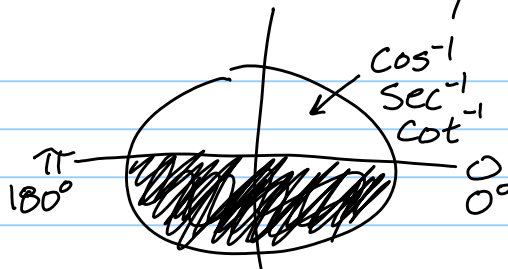
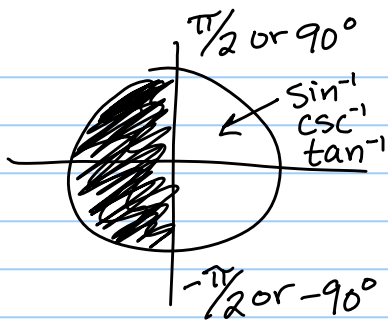


Precalculus - Unit 6 Review Key



$$1. \tan^{-1}(0) = \boxed{0}$$

$$\frac{y}{x} = 0 \quad \uparrow$$

$$2. \arcsin(\sqrt{3}/2) = \boxed{\pi/3}$$

$$\uparrow$$

$$3. \sec^{-1}(-\sqrt{2})$$

$$= \cos^{-1}(-\frac{1}{\sqrt{2}})$$

$$= \cos^{-1}(-\frac{\sqrt{2}}{2}) = \boxed{3\pi/4}$$

$$\uparrow$$

$$4. \arctan(-\sqrt{3}) = \boxed{-\pi/3}$$

$$\frac{y}{x} = -\frac{\sqrt{3}/2}{1/2} \quad \uparrow$$

$$5. \sec^{-1}(\frac{2\sqrt{3}}{3})$$

$$= \sec^{-1}(\frac{2}{\sqrt{3}})$$

$$= \cos^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\pi/6}$$

$$\uparrow$$

$$6. \cos^{-1}(-1) = \boxed{\pi} \quad \uparrow$$

$$7. \cot^{-1}(0) = \boxed{\pi/2}$$

$$\frac{x}{y} = 0 \quad \uparrow$$

$$8. \arcsin(-\sqrt{2}/2) = \boxed{-\pi/4}$$

$$\uparrow$$

$$9. \operatorname{arccsc}(-2)$$

$$= \arcsin(-\frac{1}{2}) = \boxed{-\pi/6}$$

$$\uparrow$$

$$10. \cos^{-1}(-\frac{1}{2}) = \boxed{120^\circ}$$

$$\uparrow$$

$$11. \arctan(1) = \boxed{45^\circ}$$

$$\frac{y}{x} = 1 \quad \frac{\sqrt{2}/2}{\sqrt{2}/2} \quad \uparrow$$

$$12. \cot^{-1}(-\sqrt{3}) = \boxed{150^\circ}$$

$$\frac{x}{y} = -\frac{\sqrt{3}/2}{1/2} \quad \uparrow$$

$$13. \arcsin(1/2) = \boxed{30^\circ}$$

$$\uparrow$$

$$14. \operatorname{arcsec}(2)$$

$$= \arccos(1/2) = \boxed{60^\circ} \quad \uparrow$$

$$15. \cos^{-1}(1) = \boxed{0^\circ}$$


$$\uparrow$$

$$16. \arctan(-\frac{\sqrt{3}}{3})$$

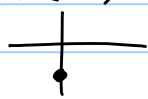
$$= \arctan(-\frac{1}{\sqrt{3}}) = \boxed{-30^\circ}$$

$$\frac{y}{x} = -\frac{1/2}{\sqrt{3}/2} \quad \uparrow$$

17. $\csc^{-1}(-\sqrt{2})$
 $= \sin^{-1}(-\frac{1}{\sqrt{2}})$
 $= \sin^{-1}(-\frac{\sqrt{2}}{2}) = \boxed{-45^\circ}$




18. $\arcsin(-1) = \boxed{-90^\circ}$

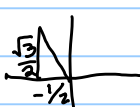


19. $\arccos(\cos(\frac{11\pi}{6}))$
 $= \arccos(\frac{\sqrt{3}}{2}) \leftarrow \text{in QI}$
 $= \boxed{\frac{\pi}{6}}$

20. $\tan^{-1}(\sin(\frac{\pi}{2}))$
 $= \tan^{-1}(1) = \boxed{\frac{\pi}{4}}$
 $\frac{y}{x} = 1$

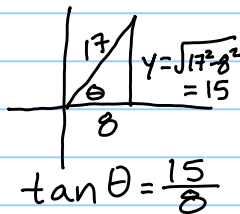


21. $\cot(\sec^{-1}(-2))$
 $= \cot(\cos^{-1}(-\frac{1}{2}))$
 $= \cot(2\pi/3)$
 $= \boxed{-1/\sqrt{3} \text{ or } -\sqrt{3}/3}$



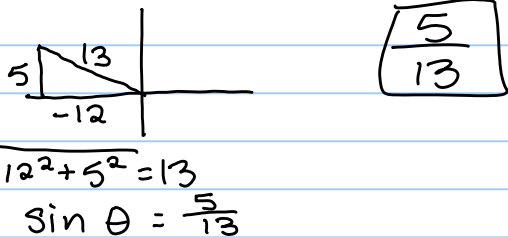
22. $\tan(\cos^{-1}(8/17)) = \boxed{\frac{15}{8}}$

What is $\tan \theta$? $\cos \theta = 8/17$
 θ is between 0 & π



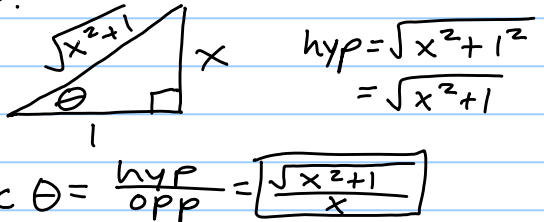
23. $\sin(\text{arccot}(-12/5))$

What is $\sin \theta$? $\cot \theta = -12/5$
 θ is between 0 & π



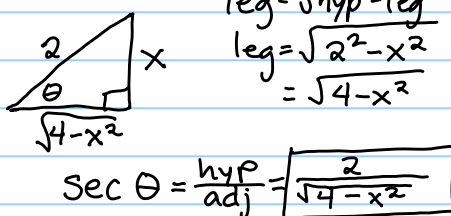
24. $\csc(\tan^{-1}(x))$

What is $\csc \theta$? $\tan \theta = \frac{x}{1}$ $\frac{\text{opp}}{\text{adj}}$



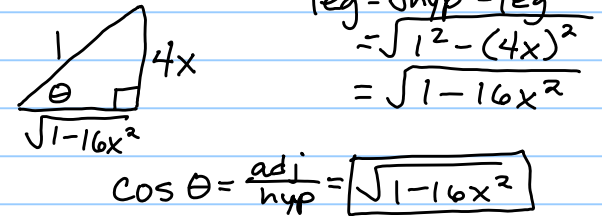
25. $\sec(\arcsin(x/2))$

What is $\sec \theta$? $\sin \theta = \frac{x}{2}$ $\frac{\text{opp}}{\text{hyp}}$




26. $\cos(\sin^{-1}(4x))$

What is $\cos \theta$? $\sin \theta = \frac{4x}{1}$ $\frac{\text{opp}}{\text{hyp}}$



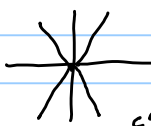
27. $2 \cos \theta - 1 = 0$
 $\cos \theta = \frac{1}{2}$
 $\theta = \boxed{\frac{\pi}{3} + 2\pi k \text{ or } \theta = \frac{5\pi}{3} + 2\pi k}$




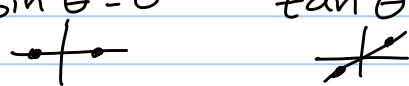
28. $2 \cos^2 \theta + \cos(2\theta) = 0$
 $2 \cos^2 \theta + 2 \cos^2 \theta - 1 = 0$
 $4 \cos^2 \theta - 1 = 0$
 $\sqrt{\cos^2 \theta} = \pm \sqrt{\frac{1}{4}}$
 $\cos \theta = \pm \frac{1}{2}$

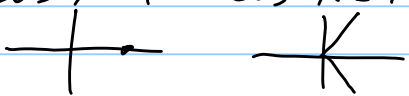
$\theta = \boxed{\frac{\pi}{3} + \pi k \text{ or } \theta = \frac{2\pi}{3} + \pi k}$

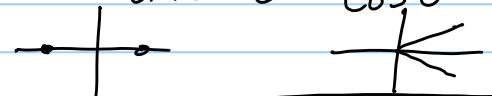
You can also write as 4 angles with $2\pi k$




29. $\tan\left(\frac{\theta}{4}\right) = \frac{\sqrt{3}}{3}$
 $\tan\left(\frac{\theta}{4}\right) = \frac{1}{\sqrt{3}} \quad \frac{1/2}{\sqrt{3}/2}$ 
 $4\left(\frac{\theta}{4}\right) = (30^\circ + 180^\circ k)(4)$
 $\theta = 120^\circ + 720^\circ k$


30. $\sin \theta \tan \theta - \sin \theta = 0$
 $\sin \theta (\tan \theta - 1) = 0$
 $\sin \theta = 0 \quad \tan \theta = 1$

 $\theta = 0^\circ + 180^\circ k \text{ or } \theta = 45^\circ + 180^\circ k$

31. $3 \cos x = 2 \cos^2 x + 1$
 $2 \cos^2 x - 3 \cos x + 1 = 0$
 $u = \cos x$
 $2u^2 - 3u + 1 = 0$ $2(1) = 2 \leftarrow \text{mult to 2}$
 $2u^2 - 2u - u + 1 = 0$ add to -3
 $2u(u-1) - 1(u-1) = 0$ $-2 \& -1$
 $(u-1)(2u-1) = 0$
 $u = 1 \quad u = 1/2$
 $\cos x = 1 \quad \cos x = 1/2$

 $x = 360^\circ k \text{ or } x = 60^\circ + 360^\circ k$
 $\text{or } x = 300^\circ + 360^\circ k$

32. $\sqrt{3} \sin \theta = \sin(2\theta)$
 $\sqrt{3} \sin \theta = 2 \sin \theta \cos \theta$
 $0 = 2 \sin \theta \cos \theta - \sqrt{3} \sin \theta$
 $0 = \sin \theta (2 \cos \theta - \sqrt{3})$
 $\sin \theta = 0 \quad \cos \theta = \sqrt{3}/2$

 $\left\{ 0, \pi, \pi/6, 11\pi/6 \right\}$

33. $1 + \sin x = -\cos x$
 $(1 + \sin x)^2 = (-\cos x)^2$
 $1 + 2 \sin x + \sin^2 x = \cos^2 x$
 $1 + 2 \sin x + \sin^2 x = 1 - \sin^2 x$
 $-1 \quad \quad \quad + \sin^2 x \quad -1 + \sin^2 x$
 $2 \sin x + 2 \sin^2 x = 0$
 $2 \sin x (1 + \sin x) = 0$
 $\sin x = 0 \quad \sin x = -1$

 $x = 0 \quad x = \pi \quad x = 3\pi/2$

squared both sides \rightarrow check answers!
 $1 + \sin 0 \stackrel{?}{=} -\cos 0 \rightarrow 1 + 0 \neq -1$
 $1 + \sin \pi \stackrel{?}{=} -\cos \pi \rightarrow 1 + 0 = -(-1) \checkmark$
 $1 + \sin(3\pi/2) \stackrel{?}{=} -\cos(3\pi/2) \rightarrow 1 + (-1) = -(-0) \checkmark$
 $\left\{ \pi, \frac{3\pi}{2} \right\}$

34. $\sin x \cos\left(\frac{\pi}{3}\right) - \cos x \sin\left(\frac{\pi}{3}\right) = \frac{1}{2}$
 $\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$
 $x - \frac{\pi}{3} = \frac{\pi}{6} + 2\pi k \text{ or } x - \frac{\pi}{3} = \frac{5\pi}{6} + \frac{2\pi}{k}$
 $+ \frac{\pi}{3} \quad + \frac{\pi}{3} \quad \quad \quad + \frac{\pi}{3} \quad + \frac{\pi}{3}$
 $\left\{ \frac{\pi}{2}, \frac{7\pi}{6} \right\}$ 

35. $\frac{\cos(3\theta)}{\sin(3\theta)} = \frac{\sqrt{3} \sin(3\theta)}{\sin(3\theta)}$

$\cot(3\theta) = \sqrt{3}$ $\frac{\sqrt{3}/2}{1/2} = \frac{x}{y}$

$\frac{3\theta}{3} = \frac{30^\circ + 180^\circ k}{3}$

$\theta = 10^\circ + 60^\circ k$

$\{10^\circ, 70^\circ, 130^\circ, 190^\circ, 250^\circ, 310^\circ\}$

36. $0 = \sin(2x) - \sin^2(2x) + \cos^2(2x)$

$0 = \sin(2x) - \sin^2(2x) + 1 - \sin^2(2x)$

$0 = -2\sin^2(2x) + \sin(2x) + 1$

$2\sin^2(2x) - \sin(2x) - 1 = 0$

$u = \sin(2x)$

$2u^2 - u - 1 = 0$ $2(-1) = -2$ \leftarrow mult to -2

$2u^2 - 2u + u - 1 = 0$ \leftarrow add to -1: -2 & 1

$2u(u-1) + 1(u-1) = 0$

$(u-1)(2u+1) = 0$

$u = 1 \quad u = -1/2$

$\sin(2x) = 1 \quad \sin(2x) = -1/2$

$\frac{2x}{2} = \frac{90^\circ + 360^\circ k}{2}$ or $\frac{2x}{2} = \frac{210^\circ + 360^\circ k}{2}$ or $\frac{2x}{2} = \frac{330^\circ + 360^\circ k}{2}$

$x = 45^\circ + 180^\circ k$ or $x = 105^\circ + 180^\circ k$ or $x = 165^\circ + 180^\circ k$

$\{45^\circ, 225^\circ, 105^\circ, 285^\circ, 165^\circ, 345^\circ\}$

37. Draw the angle in standard position & label the terminal side with the adjacent side measurement.

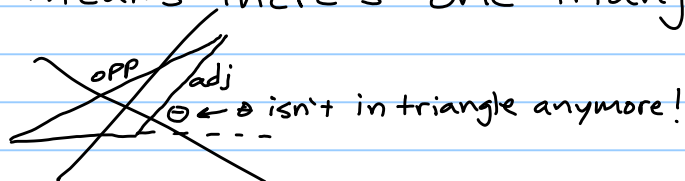
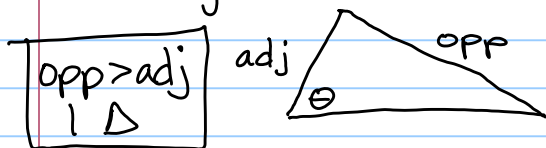
Don't label the bottom or draw the opposite side yet.

To figure out how many triangles,

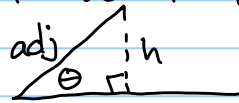
Think, "Where will the opposite side fit?"

For acute angles

Case 1: If the opposite side is longer than the adjacent side, it only fits off to the right. (If you put it to the left, you'd no longer have θ .) That means there's one triangle



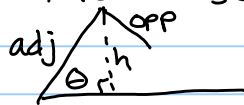
If the opposite side isn't longer than the adjacent side, draw & find the height of the triangle.



$$\sin \theta = \frac{h}{adj} \rightarrow h = adj \sin \theta$$

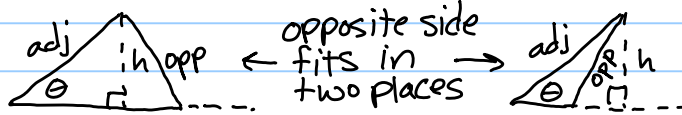
Case 2: If the opposite side is shorter than the height, it's too short to touch the base, & you can't make a triangle.

opp < height
no Δ



Case 3: If the opposite side has a length between the height & the adjacent side, then it will fit between them or to the right, so you can make two triangles.

opp between height & adj
2 Δ s



For obtuse angles

Case 1: If the opposite side is longer than the adjacent side, it's long enough to touch the base. There's one triangle.

opp > adj
1 Δ

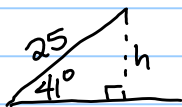


Case 2: If the opposite side is shorter than the adjacent side, it will be too short to touch the base, so no triangle will be possible.

Obtuse angle
opp < adj
no Δ



38. $\gamma = 41^\circ$ $b = 25$ $c = 17$
adj opp

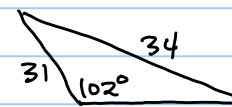


$$\sin 41^\circ = \frac{h}{25}$$

$$h = 25 \sin 41^\circ \approx 16.4$$

opp btwn height & adj
2 Δ s

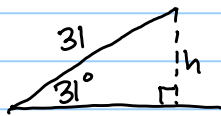
39. $\beta = 102^\circ$ $b = 34$ $c = 31$
opp adj



opp > adj

1 Δ

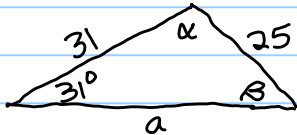
48. $\gamma = 31^\circ$ $b = 31$ $c = 25$



$$\sin 31^\circ = \frac{h}{31}$$

$$h = 31 \sin 31^\circ \approx 16.0$$

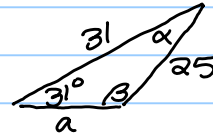
opp btwn h & adj
2 Δ s



$$\frac{\sin 31^\circ}{25} = \frac{\sin \beta}{31}$$

$$\sin \beta = \frac{31 \sin 31^\circ}{25}$$

$$\sin \beta \approx 0.6386$$



$$\beta = \sin^{-1}(0.6386)$$

$$\boxed{\beta \approx 39.7^\circ}$$

$$\alpha = 180^\circ - (31^\circ + 39.7^\circ)$$

$$\boxed{\alpha \approx 109.3^\circ}$$

$$\beta = 180^\circ - 39.7^\circ$$

$$\boxed{\beta \approx 140.3^\circ}$$

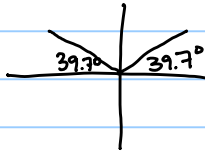
$$\alpha = 180^\circ - (31^\circ + 140.3^\circ)$$

$$\boxed{\alpha \approx 8.7^\circ}$$

$$\frac{\sin 31^\circ}{25} = \frac{\sin 109.3^\circ}{a}$$

$$\frac{a \sin 31^\circ}{\sin 31^\circ} = \frac{25 \sin 109.3^\circ}{\sin 31^\circ}$$

$$\boxed{a \approx 45.8}$$

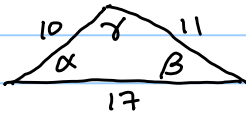


$$\frac{\sin 31^\circ}{25} = \frac{\sin 8.7^\circ}{a}$$

$$\frac{a \sin 31^\circ}{\sin 31^\circ} = \frac{25 \sin 8.7^\circ}{\sin 31^\circ}$$

$$\boxed{a \approx 7.3}$$

49. $a = 11$ $b = 10$ $c = 17$



$$17^2 = 11^2 + 10^2 - 2(11)(10) \cos \gamma$$

$$289 = 221 - 220 \cos \gamma$$

$$\frac{68}{-220} = \frac{-220 \cos \gamma}{-220}$$

$$\cos \gamma \approx -0.3091$$

$$\boxed{\gamma \approx 108.0^\circ}$$

$$\frac{\sin 108.0^\circ}{17} = \frac{\sin \alpha}{11}$$

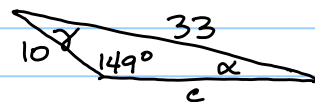
$$\sin \alpha = \frac{11 \sin 108.0^\circ}{17} \approx 0.6154$$

$$\boxed{\alpha \approx 38.0^\circ}$$

$$\beta = 180^\circ - (108.0^\circ + 38.0^\circ)$$

$$\boxed{\beta \approx 34.0^\circ}$$

50. $\beta = 149^\circ$ $a = 10$ $b = 33$
opp > adj \Rightarrow 1 Δ



$$\frac{\sin 149^\circ}{33} = \frac{\sin \alpha}{10}$$

$$\sin \alpha = \frac{10 \sin 149^\circ}{33} \approx 0.1561$$

$$\boxed{\alpha \approx 9.0^\circ}$$

$$\gamma = 180^\circ - (149^\circ + 9.0^\circ)$$

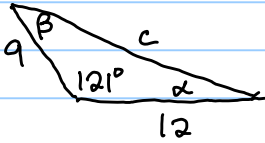
$$\boxed{\gamma \approx 22.0^\circ}$$

$$\frac{\sin 149^\circ}{33} = \frac{\sin 22.0^\circ}{c}$$

$$\frac{c \sin 149^\circ}{\sin 149^\circ} = \frac{33 \sin 22.0^\circ}{\sin 149^\circ}$$

$$\boxed{c \approx 24.0}$$

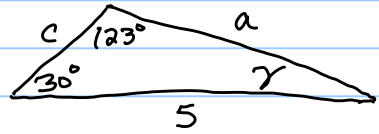
51. $a=9$ $b=12$ $\gamma=121^\circ$



$$A = \frac{1}{2}(9)(12)\sin 121^\circ$$

$$\boxed{A \approx 46.3 \text{ u}^2}$$

52. $\alpha=30^\circ$ $\beta=123^\circ$ $b=5$



$$\gamma = 180^\circ - (30^\circ + 123^\circ)$$

$$\gamma = 27^\circ$$

$$\frac{\sin 123^\circ}{5} = \frac{\sin 27^\circ}{c}$$

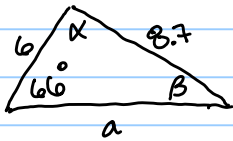
$$\frac{c \sin 123^\circ}{\sin 123^\circ} = \frac{5 \sin 27^\circ}{\sin 123^\circ}$$

$$c \approx 2.7$$

$$A = \frac{1}{2}(2.7)(5)\sin 30^\circ$$

$$\boxed{A \approx 3.4 \text{ u}^2}$$

53. $\gamma=66^\circ$ $b=6$ $c=8.7$
 $\text{opp} > \text{adj} \Rightarrow 1 \Delta$



$$\frac{\sin 66^\circ}{8.7} = \frac{\sin \beta}{6}$$

$$\sin \beta = \frac{6 \sin 66^\circ}{8.7} \approx .6300$$

$$\beta \approx 39.1^\circ$$

$$\alpha = 180^\circ - (66^\circ + 39.1^\circ)$$

$$\alpha \approx 74.9^\circ$$

$$A = \frac{1}{2}(6)(8.7)\sin 74.9^\circ$$

$$\boxed{A \approx 25.2 \text{ u}^2}$$

54. $a=4$ $b=8$ $c=10$

$$S = \frac{4+8+10}{2} = 11$$

$$A = \sqrt{11(11-4)(11-8)(11-10)}$$

$$A = \sqrt{231}$$

$$\boxed{A \approx 15.2 \text{ u}^2}$$

55. $a=4.5$ $b=6$ $c=8.7$

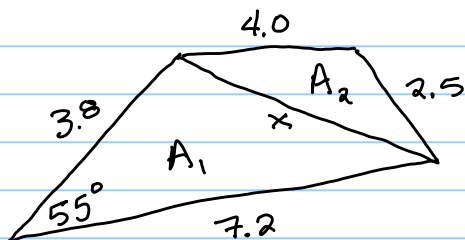
$$S = \frac{4.5+6+8.7}{2} = 9.6$$

$$A = \sqrt{9.6(9.6-4.5)(9.6-6)(9.6-8.7)}$$

$$A = \sqrt{158.6304}$$

$$\boxed{A \approx 12.6 \text{ u}^2}$$

56.



$$A_1 = \frac{1}{2}(3.8)(7.2)\sin 55^\circ$$

$$A_1 \approx 11.2 \text{ u}^2$$

$$x^2 = 3.8^2 + 7.2^2 - 2(3.8)(7.2)\cos(55^\circ)$$

$$x^2 \approx 34.89$$

$$x \approx 5.9$$

$$S = \frac{(4.0+2.5+5.9)}{2} \approx 6.2$$

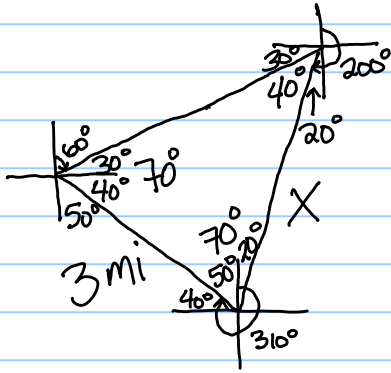
$$A_2 \approx \sqrt{6.2(6.2-4.0)(6.2-2.5)(6.2-5.9)}$$

$$A_2 \approx 3.9 \text{ u}^2$$

$$A \approx 11.2 + 3.9$$

$$\boxed{A \approx 15.1 \text{ u}^2}$$

57.



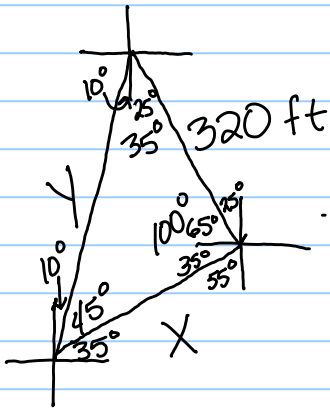
$$\frac{\sin 40^\circ}{3} = \frac{\sin 70^\circ}{X}$$

$$X \sin 40^\circ = 3 \sin 70^\circ$$

$$\frac{X \sin 40^\circ}{\sin 40^\circ} = \frac{3 \sin 70^\circ}{\sin 40^\circ}$$

$$X \approx 4.4 \text{ mi}$$

58.



$$\frac{\sin 45^\circ}{320} = \frac{\sin 35^\circ}{X}$$

$$X \sin 45^\circ = 320 \sin 35^\circ$$

$$\frac{X \sin 45^\circ}{\sin 45^\circ} = \frac{320 \sin 35^\circ}{\sin 45^\circ}$$

$$X \approx 259.6 \text{ ft}$$

$$\frac{\sin 45^\circ}{320} = \frac{\sin 100^\circ}{Y}$$

$$Y \sin 45^\circ = 320 \sin 100^\circ$$

$$\frac{Y \sin 45^\circ}{\sin 45^\circ} = \frac{320 \sin 100^\circ}{\sin 45^\circ}$$

$$Y \approx 445.7 \text{ ft}$$

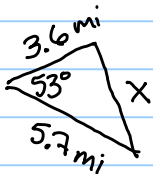
$$A = \frac{1}{2} (320)(259.6) \sin 100^\circ$$

$$A \approx 40,900.4 \text{ ft}^2$$

$$P = 320 + 259.6 + 445.7$$

$$P \approx 1025.2 \text{ ft}$$

59.

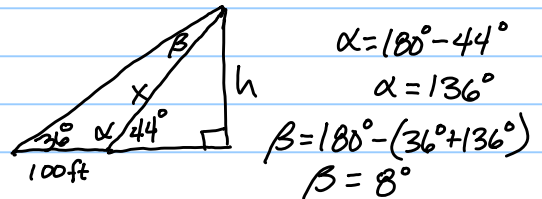


$$X^2 = 3.6^2 + 5.7^2 - 2(3.6)(5.7) \cos 53^\circ$$

$$X^2 = 20.75$$

$$X \approx 4.6 \text{ mi}$$

60.



$$\frac{\sin 8^\circ}{100} = \frac{\sin 36^\circ}{X}$$

$$X \sin 8^\circ = 100 \sin 36^\circ$$

$$\frac{X \sin 8^\circ}{\sin 8^\circ} = \frac{100 \sin 36^\circ}{\sin 8^\circ}$$

$$X \approx 422.34 \text{ ft}$$

$$\sin 44^\circ = \frac{h}{422.34 \text{ ft}}$$

$$h = 422.34 \sin 44^\circ$$

$$h \approx 293.4 \text{ ft}$$