Precalculus

Unit 2 Review

Write each function in vertex form by completing the square. State the vertex and axis of symmetry of the graph, then graph the function. Show at least 5 points (vertex and 2 on each side).

1. $f(x) = x^2 - 6x + 8$

$f(x) - 8 + \boxed{9} = x^2 - 6x + \boxed{9}$

$f(x) + 1 = (x-3)^2$

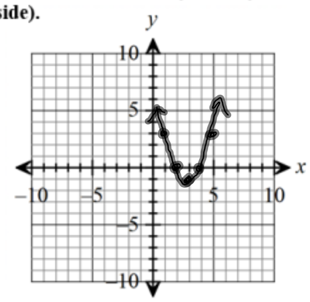
$f(x) = (x-3)^2 - 1$

h k

Vertex: $(3, -1)$

(h,k)

Axis of Symmetry: $x = 3$



2. $f(x) = -x^2 + 4x$

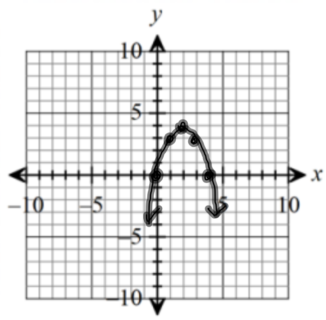
$f(x) = -(x^2 - 4x + \boxed{4})$

$f(x) + -\boxed{4} = -(x-2)^2$

$f(x) = -(x-2)^2 + 4$

Vertex: $(2, 4)$

Axis of Symmetry: $x = 2$



Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find that value using the vertex formula.

3. $f(x) = 2x^2 + 5x - 3$

$\boxed{\text{min}}$

$\frac{-b}{2a}$ $\frac{-5}{2(2)} = \frac{-5}{4}$

$f(-\frac{5}{4}) = 2(-\frac{5}{4})^2 + 5(-\frac{5}{4}) - 3$

value: -6.125 or $-\frac{49}{8}$

4. $f(x) = -\frac{1}{2}x^2 - 4x + 3$

max

$\frac{-b}{2a} = \frac{4}{2(-\frac{1}{2})} = \frac{4}{-1} = -4$

$f(-4) = -\frac{1}{2}(-4)^2 - 4(-4) + 3$

$f(-4)$

value max $= \boxed{11}$

Find the x- and y-intercepts of the graph of each quadratic function.

5. $f(x) = 6x^2 + 13x + 5$

Let $y = 0$

xint

$0 = 6x^2 + 13x + 5$

$6x^2 + 10x + 3x + 5$

$2x(3x+5) + 1(3x+5)$

$(2x+1)(3x+5)$

$2x + 1 = 0$ $3x + 5 = 0$

$\boxed{x = -\frac{1}{2}}$ $\boxed{x = \frac{-5}{3}}$

yint Let $x = 0$

$6(0)^2 + 13(0) + 5$

$\boxed{(0, 5)}$ or $\boxed{5}$

Write a quadratic function for the parabola with the given vertex that passes through the given point.

6. Vertex $(3,5)$; Passes through $(5,-3)$

$h\ k \qquad x_1\ y_1$

$y = a(x-h)^2 + k$

$y = a(x-3)^2 + 5$

$-3 = a(5-3)^2 + 5$

$-8 = a(2)^2$

$-8 = 4a$

$-2 = a \Rightarrow \boxed{y = -2(x-3)^2 + 5}$

7. Vertex $(-2,1)$; Passes through $(1,4)$

$h\ k \qquad x_1\ y_1$

$y = a(x-h)^2 + k)$

* $y = a(x+2)^2 + 1$

$4 = a(1+2)^2 + 1$

$4 = 9a + 1$

$3 = 9a$

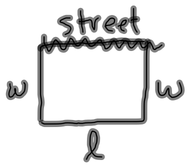
$\frac{3}{9} = a$

$\frac{1}{3} = a$

$\boxed{y = \frac{1}{3}(x+2)^2 + 1}$

For each problem, write an appropriate quadratic model, then use it to answer the question.

8. A developer wants to enclose a rectangular lot that borders a city street for parking. The developer has 864 feet of fencing and is not going to fence the side along the street.
   a) What dimensions should the lot be in order to enclose the maximum area?



864 ft = perimeter

$P = l + 2w$

$864 = l + 2w$

$864 - 2w = l$

$A = l\,w$

$A = (864 - 2w)w$

$A = 864w - 2w^2$

$A = -2w^2 + 864w$

width = 216

length

$864 - 2(216) = 432$

max $w = \frac{-b}{2a}$ $\quad \frac{-864}{2(-2)} \Rightarrow \frac{-864}{-4} \Rightarrow 216$ ft

$\boxed{\text{dimensions } 216 \text{ ft} \times 432 \text{ ft}}$

   b) What is the maximum area that can be enclosed?

$A = l \cdot w$

216×432

$\boxed{93{,}312 \text{ ft}^2}$

9. Sally is starting her own business selling glow-in-the-dark sparkly unicorn kitten lamps. The price p (in dollars) and the number of lamps sold, x, obey the demand equation $p=-\frac{1}{6}x+60$.

a) Express the revenue R as a function of x. (Remember that $R=xp$.)

$R=xp \qquad R(x)=x\left(-\frac{1}{6}x+60\right)$

$\boxed{R(x)=-\frac{1}{6}x^2+60x}$

b) How many lamps does Sally need to sell to maximize revenue?

$\frac{-b}{2a} \qquad \frac{-60}{2\left(-\frac{1}{6}\right)} \qquad \frac{-60}{-\frac{2}{6}} \qquad -60 \div \frac{-2}{6}$

$\frac{-60}{1}\cdot\frac{6}{-2}=\boxed{180}$

c) What is Sally's maximum revenue?

$R(180)=-\frac{1}{6}(180)^2+60(180)$

$\boxed{\$5{,}400}$

d) What price should Sally charge to maximize revenue?

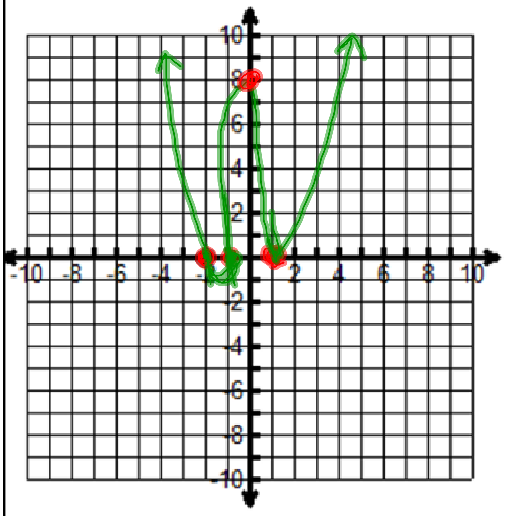
$p(180)=-\frac{1}{6}(180)+60$

$\boxed{\$30}$

For the polynomial function, do the following:
a.) List each real zero and its multiplicity.
b.) Determine whether the graph crosses or touches the *x*-axis at each *x*-intercept.
c.) Determine the end behavior
d.) SKETCH THE GRAPH!

10. $f(x)=(x+2)^3(x-1)^2(x+1)$

$x+2=0 \quad x-1=0 \quad x+1=0$



Zeros	Multiplicity	Touch/Cross
-2	3	Cross
1	2	touch
-1	1	cross

$\lim_{x\to-\infty} f(x)=\infty \qquad \lim_{x\to\infty} f(x)=\infty$

x^6

$y\text{int}=(0+2)^3(0-1)^2(0+1)$

$8(1)(1)$

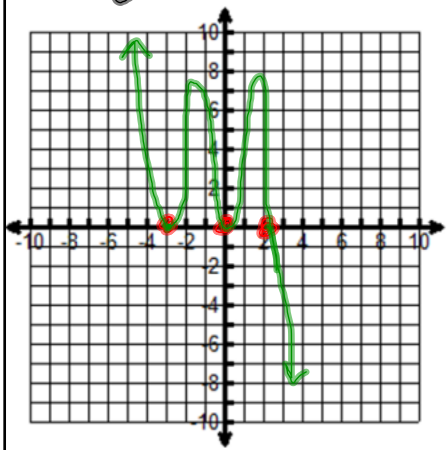
$y\text{int}=8$

11. $f(x) = -2x^2(x-2)(x+3)^2$

$-2x^2 = 0$ $x - 2 = 0$ $x + 3 = 0$

$x = 0$ $x = 2$ $x = -3$

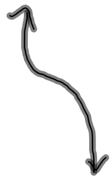
yint: $(0,0)$



Zeros	Multiplicity	Touch/Cross
0	2	Touch
2	1	Cross
-3	2	Touch

$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$

End Behavior $-2x^5$



Divide using long division.

12. $(20x^3 + 29x^2 - 16x - 8) \div (5x - 4)$

$4x^2 + 9x + 4 + \frac{8}{5x-4}$

$5x - 4 \overline{)\, 20x^3 + 29x^2 - 16x - 8}$

$\boxed{4x^2}(5x-4)$ $-20x^3 + 16x^2$

$\boxed{9x}(5x-4)$ $45x^2 - 16x$

$-45x^2 + 36x$

$\boxed{4}(5x-4)$ $20x - 8$

$-20x + 16$

8

Use synthetic division to find the quotient and the remainder. Determine whether the divisor is a factor of the dividend.

13. $\dfrac{x^4 - 3x^3 - 2x + 6}{x - 4}$

$x - 4 = 0$

$x = 4$

	x^4	x^3	x^2	x^1	x^0
4	1	-3	0	-2	6
		4	4	16	56
MULT	1	1	4	14	62

$x^3 + x^2 + 4x + 14 + \frac{62}{x-4}$

List the potential rational zeros of the polynomial function. Do not find the zeros.

14. $f(x) = -3x^3 + 5x^2 - 4x + 12$

q p

p ± 12 factors 12 $\frac{1\ 2\ 3\ 4\ 6\ 12}{1,3}$

q ± 3 factors 3

$\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \cancel{\frac{3}{3}}, \pm 4, \pm \frac{4}{3}, \pm 6$

$\cancel{\frac{6}{3}}$ ± 12, $\cancel{\frac{12}{3}}$

Find the remaining zeros of the function. Then, form a polynomial $f(x)$, with real coefficients having the given degree and zeros. Leave your answer in factored form.

15. Degree 4; Zeros: -3, 1, and $2 - i$

$x = -3$ $x = 1$ $x = 2 - i$ $x = 2 + i$

$x + 3 = 0$ $x - 1 = 0$ $x - 2 + i = 0$ $x - 2 - i = 0$

$\boxed{(x+3)(x-1)(x-2+i)(x-2-i)}$

16. Degree: 5; Zeros: 2, $3i$, and $-1 + 4i$

$x = 2$ $x = 3i$ $x = -3i$ $x = -1 + 4i$ $x = -1 - 4i$

$x - 2 = 0$ $x - 3i = 0$ $x + 3i = 0$ $x + 1 - 4i = 0$ $x + 1 + 4i = 0$

$\boxed{(x-2)(x-3i)(x+3i)(x+1-4i)(x+1+4i)}$

Find all the complex zeros of the function and write the polynomial as a product of linear factors.

17. $f(x) = 3x^3 - 11x^2 + 2x + 2$

find real zeros $\pm 2, \pm 1, \pm \frac{1}{3}, \pm \frac{2}{3}$

$-\frac{1}{3}$	3	-11	2	2
		-1	+4	-2
	3	-12	6	0

$3(x^2 - 4x + 2)$

$x = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$ $x = 2 + \sqrt{2}$, $x = 2 - \sqrt{2}$

$x = \frac{4 \pm \sqrt{8}}{2} \Rightarrow \frac{\cancel{4} \pm \cancel{2}\sqrt{2}}{\cancel{2}} \Rightarrow \boxed{2 \pm \sqrt{2}}$

$\boxed{3(x + \frac{1}{3})(x - 2 - \sqrt{2})(x - 2 + \sqrt{2})}$

18. $f(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8$

-2	2	3	6	12	-8
		-4	2	-16	8
	2	-1	8	-4	0

$\frac{1}{2}$	2	-1	8	-4
		1	0	4
	2	0	8	0

$\boxed{\begin{array}{l} x = -2 \\ x = \frac{1}{2} \\ x = 2i \\ x = -2i \end{array}}$

$2x^2 + 8 = 0$ $2(x^2 + 4) = 0$

$2x^2 = -8$

$x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$\boxed{2(x+2)(x - \frac{1}{2})(x - 2i)(x + 2i)}$

or $(x+2)(2x-1)(x-2i)(x+2i)$

Find all the complex zeros of the function and write the polynomial as a product of linear factors.

19. $f(x) = x^3 + 11x^2 + 36x + 26$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 36 & 26 \\ & & -1 & -10 & -26 \\ \hline & 1 & 10 & 26 & 0 \end{array}$$

$x^2 + 10x + 26$

$x = \dfrac{-10 \pm \sqrt{100 - 4(1)(26)}}{2(1)}$

$\dfrac{-10 \pm \sqrt{-4}}{2}$

$\dfrac{-10 \pm 2i}{2} \Rightarrow -5 \pm i$

$x = -1$, $x = -5 + i$ $x = -5 - i$

$\boxed{(x+1)(x+5-i)(x+5+i)}$

Use the given zero to find the remaining zeros (REAL AND IMAGINARY) of the function.

20. $f(x) = x^4 - 2x^3 + 13x^2 - 32x - 48$; zero: $4i$

$\boxed{\begin{array}{l} x = 4i \\ x = -4i \end{array}}$

$(x - 4i)(x + 4i)$

$x^2 + 16$

$$\begin{array}{r|l} & x^2 - 2x - 3 \\ x^2 + 16 & x^4 - 2x^3 + 13x^2 - 32x - 48 \\ x^2(x^2+16) & -x^4 \qquad\quad -16x^2 \\ \hline -2x(x^2+16) & \quad -2x^3 - 3x^2 - 32x \\ & \quad +2x^3 \qquad\quad +32x \\ \hline \boxed{-3}(x^2+16) & \qquad\quad -3x^2 \quad -48 \\ & \qquad\quad +3x^2 \quad +48 \\ \hline & \qquad\qquad\qquad 0 \end{array}$$

$x^2 - 2x - 3$

$(x-3)(x+1)$

$\boxed{x = 4i \quad x = -4i \quad x = 3 \quad x = -1}$