Precalculus
Unit 8 Review Name ____________________

Identify the type of conic, list all the key features, and accurately draw a graph.

- For circles, list the center and radius.
- For parabolas, list the vertex, focus, and directrix.
- For ellipses, list the center, vertices, and foci.
- For hyperbolas, list the center, vertices, foci, transverse axis, and asymptotes.

1. $x^2 = -12y$

$(x-h)^2 = -4ay$

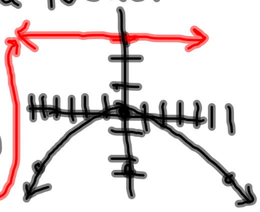
$-12 = -4a$
$3 = a$

opens down because of negative

It is a parabola because only one variable is squared.

a is the distance from the vertex to the focus.

vertex $(0,0)$
focus $(0,-3)$
directrix
$y=3$

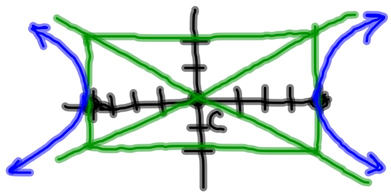


2. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

Hyperbola
x^2, y^2 minus between
opens left, right since x is first.

$\boxed{\text{center } (0,0)}$

$a^2=16$ $a=4$ $b^2=4$ $b=2$



vertices $(4,0)$ $(-4,0)$

foci $a^2+b^2=c^2$
$16+4=c^2$
$\pm\sqrt{20} = c$
$\pm 2\sqrt{5} = c$

foci $(0 \pm 2\sqrt{5}, 0)$

asymptotes $\dfrac{y}{x} = \dfrac{\pm\sqrt{4}}{\sqrt{16}}$

$\dfrac{\text{rise}}{\text{run}}$ $y = \pm\dfrac{2}{4}x$

$y = \pm\dfrac{1}{2}x$

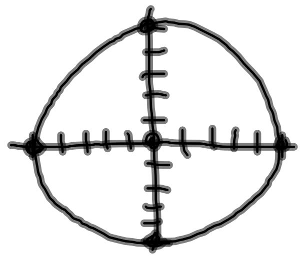
transverse axis
$y=0$

3. $(x-2)^2 + y^2 = 25$

circle $(x-h)^2+(y-k)^2=r^2$

center $(2,0)$

$r=5$



4. $\dfrac{(x-1)^2}{49} + \dfrac{(y+5)^2}{9} = 1$

stretched more $\longleftrightarrow$ since 49 is bigger than 9.

Ellipse $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

center (h,k) $\boxed{(1,-5)}$

$a^2=49$ $b^2=9$

$a^2-b^2=c^2$

$49-9=c^2$

$\pm\sqrt{40}=c$

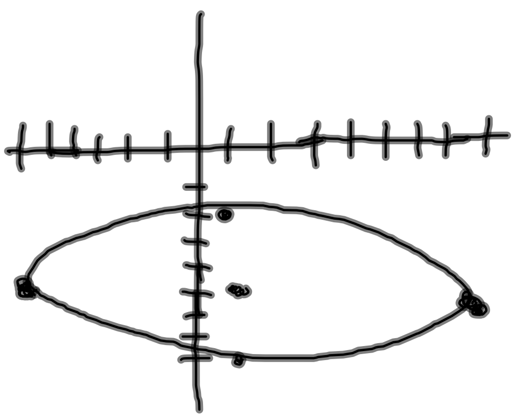
$\pm 2\sqrt{10}$

foci add $\pm 2\sqrt{10}$ to center's x coord.

foci $(1\pm 2\sqrt{10}, -5)$

vertices ± 7 from center's x coord.

$(8,-5)$ $(-6,-5)$



5. $4x^2 + y^2 = 64$

since y^2 does not have a 4 this is an ellipse.
Divide by 64

$$\frac{4x^2}{64} + \frac{y^2}{64} = \frac{64}{64}$$

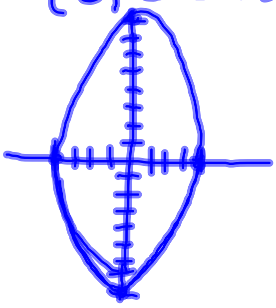
$$\frac{x^2}{16} + \frac{y^2}{64} = 1$$

$a^2 = 64$ ← biggest denominator
$b^2 = 16$ stretched more ↕

$\boxed{\text{center } (0,0)}$
$\boxed{\text{vertices } (0,8)\ (0,-8)}$ since $a = 8$

foci $a^2 - b^2 = c^2$
$64 - 16 = c^2$
$\pm\sqrt{48} = c$
$\pm 4\sqrt{3} = c$

foci $(0, 0 \pm 4\sqrt{3})$



6. $(y+3)^2 = 8(x-2)$

parabola opens right

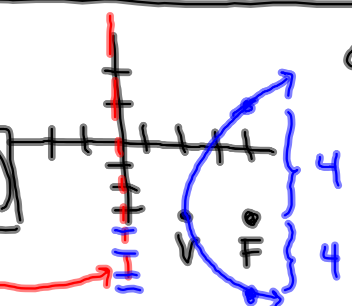
$(y-k)^2 = 4a(x-h)$

$\boxed{\text{vertex } (h,k) \text{ so } (2,-3)}$

$8 = 4a \quad a = 2$

$\boxed{\text{focus } (4,-3)}$

directrix



opens right x changes

focal distance is 8 $= 4a = 8$

subtract 2 from x coord of center

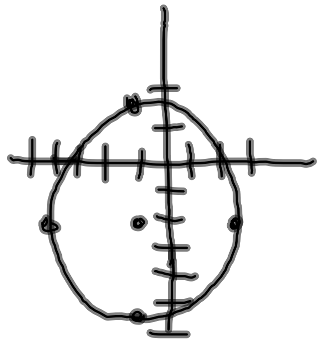
so $\boxed{x = 0}$

7. $(x+1)^2+(y+2)^2=12$

circle $(x-h)^2+(y-k)^2=12$

center $(-1,-2)$

radius $\sqrt{12}$ or $2\sqrt{3}$



8. $4(y-3)^2-36(x-4)^2=36$

hyperbola opens up down since y is first

$$\frac{4(y-3)^2}{36}-\frac{36(x-4)^2}{36}=\frac{36}{36}$$

$$\frac{(y-3)^2}{9}-\frac{(x-4)^2}{1}=1$$

$a^2=9$ $a=3$

$b^2=1$ $b=1$

Center (h,k) $\boxed{(4,3)}$

vertices ±3 to y coord of center $\boxed{(4,0)\ (4,6)}$

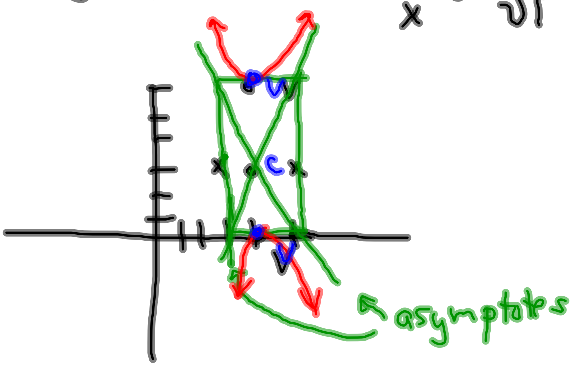
foci $a^2+b^2=c^2$

$9+1=c^2$

$\pm\sqrt{10}=c$

$(4, 3+\sqrt{10})$ $(4, 3-\sqrt{10})$

asymptotes $\frac{y}{x}=\frac{\pm\sqrt{9}}{\sqrt{1}}$



$y=\pm3x$ but need to consider center so change y to y-3 and x to (x-4)

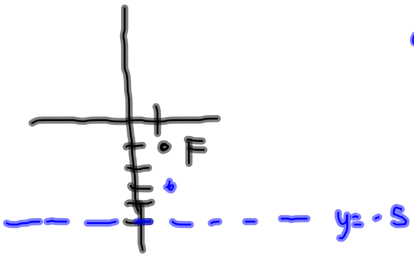
$y-3=\pm3(x-4)$

9. Write the standard form of the equation of the circle with radius $r = 4$ and center $(2,-5)$.

r h,k

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y--5)^2=4^2$$
$$(x-2)^2+(y+5)^2=16$$

10. Write the standard form of the equation of the parabola with focus $(1,-1)$ and directrix $y=-5$.

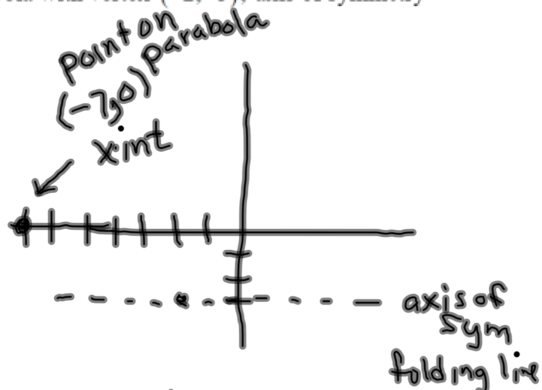


center is directly in between
from -1 to $-5 = 4$
so $\underline{2 \text{ away}}$ is $^-3$ $2=a$
vertex is $(1,-3)$
focus is in interior of parabola so opens up

equation is $(x-h)^2=4a(y-k)$
$$(x-1)^2=4(2)(y-^-3)$$
$$\boxed{(x-1)^2=8(y+3)}$$

11. Write the standard form of the equation of the parabola with vertex $(-2,-3)$, axis of symmetry $y=-3$, and x-intercept $(-7,0)$.

vertex is $(^-2,-3)$



$$(y-k)^2={}^-4a(x-h)$$
$$(y--3)^2=-4a(x--2)$$
$$(y+3)^2=-4a(x+2)$$

must open left.

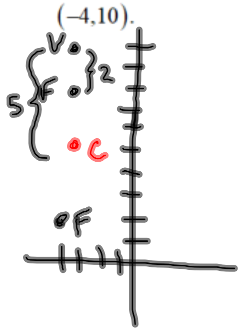
$$(0+3)^2=-4a(-7+2)$$

now use point $(-7,0)$ to find a.

$$9=-4a(-5)$$
$$9=20a$$
$$\frac{9}{20}=a$$

Equation is $(y+3)^2=-4\left(\frac{9}{20}\right)(x+2)$
$$\boxed{(y+3)^2=-\frac{9}{5}(x+2)}$$

12. Write the standard form of the equation of an ellipse with foci at $(-4,2)$ and $(-4,8)$ and vertex at $(-4,10)$.



2 to 8 = 6 $\quad c=3$

center is midpoint so 3 away $\quad a=5$

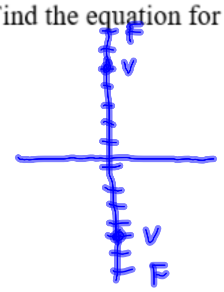
center is $(-4, 2+3)$ $\boxed{(-4,5)}$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\boxed{\frac{(x+4)^2}{16}+\frac{(y-5)^2}{25}=1}$$

$a^2-b^2=c^2$
$25-b^2=3^2$
$25-b^2=9$
$16=b^2$
$4=b$

13. Find the equation for the hyperbola with vertices at $(0,\pm5)$ and foci at $(0,\pm7)$.



center $(0,0)$ $\quad$ opens up/down

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$a=5 \quad a^2=25$
$b=?$
$c=7 \quad c^2=49$

$$\frac{(y-0)^2}{25}-\frac{(x-0)^2}{24}=1$$

$a^2+b^2=c^2$
$b^2=49-25$
$b^2=24$

$$\boxed{\frac{y^2}{25}-\frac{x^2}{24}=1}$$

14. Find the equation for the hyperbola with center $(2,3)$, focus $(0,3)$, and vertex $(1,3)$.

opens

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$c=2 \quad a=1$
$c^2=4 \quad a^2=1$

$$\frac{(x-2)^2}{a^2}-\frac{(y-3)^2}{b^2}=1$$

$a^2+b^2=c^2$
$1+b^2=4$
$b^2=3$

$$\frac{(x-2)^2}{1}-\frac{(y-3)^2}{b^2}=1$$

$$\frac{(x-2)^2}{1}-\frac{(y-3)^2}{3}=1$$