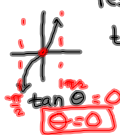
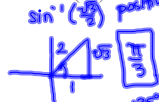

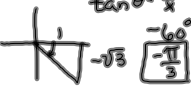


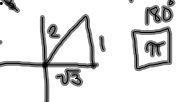
Precalculus Unit 6 Review
Part I: No Calculator
Find the exact value of each expression, in radians.


1. $\tan^{-1}(0)$
restricted $(-\frac{\pi}{2}, \frac{\pi}{2})$
 $\tan(\theta) = 0$

 $\theta = 0$


2. $\arcsin(\frac{\sqrt{3}}{2})$
 $\sin^{-1}(\frac{\sqrt{3}}{2})$
positive

 $\theta = \frac{\pi}{3}$


3. $\sec^{-1}(-\sqrt{2})$
 $\sec \theta = -\sqrt{2}$
 $\cos \theta = -\frac{1}{\sqrt{2}}$
restricted $[0, \pi]$

 $\theta = \frac{3\pi}{4}$


4. $\arctan(-\sqrt{3})$
 $\tan^{-1}(-\sqrt{3})$
 $\tan \theta = -\frac{\sqrt{3}}{1}$
 $\theta = -\frac{\pi}{3}$


5. $\sec^{-1}(\frac{2\sqrt{3}}{3})$
 $\sec \theta = \frac{2\sqrt{3}}{3}$
 $\cos \theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$
 $\cos \theta = \frac{\sqrt{3}}{2}$
positive
 $\theta = \frac{\pi}{6}$


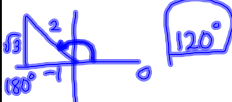
6. $\cos^{-1}(-1)$
 $\cos \theta = -1$

 $\theta = \pi$

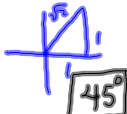
7. $\cot^{-1}(0)$
 $\cot \theta = 0$
 $\theta = \frac{\pi}{2}$

restriction $(0, \pi)$

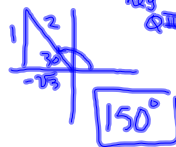
8. $\arcsin(-\frac{\sqrt{2}}{2})$
 $\sin^{-1}(-\frac{\sqrt{2}}{2})$
 $\sin \theta = -\frac{\sqrt{2}}{2}$
restricted $[-\frac{\pi}{2}, \frac{\pi}{2}]$
 $\theta = -\frac{\pi}{4}$


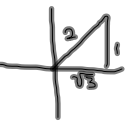
9. $\arccsc(-2)$
 $\csc^{-1}(-2)$
 $\csc \theta = -2$
 $\sin \theta = -\frac{1}{2}$
 $\theta = -\frac{\pi}{6}$


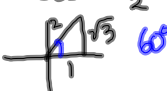
Find the exact value of each expression, in degrees.

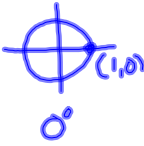
10. $\cos^{-1}(\frac{1}{2})$
 $\cos \theta = \frac{1}{2}$
 $\theta = 120^\circ$


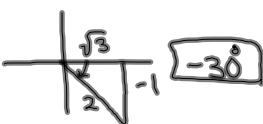
11. $\arctan(1)$
 $\tan^{-1}(1)$
 $\tan \theta = 1$
 $\theta = 45^\circ$


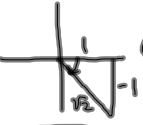
12. $\cot^{-1}(-\sqrt{3})$
 $\cot^{-1}(-\sqrt{3})$
neg
QII
 $\theta = 150^\circ$


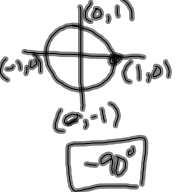
13. $\arcsin(\frac{1}{2})$
 $\sin^{-1}(\frac{1}{2})$
 $\sin \theta = \frac{1}{2}$
 $\theta = 30^\circ$


14. $\text{arcsec}(2)$
 $\sec^{-1}(2) = \theta$
 $\sec \theta = 2$
 $\cos \theta = \frac{1}{2}$
 $\theta = 60^\circ$


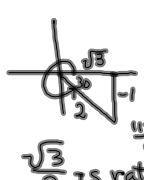
15. $\cos^{-1}(1)$
 $\cos \theta = 1$
 $\theta = 0^\circ$


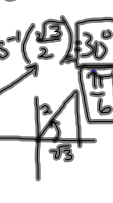
16. $\arctan(-\frac{\sqrt{3}}{3})$
 $\tan^{-1}(-\frac{\sqrt{3}}{3})$
 $\tan \theta = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$
 $\theta = -30^\circ$


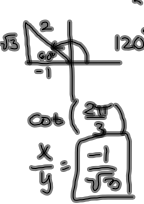
17. $\text{csc}^{-1}(-\sqrt{2})$
 $\csc^{-1}(-\sqrt{2}) = \theta$
 $\csc \theta = -\sqrt{2}$
 $\sin \theta = -\frac{1}{\sqrt{2}}$
 $\theta = -45^\circ$



18. $\arcsin(-1)$
 $\sin^{-1}(-1)$
 $\sin \theta = -1$
 $\theta = -90^\circ$


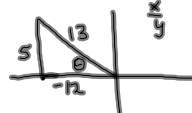
Find the exact value of each composition.

19. $\arccos\left(\cos\left(\frac{11\pi}{6}\right)\right)$
 $\cos^{-1}(\text{ratio}) = \theta$

 $\frac{\sqrt{3}}{2}$ is ratio

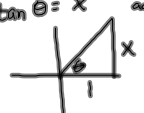
20. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$
 ratio
 $\tan^{-1}(1) = \theta$
 $\tan \theta = 1$
 QI

 45°

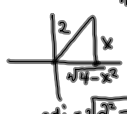
21. $\cot(\sec^{-1}(-2))$
 $\cot(\theta) = \text{ratio}$
 $\sec^{-1}(-2)$
 $\sec \theta = -2$
 $\cos \theta = -\frac{1}{2}$

 120°
 $\cot\left(\frac{2\pi}{3}\right)$
 $\frac{x}{y} = \frac{-1}{\sqrt{3}}$


22. $\tan\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$
 $\tan(\text{angle}) = \text{ratio}$
 $\text{angle} = \cos^{-1}\left(\frac{8}{17}\right)$

 $\cos \theta = \frac{8}{17} = \frac{x}{r}$
 QI
 $\tan \theta = \frac{y}{x} = \frac{15}{8}$
 $y = \sqrt{17^2 - 8^2}$
 $y = 15$

23. $\sin\left(\text{arccot}\left(\frac{-12}{5}\right)\right)$
 $\sin(\text{angle}) = \text{ratio}$
 $\cot^{-1}\left(\frac{-12}{5}\right) = \theta$

 $r = \sqrt{(-12)^2 + 5^2}$
 $r = 13$
 $\sin \theta = \frac{5}{13}$


Write an equivalent algebraic expression for each composition.


24. $\csc(\tan^{-1}(x))$
 $\csc(\theta) = \text{ratio}$
 $\text{angle} = \tan^{-1}(x)$
 $\tan \theta = x = \frac{\text{opp}}{\text{adj}} = \frac{x}{1}$

 $r = \sqrt{1^2 + x^2}$
 $\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{1+x^2}}{x}$

25. $\sec\left(\arcsin\left(\frac{x}{2}\right)\right)$
 $\sec \theta = \text{ratio}$
 $\sin^{-1}\left(\frac{x}{2}\right) = \theta$
 $\sin \theta = \frac{x}{2} = \frac{\text{opp}}{\text{hyp}}$

 $\text{adj} = \sqrt{2^2 - x^2}$
 $\sec \theta = \frac{r}{x} = \frac{2}{\sqrt{4-x^2}}$

26. $\cos(\sin^{-1}(4x))$
 $\cos \theta = \text{ratio}$
 $\sin^{-1}(4x) = \theta$

 $\text{adj} = \sqrt{5^2 - (4x)^2}$
 $\text{adj} = \sqrt{25 - 16x^2}$
 $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{25-16x^2}}{5}$

Find all real number solutions to each equation.

27. $2 \cos \theta - 1 = 0$
 $\cos \theta = \frac{1}{2}$
 positive in QI and QIV
 QI $\theta = \frac{\pi}{3} + 2\pi k$
 QIV $\theta = \frac{5\pi}{3} + 2\pi k$


28. $2 \cos^2 \theta + \cos(2\theta) = 0$
 $2 \cos^2 \theta + 2 \cos^2 \theta - 1 = 0$
 $4 \cos^2 \theta - 1 = 0$
 $4 \cos^2 \theta = 1$
 $\cos^2 \theta = \frac{1}{4}$
 $\sqrt{\cos^2 \theta} = \pm \sqrt{\frac{1}{4}}$
 $\cos \theta = \pm \frac{1}{2}$

 $\theta = \frac{\pi}{3} + 2\pi k$
 $\theta = \frac{2\pi}{3} + 2\pi k$
 $\theta = \frac{4\pi}{3} + 2\pi k$
 $\theta = \frac{5\pi}{3} + 2\pi k$

Find all angles in degrees that satisfy each equation.

29. $\tan(\theta/4) = \sqrt{3}/3$

$\tan(\theta/4) = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

positive
QI, QIII

$\frac{1}{\sqrt{3}} = \frac{y}{x}$

$\frac{\theta}{4} = 30^\circ + 180^\circ k$

$\theta = 120^\circ + 720^\circ k$

30. $\sin \theta \tan \theta - \sin \theta = 0$

$\sin \theta (\tan \theta - 1) = 0$

$\sin \theta = 0$ $\tan \theta = 1$

$\theta = 0^\circ + 180^\circ k$

$\theta = 45^\circ + 180^\circ k$

31. $3 \cos x = 2 \cos^2 x + 1$

$0 = 2 \cos^2 x - 3 \cos x + 1$

$0 = 2u^2 - 3u + 1$

$(2u - 1)(u - 1)$

$2 \cos x - 1 = 0$ $\cos x - 1 = 0$

$2 \cos x = 1$ $\cos x = 1$

$\cos x = \frac{1}{2}$ $X = 0^\circ + 360^\circ k$

$X = 60^\circ + 360^\circ k$

$X = 300^\circ + 360^\circ k$

OR $X = -60^\circ + 360^\circ k$

Find all real number solutions to each equation in the interval $[0, 2\pi)$.

32. $\sqrt{3} \sin \theta = \sin(2\theta)$ *radian*

$\sqrt{3} \sin \theta = 2 \sin \theta \cos \theta$

$\sqrt{3} \sin \theta - 2 \sin \theta \cos \theta = 0$

$\sin \theta (\sqrt{3} - 2 \cos \theta) = 0$

$\sin \theta = 0$ $\sqrt{3} - 2 \cos \theta = 0$

$0, \pi$

$-2 \cos \theta = -\sqrt{3}$

$\cos \theta = \frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}$

$\cos \theta = \frac{\sqrt{3}}{2}$

QI QIV

$\frac{\pi}{6}$ $\frac{11\pi}{6}$

33. $1 + \sin x = -\cos x$

$(1 + \sin x)^2 = (-\cos x)^2$

$1 + 2 \sin x + \sin^2 x = \cos^2 x$

$1 + 2 \sin x + \sin^2 x = 1 - \sin^2 x$

$2 \sin x + 2 \sin^2 x = 0$

$2 \sin x (1 + \sin x) = 0$

$2 \sin x = 0$ $1 + \sin x = 0$

$\sin x = 0$ $\sin x = -1$

$0, \pi$ $\frac{3\pi}{2}$

is there an extraneous root?

$1 + \sin(\pi) \neq -\cos(\pi)$

$1 + \sin(\frac{3\pi}{2}) = -\cos(\frac{3\pi}{2})$

$1 + \sin(\frac{3\pi}{2}) = -\cos(\frac{3\pi}{2})$

34. $\sin x \cos(\pi/3) = \cos x \sin(\pi/3) = 1/2$

$a = x$ $b = \pi/3$

$\sin(a - b)$

$\sin(x - \pi/3) = \frac{1}{2}$ $\theta = x - \pi/3$

$\sin \theta = \frac{1}{2}$

QI $x - \pi/3 = \frac{\pi}{6} + 2\pi k$

$x = \frac{\pi}{6} + 2\pi k + \frac{\pi}{3}$

$k=0$ $\frac{\pi}{6} + \frac{\pi}{3} = \frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6}$

$k=1$ too big $(0, 2\pi)$

QII $x - \pi/3 = \frac{5\pi}{6} + 2\pi k$

$x = \frac{5\pi}{6} + \frac{\pi}{3} + 2\pi k$

interval is $(0, 2\pi)$

so I do not need $2\pi k$

$x = \frac{7\pi}{6}$

Find all angles in the interval $[0^\circ, 360^\circ)$ that are solutions to each equation.

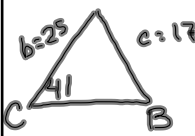
35. $\cos(3\theta) = \sqrt{3} \sin(3\theta)$
 angles are 3θ
 $\frac{\cos(3\theta)}{\sin(3\theta)} = \sqrt{3}$
 $\cot(3\theta) = \sqrt{3}$
 $3\theta = 30^\circ + 360^\circ k$
 $\theta = \frac{30}{3} + \frac{360k}{3}$
 $\theta = 10^\circ + 120^\circ k$
 $3\theta = 210^\circ + 360^\circ k$
 $\theta = 70^\circ + 120^\circ k$

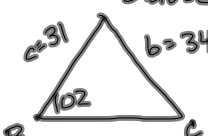
36. $0 = \sin(2x) - \sin^2(2x) + \cos^2(2x)$
 $0 = \sin(2x) - \sin^2(2x) + 1 - \sin^2(2x)$
 $0 = \sin(2x) - 2\sin^2(2x) + 1$
 $2\sin^2(2x) - \sin(2x) - 1 = 0$
 $2u^2 - u - 1 = 0$
 $(2u+1)(u-1) = 0$
 $2u+1=0 \quad u-1=0$
 $2u=-1 \quad u=1$
 $u = -\frac{1}{2} \quad \sin(2x) = 1$
 $\sin(2x) = \frac{1}{2}$
 QI $2x = 30^\circ + 360^\circ k$
 $x = 15^\circ + 180^\circ k$
 $105^\circ, 285^\circ$
 QII $2x = 150^\circ + 360^\circ k$
 $x = 75^\circ + 180^\circ k$
 $165^\circ, 345^\circ$

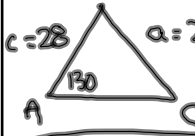
Part II: Calculator
 Answer the question in complete sentences.
 37. Explain how to determine how many triangles can be made when you are given the lengths of two sides and the measure of an angle that is not between the two sides. (SSA).

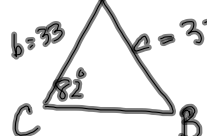
Draw \triangle with angle in left corner
 put in the other 2 sides
 Find height
 $h = b \sin A$
 zero triangle if $h > a$
 1 triangle if $h < a < b$
 2 triangles if $h < a < b$
 if angle is obtuse
 zero \triangle if $rt < left$
 or 1 \triangle if $right > left$

Determine the number of triangles with the given parts. Do not solve the triangles.

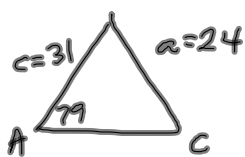
38. $\gamma = 41^\circ, b = 25, c = 17$

 $h = 25 \sin 41$
 $h = 16.4$
 $16.4 < 17 < 25$
 2 triangles

39. $\beta = 102^\circ, b = 34, c = 31$

 obtuse
 $b > c$
 1 triangle

40. $\alpha = 130^\circ, a = 25, c = 28$

 obtuse
 $25 < 28$
 no triangle

41. $\gamma = 82^\circ, b = 33, c = 37$

 $h = 33 \sin 82$
 $h = 32.6$
 1 triangle
 $h < c$ rt side $c > b$ rt > left

42. $\alpha = 79^\circ, c = 31, a = 24$



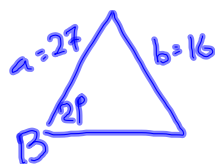
$$h = 31 \sin 79$$

$$h = 30.4$$

$$h > a$$

no triangle

43. $\beta = 21^\circ, a = 27, b = 16$



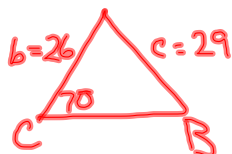
$$h = 27 \sin 21$$

$$h = 9.7$$

$$h < 16 < 27$$

2 Δ s

44. $\gamma = 70^\circ, c = 29, b = 26$

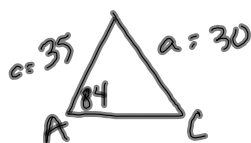


$$h = 26 \sin 70$$

$$h = 24 \quad 24 < 29 \quad 29 > 26$$

1 triangle

45. $\alpha = 84^\circ, a = 30, c = 35$



$$h = 35 \sin 84$$

$$h = 34.8$$

no triangle

Solve all possible triangles with the given parts. Round to the nearest tenth.

46. $a = 18.4, \beta = 44.4^\circ, c = 29.1$

$$\angle A = 38.9^\circ$$

$$a = 18.4$$

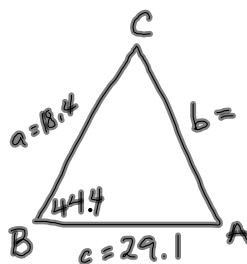
$$\angle B = 44.4$$

$$b = 20.5$$

$$\angle C = 96.7^\circ$$

$$c = 29.1$$

$$180 - 44.4 - 38.9$$



SAS
Law of Cosines

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 18.4^2 + 29.1^2 - 2(18.4)(29.1) \cos 44.4$$

$$b^2 = 420.25$$

$$b = 20.5$$

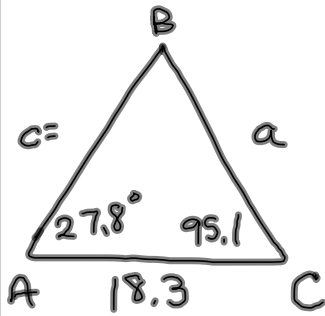
$$\frac{\sin A}{18.4} = \frac{\sin 44.4}{20.5}$$

$$\sin A = 18.4 \sin(44.4) \div 20.5$$

$$\sin A = .62799$$

$$\sin^{-1}(.62799) = 38.9^\circ$$

47. $\alpha = 27.8^\circ, \gamma = 95.1^\circ, b = 18.3$



$\angle A = 27.8$
 $\angle B = 57.1$
 ~~$180 - 27.8 - 95.1$~~
 $\angle C = 95.1$

$a = 10.2$
 $b = 18.3$
 $c = 21.7$

ASA 1 triangle

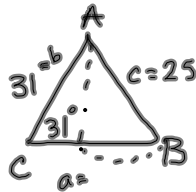
$\frac{\sin 27.8}{a} = \frac{\sin 57.1}{18.3}$

$a = 10.16$

$\frac{\sin 95.1}{c} = \frac{\sin 57.1}{18.3}$

$c = 21.7$

48. $\gamma = 31^\circ, b = 31, c = 25$



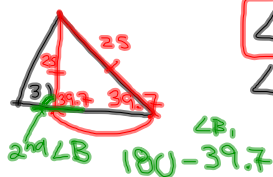
$\angle A = 109.3$
 $\angle B = 39.7^\circ$
 $\angle C = 31^\circ$
 $a_1 = 45.8$
 $b = 31$
 $c = 25$

$h = 31 \sin 31$
 $h = 15.9$
 $15.9 < 25 < 31$
 2 triangles

$\frac{\sin 31}{25} = \frac{\sin B}{31}$
 $\sin B = 31 \sin 31 \div 25$
 $\sin^{-1}(0.6386) = B$
 $B = 39.7^\circ$
 $\angle A = 180 - 39.7 - 31$
 $\frac{\sin 109.3}{a} = \frac{\sin 31}{25}$
 $a = 45.8$

2nd Δ

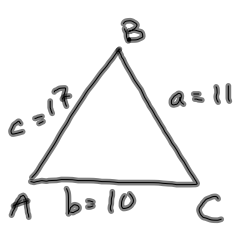
$\angle A_2 = 8.7^\circ$
 $\angle B_2 = 140.3^\circ$
 $\angle C_2 = 31^\circ$
 $a_2 = 7.3$
 $b_2 = 31$
 $c_2 = 25$



$\angle A = 180 - 31 - 140.3$

$\frac{\sin 8.7}{a} = \frac{\sin 31}{25}$
 $a = 7.3$

49. $a=11, b=10, c=17$



$\angle A = 38^\circ$
 $\angle B = 34^\circ$
 $\angle C = 108^\circ$
 $180 - 38 - 34$

$a=11$
 $b=10$
 $c=17$

SSS
Law of Cosines

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$10^2 = 11^2 + 17^2 - 2(11)(17) \cos B$$

$$100 = 121 + 289 - 374 \cos B$$

$$\frac{100 - 410}{-410} = \frac{-374 \cos B}{-374}$$

$$.8288 = \cos B$$

$$\cos^{-1}(.8288) = B$$

$\angle B = 34.0$

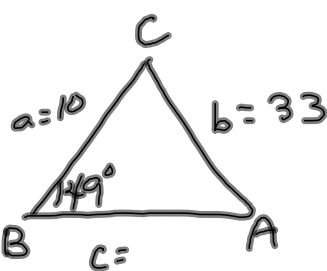
$$\frac{\sin 34}{11} = \frac{\sin A}{10}$$

$$11 \sin 34 \div 10 = \sin A$$

$$A = 38^\circ$$

Solve all possible triangles with the given parts. Round to the nearest tenth.

50. $\beta = 149^\circ, a=10, b=33$



$\angle A = 9^\circ$
 $\angle B = 149^\circ$
 $\angle C = 22^\circ$
 $180 - 149 - 9$

$a=10$
 $b=33$
 $c=24$

$h = 10 \sin 149$
 $h = 5.15$

1 Δ

$$\frac{\sin 22}{c} = \frac{\sin 149}{33}$$

$$33 \sin 22 \div \sin 149 = c$$

$$c = 24.0$$

$$\frac{\sin 149}{33} = \frac{\sin A}{10}$$

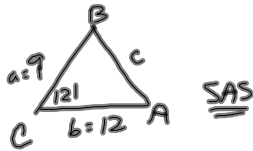
$$10 \sin 149 \div 33 = \sin A$$

$$.1561 = \sin A$$

$$A = 8.97 \approx 9.0^\circ$$

Find the area of each triangle.

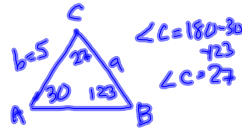
51. $a=9, b=12, \gamma=121^\circ$



Area = $\frac{1}{2} \text{side} \cdot \text{side} \cdot \sin(\text{Angle})$
 $\frac{1}{2}(12)(9)\sin 121$

A = 46.287

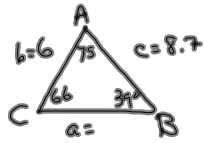
52. $\alpha=30^\circ, \beta=123^\circ, b=5$



$\frac{\sin 30}{a} = \frac{\sin 123}{5}$
 $5 \sin 30 = \sin 123$
 $2.5 = \sin 123$
 2.92

$A = \frac{1}{2}(2.92)(5)\sin 27$
A = 9.3

53. $\gamma=66^\circ, b=6, c=8.7$



$\frac{\sin 66}{8.7} = \frac{\sin B}{6}$
 $6 \sin 66 = 8.7 \sin B$
 $B = 39^\circ$

$180 - 39 - 66$
 75
 SAS
 $A = \frac{1}{2}(6)(8.7)(\sin 75)$
A = 25.2

54. $a=4, b=8, c=10$

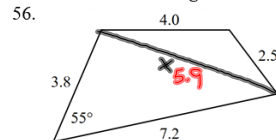
Heron!
 $\frac{(4+8+10)}{2} = S$
 $11 = S$

$A = \sqrt{11(11-4)(11-8)(11-10)}$
 $\sqrt{231}$
15.2

55. $a=4.5, b=6, c=8.7$

$\frac{(4.5+6+8.7)}{2}$
 $S = 9.6$
 $A = \sqrt{9.6(9.6-4.5)(9.6-6)(9.6-8.7)}$
 $A = \sqrt{158.6}$
12.6

Find the area of the figure below.



$x^2 = 3.8^2 + 7.2^2 - 2(3.8)(7.2)\cos 55$
 $x^2 = 34.89$
x = 5.9

SSS Heron
 $\frac{(4.0+2.5+5.9)}{2}$
 $S = 6.2$

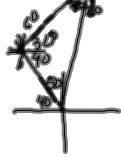
SAS
 $\text{Area} = \frac{1}{2}(3.8)(7.2)(\sin 55)$
11.2

$\sqrt{6.2(6.2-4)(6.2-2.5)(6.2-5.9)}$
 $\sqrt{15.14}$
3.89

total area $11.2 + 3.89$
15.1

Solve. Round answers to the nearest tenth.

57. A canoe travels 3 miles from the coast to an island along a course with a bearing of 310° . It then turns and travels for some distance on a new course with a bearing of 60° , and finally returns back to its starting point on a course with a bearing of 200° . How far did the canoe travel on the final leg of its journey?

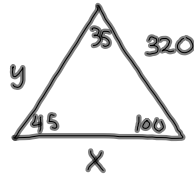


$$\frac{\sin 40}{3} = \frac{\sin 70}{X}$$

$$3 \sin 70 \div \sin 40 = X$$

$$X \approx 4.4 \text{ mi}$$

58. A surveyor locating the corners of a triangular piece of property started at one corner and walked 320 ft in the direction $S25^\circ E$ to reach the next corner. The surveyor turned and walked $S55^\circ W$ to get to the next corner of the property. Finally, the surveyor walked in the direction $N10^\circ E$ to get back to the starting point. Find the perimeter and the area of the property.



$$\frac{\sin 45}{320} = \frac{\sin 35}{X}$$

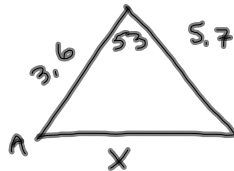
$$X = 259.6$$

$$\frac{\sin 45}{320} = \frac{\sin 100}{y}$$

$$y = 445.67$$

$$X + y = 1025.3 \text{ ft}$$

59. To find the distance between two small towns, an electronic distance measuring (EDM) instrument is placed on a hill from which both towns are visible. If the distance from the EDM to the towns is 3.6 miles and 5.7 miles and the angle between the two lines of sight is 53° , what is the distance between the towns?



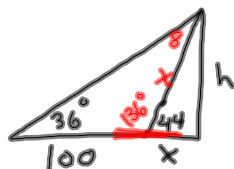
$$X^2 = 3.6^2 + 5.7^2 - 2(3.6)(5.7)\cos 53$$

$$X^2 = 20.75$$

$$X = 4.555$$

$$\approx 4.6 \text{ miles}$$

60. A tourist spots a rock climber quite high up at Devil's Tower in Wyoming. The angle of elevation of the climber is 36° . From a point that is 100 ft. closer to the climber, the angle of elevation is 44° . What is the height of the climber, to the nearest foot?



$$\frac{\sin 8}{100} = \frac{\sin 36}{X}$$

$$X = 100 \sin 36 \div \sin 8$$

$$X = 422.34$$

$$\frac{\sin 90}{422.34} = \frac{\sin 44}{h}$$

$$422.34 \sin 44 \div \sin 90$$

$$h = 293.38 \text{ ft}$$

$$\approx 294 \text{ ft}$$