Pre-calculus: 2.1A notes Factoring review and u-substitution

Factoring: The reverse of multiplying. It means figuring out what you would multiply together to get a polynomial, and writing the polynomial as the product of several factors (writing it as a multiplication problem).

Greatest Common Factor (GCF): The monomial with the largest possible coefficient and the variables with the largest possible exponents that divides evenly into every term of the polynomial.

Prime Polynomial: A polynomial that cannot be factored.

Factoring Out a Common Factor:
1. Find the GCF.
2. Use the distributive property in reverse to "factor out" the GCF:
3. Write the GCF outside a set of parentheses. Inside the parentheses, write what you are left with when you *divide* the original terms by the GCF. **Note:** If the GCF is the same as one of the terms of the polynomial, there will be a 1 left inside the parentheses.
4. When the leading coefficient is negative, factor out a common factor with a negative coefficient.

Examples: Factor the following expressions.

a) x^2+3x

$x(x+3)$

b) $-2y+6$

$-2(y-3)$

Factoring by Grouping (4 Terms):

1. Factor out any common factors from all four terms first.
2. Look at the first two terms and the last two terms of the polynomial separately.
3. Factor out the GCF from the first two terms, write a plus sign (or a minus sign if the GCF on the last two terms is negative), then factor out the GCF from the last two terms.
4. You should have the same thing left in both sets of parentheses after you take out the GCFs. Factor out this common binomial factor from the two groups.

Examples: Factor the following expressions.

a) $x^3-4x^2+3x-12$

$x^2(x-4)+3(x-4)$

$(x-4)(x^2+3)$

b) $mp+mq+np+nq$

$m(p+q)+n(p+q)$

$(m+n)(p+q)$

Factoring a Trinomial of the Form $ax^2 + bx + c$ by Grouping:

1. Always check for a GCF first! If there is a GCF, factor it out.
2. Multiply $\boldsymbol{a \cdot c}$.
3. Find two numbers that multiply to your answer $(\boldsymbol{a \cdot c})$ and add to $\boldsymbol{b}$.
4. Rewrite the middle term $\boldsymbol{bx}$ as $\mathbf{1st\,\#} \cdot x + \mathbf{2nd\,\#} \cdot x$
5. Factor the resulting polynomial by grouping.
6. If there are no numbers that multiply to $\boldsymbol{a \cdot c}$ and add to $\boldsymbol{b}$, the polynomial is prime.

Examples: Factor the following polynomials.

a) $x^2 + 11x + 30$

$(x+5)(x+6)$

b) $m^2 - 8m + 12$

$(m-6)(m-2)$

c) $q^2 - q - 56$

$(q+7)(q-8)$

d) $2y^2 - 34y + 120$

$2(y^2 - 17y + 60)$

$\boxed{2(y-5)(y-12)}$

e) $9h^2 + 9h + 2$

mult ac	add
18	9
1 · 18	19
2 · 9	11
3 · 6	9

$9h^2 + 3h + 6h + 2$

$3h(3h+1) + 2(3h+1)$

$\boxed{(3h+1)(3h+2)}$

f) $2z^2 - 11z + 12$

24	-11
-8, -3	

$2z^2 - 8z - 3z + 12$

$2z(z-4) - 3(z-4)$

$(z-4)(2z-3)$

Factoring a Difference of Squares:

- A polynomial of the form $A^2 - B^2$ is called a ***difference of squares.***
- Differences of squares always factor as follows: $A^2 - B^2 = (A+B)(A-B)$

★ This only works if ***both terms are perfect squares*** ***and*** ***you are subtracting.*** Don't forget to check for a GCF first!

Examples: Factor the following polynomials.

a) $x^2 - 25$

$(x-5)(x+5)$

b) $m^2 - 81$

$(m+9)(m-9)$

c) $w^2 + 36$

prime

Factoring Sum and Difference of Cubes:

- A polynomial of the form $A^3 - B^3$ *or* $A^3 + B^3$ is called a ***difference or sum of cubes.***
- Differences or sum of cubes always factor as follows: $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$ and

$A^3 + B^3 = (A \underset{S}{+} B)(A^2 \underset{O}{-} AB \underset{AP}{+} B^2)$

★ This only works if ***both terms are perfect cubes.*** Don't forget to check for a GCF first!

a) $x^3 - 125$

$a = x$ $\quad b = 5$

$a^3 = x^3$ $\quad b^3 = 125$

$(x-5)(x^2+5x+25)$

b) $m^3 - 27$

$a = m$ $\quad b = 3$

$(m-3)(m^2+3m+9)$

c) $w^3 + 64$

$a = w$ $\quad b = 4$

$(w+4)(w^2-4w+16)$

Equations Quadratic in Form

An equation is ***quadratic in form*** if it can be written as $au^2 + bu + c = 0$, where $a \neq 0$ and u is a variable expression.

Solve Equations Quadratic in Form

1. Check to see if the equation is quadratic in form. If it is quadratic in form, the equation will have two variable expressions, and one will be the square of the other.
2. If the equation is quadratic in form, use "u-substitution." Let u = variable expression of the 2nd term.
3. Solve the equation for "u" using quadratic methods (factoring, square root principle, completing the square, quadratic formula).
4. Plug the substitution back in for u and solve for the original variable. (Don't forget the ± if you take an even root of both sides).
5. Check for extraneous solutions. (If you raise both sides of the equation to an even power, there may be extraneous solutions.)

Examples:

a) $x^4 - 10x^2 + 25 = 0$

$u = x^2$

$u^2 = x^4$

$u^2 - 10u + 25 = 0$

$(u-5)(u-5) = 0$

$u - 5 = 0 \quad u - 5 = 0$

$u = 5$

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$\boxed{x = \pm\sqrt{5}}$

b) $2y^6 - 7y^3 + 3 = 0$

$u = y^3$

$2u^2 - 7u + 3 = 0$

$6 \mid -7$

$-1 \cdot -6$

$2u^2 - 1u - 6u + 3$

$u(2u-1) - 3(2u-1)$

$(2u-1)(u-3) = 0$

$2u - 1 = 0 \quad u - 3 = 0$

$u = \frac{1}{2} \quad u = 3$

$y^3 = \frac{1}{2} \quad y^3 = 3$

$\boxed{y = \sqrt[3]{\frac{1}{2}}} \quad y = \sqrt[3]{3}$

c) $(2z+5)^2 - 4(2z+5)^1 = 1$

$u = 2z + 5$

$u^2 - 4u - 1 = 0$

$u = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$

$u = \frac{4 \pm 2\sqrt{5}}{2} \quad u = 2 \pm 1\sqrt{5}$

$2z + 5 = 2 + \sqrt{5} \quad 2z + 5 = 2 - \sqrt{5}$

$2z = -3 + \sqrt{5} \quad -5 \; -5$

$\boxed{z = \frac{-3+\sqrt{5}}{2}} \quad \boxed{z = \frac{-3-\sqrt{5}}{2}}$

d) $(x^2-1)^2 - (x^2-1) - 2 = 0$

$u = x^2 - 1$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$x^2 - 1 = 2 \quad x^2 - 1 = -1$

$x^2 = 3 \quad x^2 = 0$

$\boxed{x = \pm\sqrt{3}} \quad \boxed{x = 0}$

e) $n - 3\sqrt{n} - 4 = 0$

$u = \sqrt{n}$

$u^2 = n$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4 \quad u = -1$

$\sqrt{n} = 4 \quad \sqrt{n} = -1$

$\boxed{n = 16} \quad \cancel{n = 1}$

f) $x^{2/3} + 6x^{1/3} + 8 = 0$

$u = x^{\frac{1}{3}}$

$u^2 + 6u + 8 = 0$

$(u+4)(u+2) = 0$

$u = -4 \quad u = -2$

$x^{\frac{1}{3}} = -4 \quad x^{\frac{1}{3}} = -2$

$\boxed{x = -64} \quad \boxed{x = -8}$

g) $2m^{-2} + m^{-1} = 15$

$m^{-1} = \frac{1}{m}$

$u = m^{-1}$

$2u^2 + u - 15 = 0$

$-30 \mid 1$

$+6 \cdot -5$

$2u^2 + 6u - 5u - 15 = 0$

$2u(u+3) - 5(u+3) = 0$

$(u+3)(2u-5) = 0$

$u = -3 \quad 2u - 5 = 0$

$\frac{1}{m} = \frac{-3}{1} \quad u = \frac{5}{2}$

$1 = -3m \quad \frac{1}{m} = \frac{5}{2}$

$\boxed{-\frac{1}{3} = m} \quad 2 = 5m$

$\boxed{\frac{2}{5} = m}$

h) $\frac{1}{(x-1)^2} + \frac{4}{x-1} = 12$

$u = \frac{1}{x-1}$

$u^2 + 4u - 12 = 0$

$(u+6)(u-2) = 0$

$u = -6 \quad u = 2$

$\frac{1}{x-1} = \frac{-6}{1} \quad \frac{1}{x-1} = \frac{2}{1}$

$1 = -6x + 6 \quad 1 = 2x - 2$

$-5 = -6x \quad 3 = 2x$

$\boxed{\frac{5}{6} = x} \quad \boxed{\frac{3}{2} = x}$