet away by land? (Round your answer to the nearest tenth of a degree.)



$$\tan\theta = \frac{1500}{13{,}000}$$

$$\tan^{-1}(1500 \div 13000)$$

$$6.58^\circ$$

$$\boxed{\approx 6.6^\circ}$$

Graph the function over a one-period interval. Clearly label your axes. Label or list the five key points on the graph. List the amplitude, period, phase shift, and range.

18. $y = 2\sin\left[2\pi\left(x - \frac{1}{2}\right)\right] - 1$

$a = 2 \quad b = 2\pi \quad c = \frac{1}{2} \quad d = -1$

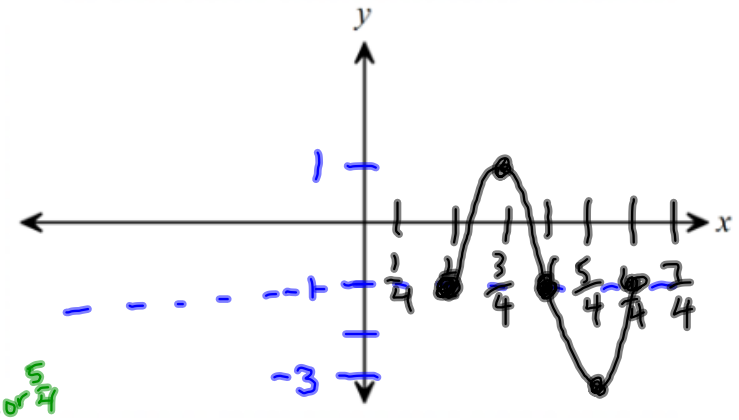
Amplitude: 2

Period: $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$

Phase shift: $\frac{1}{2}$

Vertical shift: -1

Range: $[-3, 1]$



divide by 2π / add $\frac{1}{2}$

θ	0	$\frac{\pi}{2}$	π	$3\pi/2$	2π
$\sin x$	0	1	0	-3	0
	-1	1	-1	-3	-1

$\frac{1}{2}$ $\frac{3}{4}$ 1 $\frac{1}{4}$ or $\frac{5}{4}$ $\frac{3}{2}$

mult by $a + d$

$a = 2 \quad d = -1$

$1(2) + -1 \quad -(2) - 1$

$\frac{0}{2\pi} + \frac{1}{2} \qquad \frac{2\pi}{2\pi} + \frac{1}{2} = 1\frac{1}{2}$

$\frac{\pi}{2} \div 2\pi + \frac{1}{2}$

$\frac{\pi}{2} \cdot \frac{1}{2\pi}$

$\frac{1}{4} + \frac{1}{2}$

$\frac{3}{4}$

Find the equations of the asymptotes.

19. $y = 3\csc\left(\frac{1}{4}x - \frac{\pi}{3}\right)$

$y_1 = 3\sin\left[\frac{1}{4}\left(x - \frac{4\pi}{3}\right)\right]$

period $\frac{2\pi}{b}$ $b = \frac{1}{4}$

$\frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$

÷ $\frac{1}{4}$ + $\frac{4\pi}{3}$

θ	~~0~~	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	~~0~~	~~1~~	~~0~~	~~-1~~	~~0~~
mult by 3 add 0	0	3	0	-3	0

$\frac{\pi}{2} \cdot \frac{2}{4} + \frac{4\pi}{3} = \frac{10\pi}{3}$

$0 \div \frac{1}{4} + \frac{4\pi}{3} = \frac{4\pi}{3}$

Asymptotes are when sin = 0

$X = \frac{4\pi}{3} + \left(\frac{1}{2}\text{period}\right)k$

$X = \frac{4\pi}{3} + 4\pi k$ where k is an integer

Graph the function. Label or list the key points. List the period and the equations of the asymptotes.

20. $y = -\frac{2}{3}\tan(x)$

$a = \frac{2}{3}$ flip

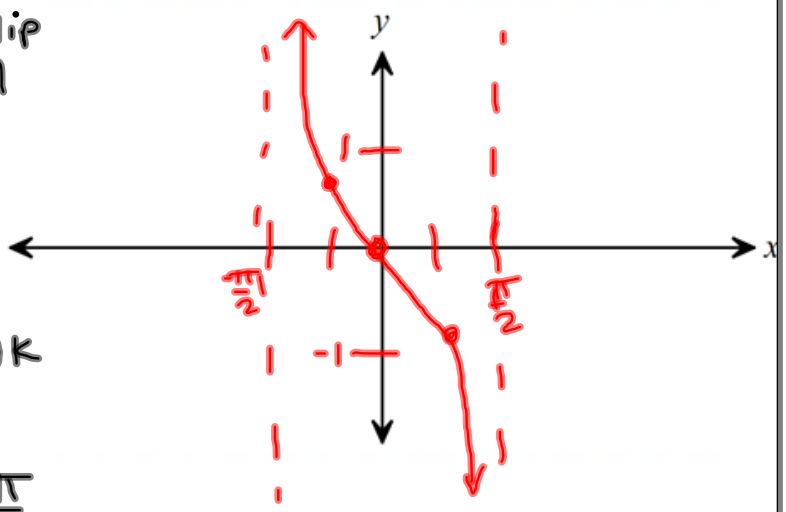
$b = 1$

$\frac{\pi}{b}$

Period π

Asymptotes $x = -\frac{\pi}{2} + \pi k$

$+(\text{period})k$



θ	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\tan x$	und	-1	0	1	UND
mult by $-\frac{2}{3}$		$\frac{2}{3}$	0	$-\frac{2}{3}$	

Solve the problem.

21. The voltage E in an electrical circuit is given by $E = 4\cos 30\pi t$ where t is time measured in seconds. Find the frequency of the function (that is, find the number of cycles or periods completed in one second.)

reciprocal of period

$E = 4\cos 30\pi t$

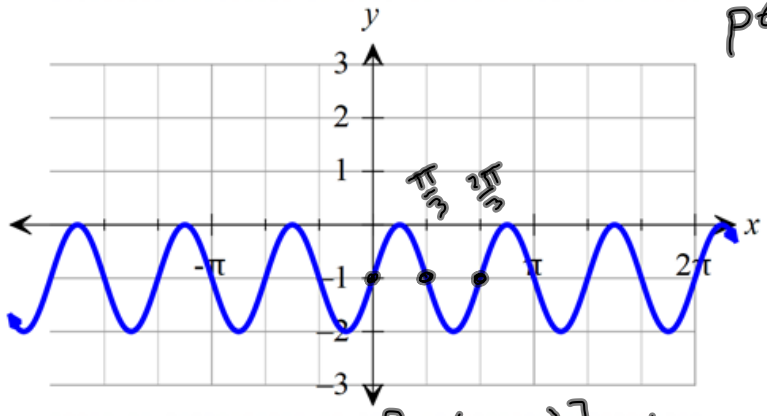
period: $\frac{2\pi}{b}$

$b = 30\pi$

$\frac{2\pi}{30\pi} = \frac{1}{15}$

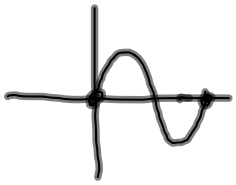
freq = 15 cycles per sec

22. Determine an equation of the function that is graphed in the form y = a sin[b(x-c)] + d.



period is $\frac{2\pi}{3}$

sin



$y = a\sin[b(x-c)] + d$

$y = 1\sin[3(x-0)] - 1$

mid line $d = -1$

$a = 1$

phase shift $= 0$

period $\frac{2\pi}{3}$

$\frac{2\pi}{b} = \frac{2\pi}{3}$

$6\pi = 2\pi b$

$\frac{6\pi}{2\pi} = b$

$3 = b$

$y = \sin(3x) - 1$