PreCalculus Unit 3 Review

Solve the equation.

1) $4^{1+2x} = 64$

$4^{1+2x} = 4^3$

$1+2x=3$

$-1 \quad -1$

$2x=2$

$\boxed{x=1}$

2) $27^{4x+5} = 9^{4x}$

$27^{4x+5} = 9^{4x}$

$3^{3(4x+5)} = 3^{2(4x)}$

$12x+15=8x$

$-4x = 15$

$\frac{-4x}{-4} = \frac{15}{-4}$ $\quad x = \frac{-15}{4}$

Change the exponential expression to an equivalent expression involving a logarithm.

3) $5^x = 125$

3 $5^x = 125$ $\quad \log_5 125 = x$

Change the logarithmic expression to an equivalent expression involving an exponent.

4) $\log_b 16 = 4$

$b^4 = 16$

Find the exact value of the logarithmic expression.

5) $\cancel{\ln} e^3$

3

$\log_e e^3 \quad 3\cancel{\log_e e}$

$3(1)$

6) $\log_4 \frac{1}{64}$

$\mathrm{Log}_{\cancel{4}} \cancel{4}^{-3} = -3$

7) $\log_3 \sqrt{3}$

$\log_{\cancel{3}} \cancel{3}^{1/2}$ $\boxed{=1/2}$

Use the properties of logarithms to find the exact value of the expression. Do not use a calculator.

8) $\log_{144} 8 + \log_{144} 18$

$\log_{144}(8 \cdot 18)$

$\log_{144} 144$

$\boxed{1}$

9) $10^{\log 21 - \log 3}$

$10^{\log_{10}\left(\frac{21}{3}\right)}$

$10^{\log_{10} 7}$

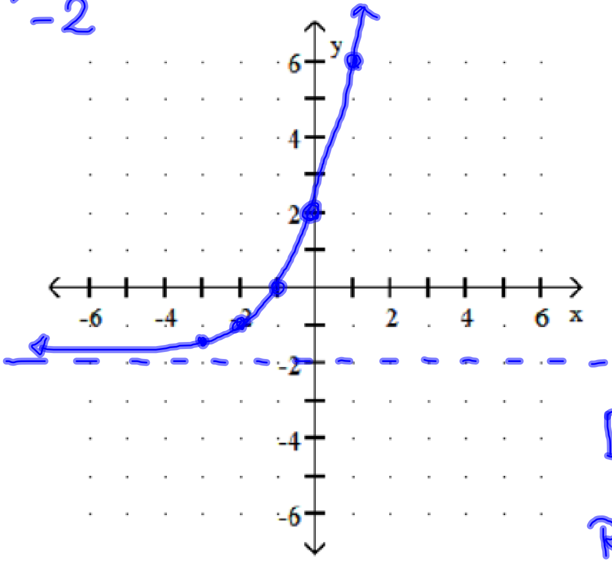
$\boxed{7}$

Graph the function.

10) $f(x) = 2^{(x+2)} - 2$.

$2^{(x+2)} - 2$

$y = -2$



$2^{x+2} - 2$

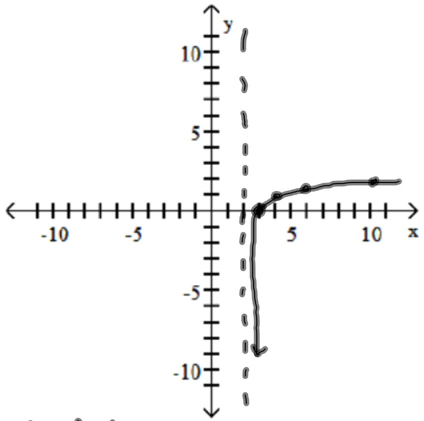
x	y
-2	$2^0 - 2 = -1$
-1	0
0	2
1	$2^3 - 2 = 6$

Domain: $(-\infty, \infty)$

Range $(-2, \infty)$

H.A. $y = -2$

11) $f(x) = \log_4 (x - 2)$



x	$\log_4(x-2)$
0	error
4	.5
3	0
6	1
10	1.5

Domain

$x-2>0$

$x>2$

V.A. $x=2$

Range $(-\infty,\infty)$

calculator

$y_1 = \frac{\ln(x-2)}{\ln(4)}$

Change of base

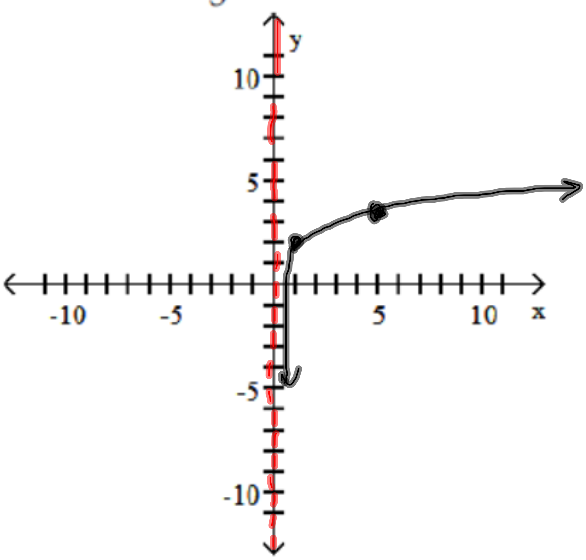
$\log_4(x-2)=y$

$4^y = x-2$

$4^y+2=x$

put in numbers for y to find x.

12) $f(x) = 2 + \log_5 x$



$x=0$

$y_1 = 2 + \frac{\ln x}{\ln 5}$

x	$2+\log_5 x$
0	error
1	$2+\log_5 1=2$
5	$2+\log_5 5=3$

Domain

$x>0$

$(0,\infty)$

Range $(-\infty,\infty)$

V.A. $x=0$

Write as the sum and/or difference of logarithms. Express powers as factors.

13) $\log_4 \sqrt{7x}$

$\log_4 (7x)^{1/2}$ $\quad 1/2 \log_4 7x$

$\frac{1}{2}\log_4 7 + 1/2 \log_4 x$ or $\frac{1}{2}(\log_4 7 + \log_4 x)$

$\frac{1}{2}\log_4 7 + \frac{1}{2}\log_4 x$

14) $\log_3 \frac{\sqrt[2]{p}\,\sqrt[5]{q}}{t^2}$

$\log_3(\sqrt[2]{p}) + \log_3(\sqrt[5]{q}) - \log_3(t^2)$

$\frac{1}{2}\log_3 p + \frac{1}{5}\log_3 q - 2\log_3 t$

Express as a single logarithm.

15) $5\log_c q - \frac{2}{3}\log_c r + \frac{1}{4}\log_c f - 3\log_c p$

$\log_c q^5 - \log_c r^{2/3} + \log_c f^{1/4} - \log_c p^3$

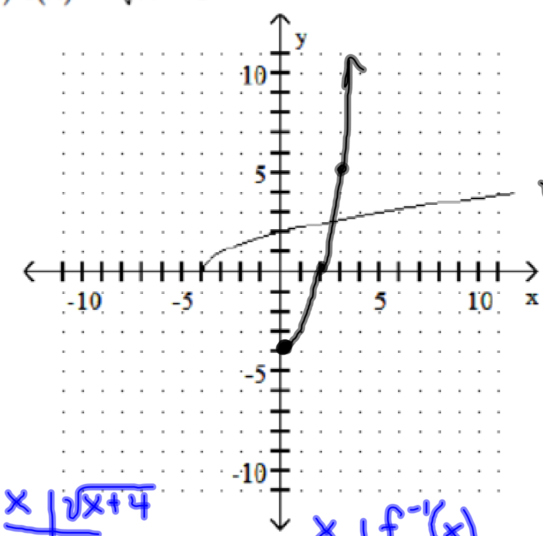
$\log_c \frac{q^5}{r^{2/3}} + \log_c \frac{f^{1/4}}{p^3}$

$\log_c \frac{q^5 \cdot f^{1/4}}{r^{2/3} \cdot p^3}$

$\log_c \frac{q^5\ \sqrt[4]{f}}{\sqrt[3]{r^2} \cdot p^3}$ $\qquad \log_c\left(\frac{q^5}{r^{2/3}}\right)\left(\frac{f^{1/4}}{p^3}\right)$

The graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1} as a dashed line or curve.

16) $f(x) = \sqrt{x + 4}$



$\sqrt{x+4}$

$x = \sqrt{y+4}$

$x^2 = y + 4$

$y = x^2 - 4$

x	$\sqrt{x+4}$
-4	0
0	2
5	3

x	$f^{-1}(x)$
0	-4
2	0
3	5

For the given functions f and g, find the requested composite function.

17) $f(x) = \sqrt{x} + 6$, $g(x) = 8x - 10$; Find $(f \circ g)(x)$.

inside ℝ

$f \circ g(x)$ $\sqrt{8x-10+6}$ $\sqrt{8x-4}$

$f(g(x))$ $\sqrt{4(2x-1)}$

$2\sqrt{2x-1}$

Replace x in f(x) with g(x)

Domain:

$8x-4 \geq 0$

$8x \geq 4$ $x \geq \frac{1}{2}$

$2x-1 \geq 0$ $x \geq \frac{1}{2}$

Decide whether the composite functions, $f \circ g$ and $g \circ f$, are equal to x.

18) $f(x) = \frac{x-2}{2}$, $g(x) = 2x + 2$

$\frac{(2x+2-2)}{2} = \frac{2x}{2} = x$

$2\left(\frac{x-2}{2}\right)+2 = \frac{2x-4}{2}+2 = x-2+2 = x$

$\frac{\cancel{2}(x-2)}{\cancel{2}} + 2$

$x-2+2 = x$

Find the inverse function of f. State the domain and range of f.

19) $f(x) = \frac{3x-2}{x+5}$

$x = \frac{3y-2}{y+5}$

$x(y+5) = 3y-2$

$xy+5x = 3y-2$

$xy-3y = -2-5x$

$y(x-3) = -2-5x$

$y = \frac{-2-5x}{x-3}$

$\boxed{f^{-1}(x) = \frac{-2-5x}{x-3}}$

denominator $x+5 \neq 0$ $x \neq -5$

$Df(x)$ $x \neq -5$

$Rf(x)$ $y \neq 3$

$Df^{-1}(x)$ $x \neq 3$

$Rf^{-1}(x)$ $y \neq -5$

⇐ denominator $x-3 \neq 0$ $x \neq 3$

Solve the equation.

20) $\log(4x) = \log 5 + \log(x-1)$

$\log_{10}(4x) = \log_{10}5 + \log_{10}(x-1)$

$\log_{10}(4x) = \log_{10}5(x-1)$

$4x = 5x-5$

$-5x \quad -5x$

$-x = -5$

$\boxed{x = 5}$

Solve the equation. Express irrational answers in exact form and as a decimal rounded to 3 decimal places.

21) $(4)^x = 6^{1 - x}$

$$4^x = 6^{1-x}$$
$$\ln 4^x = (1-x)\ln 6$$
$$x\ln 4 = (1-x)\ln 6$$
$$x\ln 4 = 1\ln 6 - x\ln 6$$
$$+x\ln 6 \quad +x\ln 6$$
$$x\ln 4 + x\ln 6 = 1\ln 6$$
$$\frac{x(\ln 4 + \ln 6)}{(\ln 4 + \ln 6)} = \frac{\ln 6}{(\ln 4+\ln 6)}$$
$$x = \frac{\ln 6}{(\ln 4 + \ln 6)} \qquad x \approx .564$$

Find the present value. Round to the nearest cent.

22) To get $25,000 after 10 years at 11% compounded semiannually

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad A = 25{,}000$$
$$25{,}000 = P\left(1 + \frac{.11}{2}\right)^{2(10)} \qquad r = .11$$
$$t = 10$$
$$n = 2$$
$$\frac{25{,}000}{\left(1 + \frac{.11}{2}\right)^{20}} = P$$
$$\frac{25{,}000}{(1.055)^{20}} \qquad \boxed{P = \$8568.23}$$

Solve the problem.

23) The half-life of silicon-32 is 710 years. If 80 grams is present now (A_0), how much will be present in 200 years? (Round your answer to three decimal places.)

Part I Find K

$$A = A_0 e^{kt}$$
$$\tfrac{1}{2}(80) = 80e^{k(710)}$$
$$\frac{40}{80} = e^{710k}$$
$$\ln .5 = 710k \cancel{\ln e} \qquad \ln e = 1$$
$$\frac{\ln(.5)}{710} = k$$

Part II

$$A = 80e^{k(200)}$$
$$A = 80e^{\left(\frac{\ln .5}{710} * 200\right)}$$
$$\Downarrow$$
$$80e \wedge (\ln(.5) \div 710 * 200)$$
$$\boxed{65.810}$$

Find the domain of the function.

24) $f(x) = \ln(7 - x)$

$$7 - x > 0$$
$$-x > -7$$
$$\boxed{x < 7} \text{ or } \boxed{(-\infty, 7)}$$

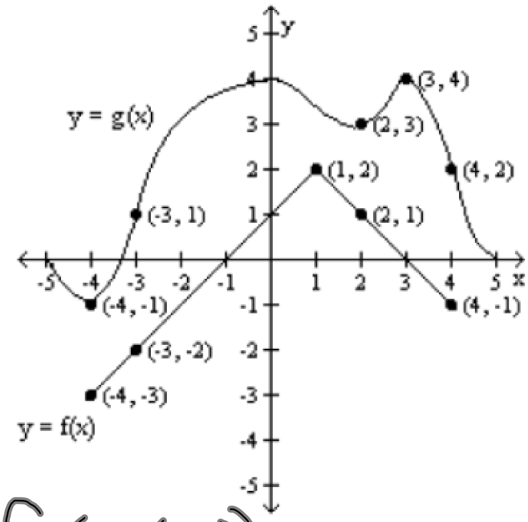
Solve the equation.

25) $\log_2(3x - 2) - \log_2(x - 5) = 4$

$$\log_2(3x-2) - \log_2(x-5) = 4$$
$$\log_2 \frac{3x-2}{x-5} = 4 \qquad \text{Domain}$$
$$2^4 = \frac{3x-2}{x-5} \qquad x > \tfrac{2}{3}$$
$$x > 5$$
$$\frac{16}{1} = \frac{3x-2}{x-5}$$
$$16(x-5) = 3x-2$$
$$16x - \cancel{80} = \cancel{3x} - 2$$
$$-3x \ \cancel{+80} \quad -\cancel{3x} \ +80$$
$$13x = 78$$
$$\boxed{x = 6}$$

Evaluate the expression using the values given in the table.

26)



f(g(-3))

$f(\underbrace{g(-3)}_{f(1)})$

$f(1) = \boxed{2}$

Find the amount that results from the investment.

27) $12,000 invested at 9% compounded quarterly after a period of 3 years

$n=4 \quad t=3$

$r=.09$

$P=12{,}000$

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$A=12{,}000\left(1+\frac{.09}{4}\right)^{4(3)}$

$A=12{,}000\left(1+\frac{.09}{4}\right)^{12}$

$\boxed{A=\$15{,}672.60}$

Solve the problem. Round your answer to three decimals.

28) How long will it take for an investment to double in value if it earns 7.25% compounded continuously?

$r=7.25\%$

$r=.0725$

$t=?$

$A=Pe^{rt}$

$2=1e^{.0725t}$

$\ln 2=\ln e^{.0725t}$

$\ln 2= .0725t \cancel{\ln e}$ (1)

$\ln 2 = .0725t$

$\frac{\ln 2}{.0725}=t$

$\boxed{9.561 \text{ yrs} \approx t}$