


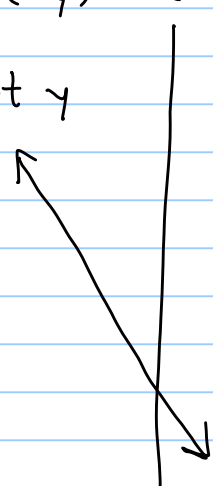
Precalculus - Exam 5 Review

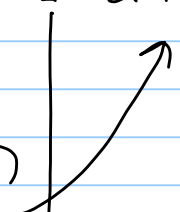
- $$1. \frac{\tan x \csc x}{\sec x} = \frac{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}}{\frac{1}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}} = \boxed{1}$$
- $$2. \tan^2 x - \frac{\sin(-x)}{\sin x} = \tan^2 x - \frac{(-\sin x)}{\sin x} = \tan^2 x + 1 = \boxed{\sec^2 x}$$
- $$3. \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\cos(2x)}{\sin(2x)} = \boxed{\cot(2x)}$$
- $$4. \frac{1}{1 + \sin \alpha} + \frac{\sin \alpha}{\cos^2 \alpha} = \frac{1}{1 + \sin \alpha} + \frac{\sin \alpha}{1 - \sin^2 \alpha}$$
$$= \frac{1}{1 + \sin \alpha} + \frac{\sin \alpha}{(1 + \sin \alpha)(1 - \sin \alpha)}$$
$$= \left(\frac{1 - \sin \alpha}{1 - \sin \alpha} \right) \left(\frac{1}{1 + \sin \alpha} \right) + \frac{\sin \alpha}{(1 + \sin \alpha)(1 - \sin \alpha)}$$
$$= \frac{1 - \sin \alpha + \sin \alpha}{(1 - \sin \alpha)(1 + \sin \alpha)} = \frac{1}{1 - \sin^2 \alpha} = \frac{1}{\cos^2 \alpha} = \boxed{\sec^2 \alpha}$$
- $$5. \frac{2 \tan(2\theta)}{1 - \tan^2(2\theta)} = \tan(2 \times 2\theta) = \boxed{\tan(4\theta)} \leftarrow \text{Using } \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
- $$6. \sin \theta \cos \theta (\tan \theta + \cot \theta)$$
$$= \sin \theta \cos \theta \tan \theta + \sin \theta \cos \theta \cot \theta$$
$$= \sin \theta \cos \theta \frac{\sin \theta}{\cos \theta} + \sin \theta \cos \theta \frac{\cos \theta}{\sin \theta}$$
$$= \sin^2 \theta + \cos^2 \theta = \boxed{1}$$
- $$7. \sin^2 x \tan^2 x + \sin^2 x$$
$$= \sin^2 x (\tan^2 x + 1)$$
$$= \sin^2 x (\sec^2 x)$$
$$= \sin^2 x \left(\frac{1}{\cos^2 x} \right) = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$
- $$8. \cos 75^\circ \cos 60^\circ + \sin(-75^\circ) \sin 60^\circ$$
$$= \cos 75^\circ \cos 60^\circ - \sin 75^\circ \sin 60^\circ \leftarrow \text{odd/even}$$
$$= \cos(75^\circ + 60^\circ)$$
$$= \cos(135^\circ) = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\begin{aligned}
 9. \quad & \sin 80^\circ \cos(-50^\circ) - \sin 10^\circ \sin 50^\circ \\
 & = \sin 80^\circ \cos 50^\circ - \sin 10^\circ \sin 50^\circ \quad \leftarrow \text{odd/even} \\
 & = \sin 80^\circ \cos 50^\circ - \cos 80^\circ \sin 50^\circ \quad \leftarrow \text{cofunction: } \sin 10^\circ = \cos 80^\circ \\
 & = \sin(80^\circ - 50^\circ) \\
 & = \sin(30^\circ) = \boxed{\frac{1}{2}}
 \end{aligned}$$

$$10. \quad \frac{\sin(4y)}{1 + \cos(4y)} = \tan\left(\frac{4y}{2}\right) = \boxed{\tan(2y)} \quad \left(\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x} \right)$$

$$\begin{aligned}
 11. \quad & \frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x \\
 & = \frac{\sin x \cos x}{\frac{\sin x}{\cos x}} \\
 & = \cancel{\sin x} \cos x \left(\frac{\cos x}{\cancel{\sin x}} \right) \\
 & = \cos^2 x \\
 & = 1 - \sin^2 x
 \end{aligned}$$


$$\begin{aligned}
 12. \quad & \cot(-y) = \frac{1 - \sin^2 y}{\cos(-y) \sin(-y)} \\
 & = -\cot y = \frac{1 - \sin^2 y}{\cos y (-\sin y)} \\
 & = \frac{1 - \sin^2 y}{-\cos y \sin y} \\
 & = \frac{\cos^2 y}{-\cos y \sin y} \\
 & = -\frac{\cos y}{\sin y} \\
 & = -\cot y
 \end{aligned}$$


$$\begin{aligned}
 13. \quad & \frac{\sin(2\beta)}{2 \csc \beta} = \sin^2 \beta \cos \beta \\
 & = \frac{2 \sin \beta \cos \beta}{2 \left(\frac{1}{\sin \beta} \right)} \\
 & = \sin \beta \cos \beta (\sin \beta) \\
 & = \sin^2 \beta \cos \beta
 \end{aligned}$$


$$\begin{aligned}
 14. \quad & \frac{1}{\sec \theta - 1} - \frac{1}{\sec \theta + 1} = 2 \cot^2 \theta \\
 & = \frac{(\sec \theta + 1)}{(\sec \theta + 1)} \left(\frac{1}{\sec \theta - 1} \right) + \left(\frac{-1}{\sec \theta + 1} \right) \left(\frac{\sec \theta - 1}{\sec \theta - 1} \right) \\
 & = \frac{\sec \theta + 1 - \sec \theta + 1}{\sec^2 \theta - 1} \\
 & = \frac{2}{\sec^2 \theta - 1} \\
 & = \frac{2}{\tan^2 \theta} \\
 & = 2 \cot^2 \theta
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \cos(3x) = \cos x (1 - 4 \sin^2 x) \\
 & = \cos(2x + x) \\
 & = \cos(2x) \cos x - \sin(2x) \sin x \\
 & = (1 - 2 \sin^2 x) \cos x - 2 \sin x \cos x \sin x \\
 & = \cos x - 2 \sin^2 x \cos x - 2 \sin^2 x \cos x \\
 & = \cos x - 4 \sin^2 x \cos x \\
 & = \cos x (1 - 4 \sin^2 x)
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \sin^2 \left(\frac{x}{2} \right) = \frac{\csc^2 x - \cot^2 x}{2 \csc^2 x + 2 \csc x \cot x} \\
 & = \left(\pm \sqrt{\frac{1 - \cos x}{2}} \right)^2 \\
 & = \frac{1 - \cos x}{2} \\
 & = \frac{1}{\frac{2}{\sin^2 x} + \frac{2}{\sin x} \cdot \frac{\cos x}{\sin x}} \\
 & = \frac{1}{\frac{2 + 2 \cos x}{\sin^2 x}} \\
 & = \frac{\sin^2 x}{2 + 2 \cos x} \\
 & = \frac{1 - \cos^2 x}{2 + 2 \cos x} \\
 & = \frac{(1 + \cos x)(1 - \cos x)}{2(1 + \cos x)} \\
 & = \frac{1 - \cos x}{2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{\sin(2\alpha)}{1 + \cos(2\alpha)} &= \tan \alpha \\
 &= \frac{2 \sin \alpha \cos \alpha}{1 + 2 \cos^2 \alpha - 1} \\
 &= \frac{2 \sin \alpha \cos \alpha}{2 \cos^2 \alpha} \\
 &= \frac{\sin \alpha}{\cos \alpha} \\
 &= \tan \alpha
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \sec(2\theta) &= \frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} \\
 &= \frac{1}{\cos(2\theta)} \\
 &= \left(\frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} \right) \left(\frac{\cos^2 \theta}{\cos^2 \theta} \right) \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta} \\
 &= \frac{1}{\cos(2\theta)}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) \\
 &= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\
 &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \sin\left(-\frac{5\pi}{12}\right) &= \sin\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right) \\
 &= \sin\left(\frac{\pi}{4} - \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right) \\
 &= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}
 \end{aligned}$$

$$21. \quad \tan(195^\circ) = \tan(60^\circ + 135^\circ) = \frac{\tan 60^\circ + \tan 135^\circ}{1 - \tan 60^\circ \tan 135^\circ} = \frac{\sqrt{3} + (-1)}{1 - (\sqrt{3})(-1)} = \boxed{\frac{\sqrt{3}-1}{1+\sqrt{3}}}$$

$$22. \quad \sin\left(-\frac{\pi}{8}\right) = \sin\left(-\frac{\pi/4}{2}\right) = -\sqrt{\frac{1 - \cos(-\pi/4)}{2}} = -\sqrt{\left(\frac{1 - \sqrt{2}/2}{2}\right)\left(\frac{2}{2}\right)} = -\sqrt{\frac{2 - \sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$$

\uparrow
 QIV
 sine is negative

$$23. \quad \cos(75^\circ) = \cos\left(\frac{150^\circ}{2}\right) = \sqrt{\frac{1 + \cos(150^\circ)}{2}} = \sqrt{\left(\frac{1 + \sqrt{3}/2}{2}\right)\left(\frac{2}{2}\right)} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

\uparrow
 QI
 cosine is positive

$$24. \quad \tan\left(\frac{5\pi}{8}\right) = \tan\left(\frac{5\pi/4}{2}\right) = \frac{1 - \cos(5\pi/4)}{\sin(5\pi/4)} = \frac{1 - (-\frac{\sqrt{2}}{2})}{-\frac{\sqrt{2}}{2}} = \frac{(1 + \frac{\sqrt{2}}{2})(-\frac{2}{\sqrt{2}})}{-\sqrt{2}} = \boxed{-\sqrt{2}-1}$$

$$\begin{aligned}
 25. \sin(A+B) &= \sin A \cos B + \cos A \sin B \\
 &= \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right) + \left(-\frac{12}{13}\right)\left(\frac{4}{5}\right) \\
 &= -\frac{15}{65} - \frac{48}{65} \\
 &= \boxed{-\frac{63}{65}}
 \end{aligned}$$

$$\begin{aligned}
 \sin A &= -\frac{5}{13} & \cos B &= \frac{3}{5} \\
 A \text{ in QIII} & & B \text{ in QI} & \\
 \begin{array}{|c|} \hline -12 \\ \hline \end{array} & \begin{array}{|c|} \hline 5 \\ \hline \end{array} & \begin{array}{|c|} \hline 4 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 13 \\ \hline \end{array} & & \begin{array}{|c|} \hline 3 \\ \hline \end{array} \\
 x = -\sqrt{13^2 - 5^2} = -12 & & y = \sqrt{5^2 - 3^2} = 4 \\
 \cos A = -\frac{12}{13} & & \sin B = \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 26. \cos(\alpha+\beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
 &= \left(-\frac{8}{17}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{15}{17}\right)\left(-\frac{1}{3}\right) \\
 &= -\frac{16\sqrt{2}}{51} + \frac{15}{51} \\
 &= \boxed{\frac{-16\sqrt{2} + 15}{51}}
 \end{aligned}$$

$$\begin{aligned}
 \sin \alpha &= \frac{15}{17} & \sin \beta &= -\frac{1}{3} \\
 \alpha \text{ in QII} & & \beta \text{ in QIII} & \\
 \begin{array}{|c|} \hline 15 \\ \hline \end{array} & \begin{array}{|c|} \hline 17 \\ \hline \end{array} & \begin{array}{|c|} \hline 2\sqrt{2} \\ \hline \end{array} & \begin{array}{|c|} \hline -1 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline -8 \\ \hline \end{array} & & \begin{array}{|c|} \hline 3 \\ \hline \end{array} & \\
 x = -\sqrt{17^2 - 15^2} = -8 & & x = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2} \\
 \cos \alpha = -\frac{8}{17} & & \cos \beta = \frac{2\sqrt{2}}{3}
 \end{aligned}$$

$$\begin{aligned}
 27. \cos\left(\frac{\alpha}{2}\right) &= -\sqrt{\frac{1 + \cos \alpha}{2}} \\
 &= -\sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} \left(\frac{4}{4}\right) \\
 &= -\sqrt{\frac{4 + \sqrt{15}}{8}} \\
 &= \boxed{-\frac{\sqrt{4 + \sqrt{15}}}{2\sqrt{2}}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{270^\circ}{2} < \frac{\alpha}{2} < \frac{360^\circ}{2} &\rightarrow 135^\circ < \frac{\alpha}{2} < 180^\circ \\
 &\frac{\alpha}{2} \text{ in QII} \\
 &\cos\left(\frac{\alpha}{2}\right) \text{ is negative} \\
 \sin \alpha &= -\frac{1}{4} \\
 \alpha \text{ in QIII} & \\
 \begin{array}{|c|} \hline \sqrt{15} \\ \hline \end{array} & \begin{array}{|c|} \hline -1 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 4 \\ \hline \end{array} & \\
 x = \sqrt{4^2 - 1^2} = \sqrt{15} \\
 \cos \alpha &= \frac{\sqrt{15}}{4}
 \end{aligned}$$

$$\begin{aligned}
 28. \sin(2\theta) &= 2 \sin \theta \cos \theta \\
 \sin(2\theta) &= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) = \boxed{-\frac{120}{169}}
 \end{aligned}$$

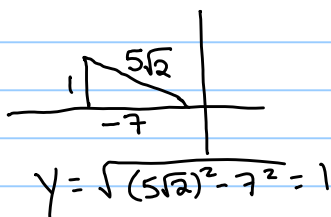
$$\begin{aligned}
 \cos \theta &= -\frac{12}{13} \\
 \theta \text{ in QII} & \\
 \begin{array}{|c|} \hline 5 \\ \hline \end{array} & \begin{array}{|c|} \hline 13 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline -12 \\ \hline \end{array} & \\
 y = \sqrt{13^2 - 12^2} = 5 \\
 \sin \theta &= \frac{5}{13}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \cos(2\beta) &= \frac{24}{25} \\
 2\cos^2\beta - 1 &= \frac{24}{25} \\
 2\cos^2\beta &= \frac{49}{25} \\
 \cos^2\beta &= \frac{49}{50} \\
 \cos\beta &= -\sqrt{\frac{49}{50}} \\
 \cos\beta &= -\frac{7}{5\sqrt{2}}
 \end{aligned}$$

$$\frac{180^\circ}{2} < \frac{2\beta}{2} < \frac{360^\circ}{2} \Rightarrow 90^\circ < \beta < 180^\circ$$

β in QII

$\cos\beta$ is negative

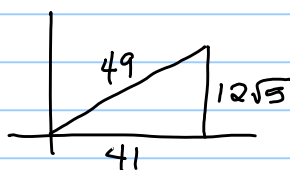


$$\begin{aligned}
 \sin\beta &= \frac{1}{5\sqrt{2}} \\
 \tan\beta &= -\frac{1}{7}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad \sin(\alpha/2) &= -\frac{2}{7} \\
 -\sqrt{\frac{1-\cos\alpha}{2}} &= -\frac{2}{7} \\
 \left(\sqrt{\frac{1-\cos\alpha}{2}}\right)^2 &= \left(\frac{2}{7}\right)^2 \\
 \frac{1-\cos\alpha}{2} &= \frac{4}{49} \\
 1-\cos\alpha &= \frac{8}{49} \\
 -\cos\alpha &= -\frac{41}{49} \\
 \cos\alpha &= \frac{41}{49}
 \end{aligned}$$

$$2(\pi) < 2(\alpha/2) < 2(5\pi/4) \Rightarrow 2\pi < \alpha < 5\pi/2$$

α in QI



$$\sin\alpha = \frac{12\sqrt{5}}{49}$$

$$\tan\alpha = \frac{12\sqrt{5}}{41}$$

$$y = \sqrt{49^2 - 41^2} = \sqrt{720} = 12\sqrt{5}$$

