Precalculus

Unit 7 Review

Find the component form of the vector with the given magnitude and direction angle

1. $|\mathbf{v}| = 27.3,\ \theta = 214.9°$

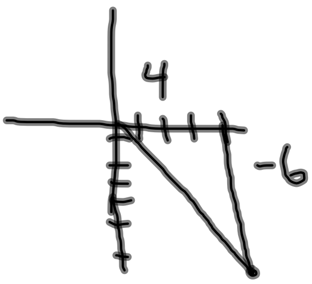
$x = r\cos\theta \quad y = r\sin\theta$

$x = 27.3(\cos 214.9°) \quad y = 27.3(\sin 214.9°)$

$\langle -22.4, -15.6 \rangle$

Find the magnitude and direction angle of the vector. Give the measure of the direction angle as an angle in $[0°, 360°)$.

2. $\langle 4, -6 \rangle$



$\sqrt{4^2 + (-6)^2}$

$\sqrt{16 + 36}$

$\sqrt{52}$ or $2\sqrt{13}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) \quad \tan^{-1}\left(\frac{-6}{4}\right)$

$\theta = -56.3°$

$360 - 56.3° \approx \boxed{303.7°}$

Perform the indicated operation. Use the form $\langle a, b \rangle$ for vectors. $\mathbf{u} = \langle -1, 5 \rangle$, $\mathbf{v} = \langle 4, -7 \rangle$

3. Find $3\mathbf{u} - \mathbf{v}$

$3\langle -1, 5 \rangle - \langle 4, -7 \rangle$

$\langle -3, 15 \rangle - \langle 4, -7 \rangle$

$\langle -3 - 4, 15 - -7 \rangle$

$\boxed{\langle -7, 22 \rangle}$

4. Find $\mathbf{u} \cdot \mathbf{v}$

$u \cdot v$

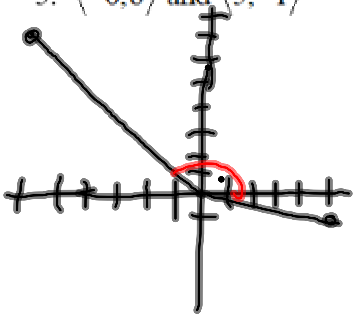
$(-1 \cdot 4) + (5 \cdot -7)$

$-4 + -35$

$\boxed{-39}$

Find the smallest positive angle between the given vectors to the nearest tenth of a degree.

5. $\langle -6, 8 \rangle$ and $\langle 5, -1 \rangle$



$\theta = \cos^{-1} \dfrac{u \cdot v}{|u||v|}$

$\dfrac{(-6 \cdot 5) + (8 \cdot -1)}{\left(\sqrt{(-6)^2 + 8^2}\right)\left(\sqrt{5^2 + (-1)^2}\right)}$

$\dfrac{-30 + -8}{\sqrt{100} \cdot \sqrt{26}}$

$\theta = \cos^{-1}\left(\dfrac{-38}{\sqrt{2600}}\right)$

$\boxed{\theta \approx 138.2^\circ}$

Determine whether the vectors are parallel, perpendicular, or neither.

6. $\langle 2,-4\rangle$ and $\langle 6,3\rangle$ 7. $\langle 9,1\rangle$ and $\langle 1,9\rangle$ 8. $\langle -1,7\rangle$ and $\langle 3,-21\rangle$

Same slope means parallel

$\frac{y}{x}$ $\frac{-4}{2} \neq \frac{3}{6}$

perpendicular $u \cdot v = 0$

$2(6) + -4(3)$

$12 + -12 = 0$

$\boxed{\text{perpendicular}}$

$\frac{1}{9} \neq \frac{9}{1}$

$9(1) + 1(9) \neq 0$

$\boxed{\text{Neither}}$

$\frac{7}{-1} = \frac{-21}{3}$

$\frac{7}{-1} = \frac{-7}{1}$

parallel

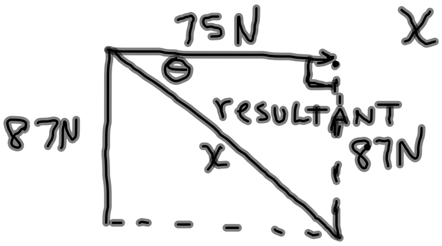
Solve the problems.

9. One rope pulls a barge due east with a force of 75 N, and another rope pulls the barge due south with a force of 87 N. Find the magnitude of the resultant force acting on the barge and the angle between the resultant force and the smaller force.

SAS

Law of Cosines

$x^2 = 75^2 + 87^2 - 2(75)(87)\cos(90^\circ)$



$x^2 = 13194$

$x = \sqrt{13194}$

$\approx 114.9\ N$

Law of sines

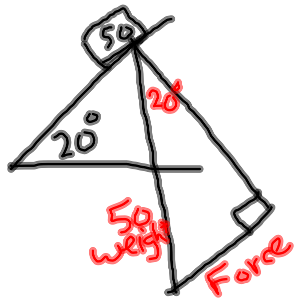
$$\frac{\sin 90}{114.9} = \frac{\sin \theta}{87}$$

$\sin\theta = 87 \sin 90^\circ \div 114.9$

$\theta = \sin^{-1}(.7572) \approx 49.2^\circ$

10. Find the force required to keep a 50-lb wagon from sliding down a ramp inclined at 20° to the horizontal.

weight



$$\frac{\sin\theta}{1} = \frac{\text{Force}}{\text{weight}}$$

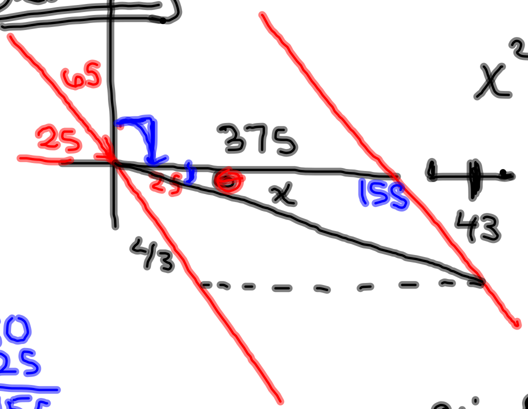
$$\frac{\sin 20^\circ}{1} = \frac{F}{50}$$

$$F = 50\sin 20^\circ$$

$$\boxed{F \approx 17.1 \text{ lb}}$$

11. An airplane flies due east at 375 mph. The wind affecting the plane is blowing from 295° at 43 mph. What is the true course and ground speed of the airplane? Round to the nearest tenth.

Bearing



$$\begin{array}{r} 270 \\ +\ 25 \\ \hline 295 \end{array}$$

$$\begin{array}{r} 180 \\ -\ 25 \\ \hline 155 \end{array}$$

Law of cosines

$$x^2 = 375^2 + 43^2 - 2(375)(43)\cos 155^\circ$$

$$x^2 = \sqrt{171702.4261}$$

$$x = 414.3699 \text{ mph}$$

$$\boxed{414.4 \text{ mph}}$$

$$\frac{\sin\theta}{43} = \frac{\sin 155}{414.4}$$

$$\theta = \sin^{-1}(43\sin 155 \div 414.4)$$

drift angle 2.5°

true course $90 + 2.5 = \boxed{92.5^\circ}$

due East

Perform the indicated operation. Write the answer in the form $a+bi$.

12. $4(\cos 80° + i\sin 80°)\cdot 3(\cos 130° + i\sin 130°)$

$4\cdot 3\left(\cos(80+130) + i\sin(80+130)\right)$

$12\left(\cos 210° + i\sin 210°\right)$

$12\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$\boxed{-6\sqrt{3} - 6i}$

13. $\dfrac{7\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

$\frac{7}{2}\left(\cos\left(\frac{5\pi}{6} - \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{6} - \frac{\pi}{3}\right)\right)$

$3.5\left(\cos\left(\frac{5\pi}{6} - \frac{2\pi}{6}\right) + i\sin\left(\frac{3\pi}{6}\right)\right)$

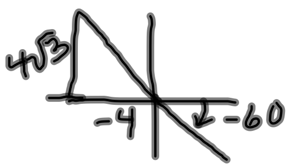
$3.5\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$3.5\left(0 + i\cdot 1\right)$

$3.5i$ or $\frac{7}{2}i$

Use De Moivre's Theorem to simplify the expression. Write the answer in $a+bi$ form.

14. $\left(-4+4i\sqrt{3}\right)^4$



$r=\sqrt{(-4)^2+(4\sqrt{3})^2}$

$\sqrt{16+16(3)}$

$\sqrt{64}$

$\tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right)=-60°$

$+180$

$120°$

$r=8\ (\cos 120° + i\sin 120°)$

$r^4=8^4\ (\cos (120\cdot 4)+i\sin 480°)$

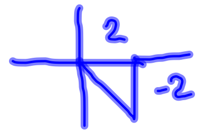
480
-360
120

$r^4=8^4\ (\cos 120° + i\sin 120°)$

$4096\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$\boxed{-2048+2048\sqrt{3}\,i}$

15. $(2-2i)^5$



$\sqrt{2^2+(-2)^2}$

$2\sqrt{2}\ \text{cis}(-45°)$

$(2\sqrt{2})^5\ \text{cis}(-45°\cdot 5)$

$2\sqrt{2}\cdot 2\sqrt{2}\cdot 2\sqrt{2}\cdot 2\sqrt{2}\cdot 2\sqrt{2}$

$2^7\sqrt{2}$

$128\sqrt{2}\ \text{Cis}(-225)$

$360-225$

$128\sqrt{2}\ \text{cis}\ 135°$

$128\sqrt{2}\ (\cos 135+i\sin 135)$

$128\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)$

$\boxed{-128+128i}$

Find the indicated roots. Write the answers in trigonometric form.

16. Fourth roots of $81(\cos 280° + i \sin 280°)$

$$\sqrt[4]{81}\left(\cos\frac{280+360k}{4} + i\sin\frac{280+360k}{4}\right)$$

$$3(\cos(70+90k) + i\sin(70+90k))$$

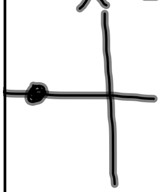
add 90 each time

$$\boxed{\begin{array}{ll} k=0 & 3(\cos 70° + i\sin 70°) \\ k=1 & 3(\cos 160° + i\sin 160°) \\ k=2 & 3(\cos 250° + i\sin 250°) \\ k=3 & 3(\cos 340° + i\sin 340°) \end{array}}$$

Solve the equation. Write the answer in $a+bi$ form. Give exact answers (no calculator).

17. $x^3 + 64 = 0$

$x^3 = -64$ 3 roots



$64(\cos 180° + i\sin 180°)$

$$\sqrt[3]{64}\left(\cos\frac{180°+360°k}{3} + i\sin\frac{180°+360°k}{3}\right)$$

$4(\cos 60° + 120°k + i\sin 60° + 120°k)$

add 120° each time

$k=0$ $4(\cos 60° + i\sin 60°)$

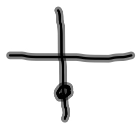
$k=1$ $4(\cos 180° + i\sin 180°)$

$k=2$ $4(\cos 300° + i\sin 300°)$

$$\boxed{\begin{array}{l} 2+2\sqrt{3}i \\ -4 \\ 2-2\sqrt{3}i \end{array}}$$

18. $x^2 + 18i = 0$

$x^2 = -18i$



$r = 18$

$18(\cos 270° + i\sin 270°)$

$$\sqrt{18}\ \text{cis}\left(\frac{270°+360°k}{2}\right)$$

$3\sqrt{2}\ \text{cis}(135° + 180°k)$

$k=0$

$3\sqrt{2}\ \text{cis}\ 135° =$

$3\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$\boxed{-3+3i}$

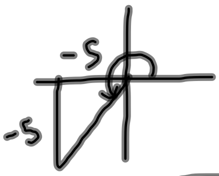
$k=1$ $3\sqrt{2}\ \text{cis}\ 315°$

$3\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{-\sqrt{2}}{2}\right)$

$\boxed{3-3i}$

Convert the rectangular coordinates to polar coordinates, using radian measure for the angle.

19. $(-5,-5)$



$\tan^{-1}\left(\frac{-5}{-5}\right) = 45^\circ$

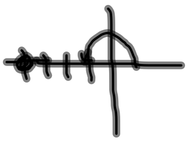
$\frac{180}{225^\circ}$

$r = \sqrt{(-5)^2 + (-5)^2}$

$\sqrt{50}$

$\boxed{\left(5\sqrt{2}, \frac{5\pi}{4}\right)}$

20. $(-4,0)$



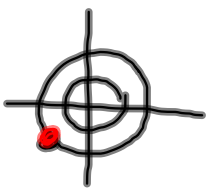
$r = 4$

$\theta = 180^\circ \quad \pi$

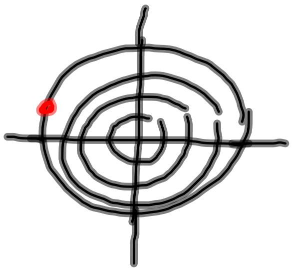
$\boxed{(4, \pi)}$

Plot the point whose polar coordinates are given.

21. $\left(2, -\frac{2\pi}{3}\right)$



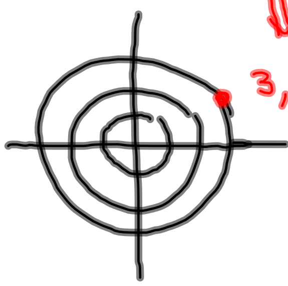
22. $\left(4, \frac{5\pi}{6}\right)$



23. $(-3, 210^\circ)$

$\Downarrow$

$3, 30^\circ$



Convert to rectangular coordinates.

24. $\left(-2, \frac{3\pi}{4}\right)$ 25. $\left(3, -\frac{1}{2}\pi\right)$ 26. $\left(-4, \frac{4\pi}{3}\right)$

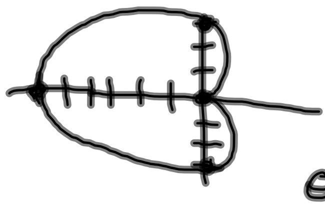
$x = r\cos\theta$
$x = -2\cos\frac{3\pi}{4}$
$x = -2\left(-\frac{\sqrt{2}}{2}\right)$
$x = \sqrt{2}$
$y = r\sin\theta$
$y = -2\sin\frac{3\pi}{4}$
$-2\left(\frac{\sqrt{2}}{2}\right)$
$-\sqrt{2}$
$\boxed{(\sqrt{2}, -\sqrt{2})}$

$x = r\cos\theta$
$3(\cos(-\frac{\pi}{2}))$
$x = 0$
$y = r\sin\theta$
$3(\sin(-\frac{\pi}{2}))$
$3(-1)$
-3
$\boxed{(0, -3)}$

$x = r\cos\theta$
$x = -4\cos\frac{4\pi}{3}$
$-4(-\frac{1}{2})$
$x = 2$
$y = -4\sin(\frac{4\pi}{3})$
$-4\left(-\frac{\sqrt{3}}{2}\right)$
$y = 2\sqrt{3}$
$\boxed{(2, 2\sqrt{3})}$

Graph the polar equation.

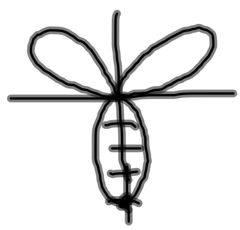
27. $r = 3 - 3\cos\theta$ cardiod



θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	0	3	6	3	0

28. $r = 4\sin(3\theta)$

rose
3 petals
Length 4
sym with y



For the given polar equation, write an equivalent rectangular equation.

29. $r = 5\cos\theta$

$r^2 = 5\cos\theta \cdot r$ MULT bothsides by r

$r^2 = 5r\underbrace{\cos\theta}$

$\boxed{x^2+y^2 = 5x}$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$ } use most

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\tan\theta = \left(\frac{y}{x}\right)$

For the given rectangular equation, write an equivalent polar equation.

30. $x = 5$

$r\cos\theta = 5$

$\boxed{r = \frac{5}{\cos\theta}}$

or $\boxed{r = 5\sec\theta}$

get rid of x and y

use r θ

Must be r= ____

Eliminate the parameter of the pair of parametric equations.

31. $x = t + 5,\ y = t^2 + 3$

$x = t + 5$

$x - 5 = t$

$y = t^2 + 3$

$\boxed{y = (x-5)^2 + 3}$

OR

$y = x^2 - 10x + 25 + 3$

$\boxed{y = x^2 - 10x + 28}$

32. $x = 4\cos\theta,\ y = \sin\theta$

$x = 4\cos\theta \quad y = \sin\theta$

$\frac{x}{4} = \cos\theta \quad y^2 = \sin^2\theta$

$\left(\frac{x}{4}\right)^2 = \cos^2\theta$

$\frac{x^2}{16} = \cos^2\theta$

Add left sides = add right sides

$\frac{x^2}{16} + y^2 = \cos^2\theta + \sin^2\theta$

$\boxed{\frac{x^2}{16} + y^2 = 1}$

Write a pair of parametric equations that will produce the indicated graph.

33. The line segment starting at $(1,-2)$ with $t=0$ and ending at $(13,1)$ with $t=3$.

t	x	y
0	1	-2
3	13	1

$x = mt + b$

$1 = m(0) + b$

$1 = b$

$x = mt + 1$

$13 = m(3) + 1$

$-1 \qquad -1$

$12 = 3m$

$4 = m$

$\boxed{x = 4t + 1}$

$\boxed{y = 1t - 2}$

$\boxed{0 \le t \le 3}$

$y = mt + b$

$-2 = m(0) + b$

$-2 = b$

$y = mt + -2$

$1 = m(3) + -2$

$+2 \qquad +2$

$3 = 3m$

$1 = m$

$y = 1t - 2$