Precalculus – Exam 5 Review

Use identities to simplify each expression:

1. $\dfrac{\tan x \csc x}{\sec x}$

$\tan x \csc x \div \sec x$

$\dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} \div \dfrac{1}{\cos x}$

$\dfrac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \dfrac{1}{\cancel{\sin x}} \cdot \dfrac{\cancel{\cos x}}{1} = \boxed{1}$

2. $\tan^2 x - \dfrac{\sin(-x)}{\sin x}$

$\tan^2 x - \dfrac{-\sin x}{\sin x}$

$\tan^2 x + 1$

$\boxed{\sec^2 x}$

3. $\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x}$

$\dfrac{\cos(2x)}{\sin(2x)}$

$\boxed{\cot(2x)}$

4. $\dfrac{1}{1+\sin\alpha} + \dfrac{\sin\alpha}{\cos^2\alpha}$

$\dfrac{1}{1+\sin a} + \dfrac{\sin a}{1-\sin^2 a}$

$\dfrac{(1-\sin a)\,1}{(1-\sin a)(1+\sin a)} + \dfrac{\sin a}{(1-\sin a)(1+\sin a)}$

$\dfrac{1-\sin a+\sin a}{1-\sin^2 a}$

$\dfrac{1}{\cos^2 a}$

$\boxed{\sec^2 a}$

5. $\dfrac{2\tan(2\theta)}{1-\tan^2 2\theta}$

Let $2\theta = x$

$\boxed{\dfrac{2\tan(x)}{1-\tan^2(x)}}$

trig id $\tan(2x)$

$\tan(2\cdot 2\theta)$

$\boxed{\tan(4\theta)}$

6. $\sin\theta\cos\theta(\tan\theta+\cot\theta)$

$\dfrac{\sin\theta}{1}\cdot\dfrac{\cancel{\cos\theta}}{1}\cdot\dfrac{\sin\theta}{\cancel{\cos\theta}} + \dfrac{\cancel{\sin\theta}}{1}\cdot\dfrac{\cos\theta}{1}\cdot\dfrac{\cos\theta}{\cancel{\sin\theta}}$

$\sin^2\theta + \cos^2\theta$

$\boxed{1}$

7. $\sin^2 x \tan^2 x + \sin^2 x$

$\sin^2 x(\tan^2 x + 1)$

$\sin^2 x(\sec^2 x)$

$\dfrac{\sin x}{1}\cdot\dfrac{\sin x}{1}\cdot\dfrac{1}{\cos x}\cdot\dfrac{1}{\cos x}$

$\dfrac{\sin^2 x}{\cos x^2} = \boxed{\tan^2 x}$

8. $\cos 75^\circ \cos 60^\circ + \sin(-75^\circ)\sin 60^\circ$

$a=75$ $b=60$ $a=-75$ $b=60$

$\cos 75 \cos 60 - \sin 75 \sin 60$

$\cos(a+b)$

$\cos(75+60)$

$\cos(135^\circ)$

$\boxed{\dfrac{-\sqrt{2}}{2}}$

9. $\sin 80° \cos(-50°) - \sin 10° \sin 50°$

even

$a = 80 \quad b = -50 \quad a = 10 \quad b = 50$

$\sin 80 \cos 50 - \sin 10 \sin 50$

sin10 is a cofunction of cos 80

$\sin 80 \cos 50 - \cos 80 \sin 50$

$\sin(80 - 50)$

$\sin 30$

$\boxed{\frac{1}{2}}$

10. $\frac{\sin(4y)}{1+\cos(4y)}$

let $x = 4y$

$\frac{\sin(x)}{1+\cos(x)}$

$\tan\left(\frac{x}{2}\right)$

$\tan\left(\frac{4y}{2}\right)$

$\boxed{\tan(2y)}$

Verify each identity:

11. $\frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

$\sin x \cos x \div \tan x$

$\sin x \cos x \div \frac{\sin x}{\cos x}$

$\frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{1} \cdot \frac{\cos x}{\cancel{\sin x}}$

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x \stackrel{\checkmark}{=} \cos^2 x$

12. $\cot(-y) = \frac{\sin^2 y}{\cos(-y)\sin(-y)}$

$-\cot(y) = \frac{\cos^2 y}{\cos(y) \cdot -\sin y}$

$= \frac{\cos y}{\cos y} \cdot \frac{\cos y}{-\sin y}$

$1 \cdot -\cot y$

$-\cot y \stackrel{\checkmark}{=} -\cot y$

13. $\frac{\sin(2\beta)}{2\csc\beta} = \sin^2\beta\cos\beta$

trig id

$2\sin\beta\cos\beta \div 2\csc\beta$

$2\sin\beta\cos\beta \div \frac{2}{\sin\beta}$

$\frac{\cancel{2}\sin\beta\cos\beta}{1} \cdot \frac{\sin\beta}{\cancel{2}}$

$\sin^2\beta\cos\beta \stackrel{\checkmark}{=} \sin^2\beta\cos\beta$

14. $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2\cot^2\theta$

$\frac{(\sec\theta+1)}{\sec\theta+1} \cdot \frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta+1} \cdot \frac{\sec\theta - 1}{\sec\theta - 1}$

$\frac{\sec\theta + 1 - (\sec\theta - 1)}{(\sec\theta + 1)(\sec\theta - 1)}$

$\frac{\cancel{\sec\theta} + 1 \cancel{- \sec\theta} + 1}{\sec^2\theta - 1}$

$\frac{2}{\sec^2\theta - 1}$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\frac{2}{\tan^2\theta}$

$2\cot^2\theta \stackrel{\checkmark}{=} 2\cot^2\theta$

15. $\cos(3x) = \cos x(1 - 4\sin^2 x)$

$\cos(2x + x)$

$\cos(2x)\cos x - \sin(2x)\sin x$

trig $\qquad$ trig id

$(1 - 2\sin^2 x)\cos x - 2\sin x\cos x \cdot \sin x$

$\cos x - 2\sin^2 x\cos x - 2\sin^2 x\cos x$

$\cos x - 4\sin^2 x\cos x$

$\cos x(1 - 4\sin^2 x) \overset{\checkmark}{=} \cos x(1 - 4\sin^2 x)$

16. $\sin^2\left(\frac{x}{2}\right) = \frac{\csc^2 x - \cot^2 x}{2\csc^2 x + 2\csc x\cot x}$

$= \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \div$

$\frac{1 - \cos^2 x}{\sin^2 x} \div \left[\frac{2}{\sin^2 x} + \frac{2\cos x}{\sin^2 x}\right]$

$\frac{1 - \cos^2 x}{\sin^2 x} \div \frac{2 + 2\cos x}{\sin^2 x}$

$\frac{(1 - \cos x)(1 + \cos x)}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{2(1 + \cos x)}$

$\frac{(1 - \cos x)\cancel{(1 + \cos x)}}{2\cancel{(1 + \cos x)}}$

$\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$

$\left(\sqrt{\frac{1 - \cos x}{2}}\right)^2$

$\frac{1 - \cos x}{2} \overset{\checkmark}{=} \frac{1 - \cos x}{2}$

17. $\frac{\sin(2\alpha)}{1 + \cos(2\alpha)} = \tan(\alpha)$

trig id

$\frac{2\sin a\cos a}{1 + (2\cos^2 a - 1)}$

$\frac{2\sin a\cos a}{\cancel{1} + 2\cos^2 a \cancel{- 1}}$

$\frac{\cancel{2}\sin a\cancel{\cos a}}{\cancel{2}\cos a\cancel{\cos a}}$

$\frac{\sin a}{\cos a}$

$\tan a \overset{\checkmark}{=} \tan a$

18. $\sec(2\theta) = \frac{1 + \tan^2\theta}{1 - \tan^2\theta}$

$\frac{\sec^2\theta}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$

$\sec^2\theta \div \left(\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\right)$

$\frac{1}{\cos^2\theta} \div \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$\frac{1}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{\cos^2\theta - \sin^2\theta}$

$\frac{1}{\cos^2\theta - \sin^2\theta}$ trig id

$\frac{1}{\cos(2\theta)}$

$\sec 2\theta \overset{\checkmark}{=} \sec(2\theta)$

Find the exact value by using a sum or difference identity:

19. $\cos\left(\frac{7\pi}{12}\right)$

$\cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$\boxed{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}$ or $\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

20. $\sin\left(-\frac{5\pi}{12}\right)$

$\sin\left(-\frac{3\pi}{12} + -\frac{2\pi}{12}\right)$

$\sin\left(-\frac{\pi}{4} - \frac{\pi}{6}\right)$

$\sin\left(-\frac{\pi}{4}\right)\cos\frac{\pi}{6} - \cos\left(-\frac{\pi}{4}\right)\sin\frac{\pi}{6}$

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\boxed{-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}}$

OR $\boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$

21. $\tan(195°)$

$\tan(60 + 135)$

$\frac{\tan 60 + \tan 135}{1 - \tan 60\tan 135}$

$\frac{\sqrt{3} + (-1)}{1 - \sqrt{3}(-1)}$

$\boxed{\frac{\sqrt{3} - 1}{1 + \sqrt{3}}}$

Find the exact value by using a half-angle identity.

22. $\sin\left(-\frac{\pi}{8}\right)$ $\quad -\frac{\pi}{8}\cdot\frac{180}{\pi}$

$\sin(-22.5^\circ)$ QIV

sin is neg in QIV

$\sin\left(\frac{-45^\circ}{2}\right)$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$

$\sin\left(\frac{-45}{2}\right) = -\sqrt{\frac{1-\cos(-45)}{2}}$

$\sin\left(-\frac{\pi}{8}\right) = \boxed{-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}$

23. $\cos 75^\circ$ QI

$\cos\left(\frac{150}{2}\right)$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\cos\left(\frac{150}{2}\right) = \sqrt{\frac{1+\cos 150}{2}}$

$\boxed{\sqrt{\frac{1+-\frac{\sqrt{3}}{2}}{2}}}$

OR $\frac{\sqrt{2-\sqrt{3}}}{2}$

24. $\tan\left(\frac{5\pi}{8}\right)$ $\quad \frac{5\pi}{8}\cdot\frac{180}{\pi} = 112.5$ QII tan neg

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$

$\tan\left(\frac{225^\circ}{2}\right) = \frac{1-\cos 225^\circ}{\sin 225^\circ}$

$\frac{1--\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{\frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}}$

$\frac{2+\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{-\sqrt{2}}$

OR $\boxed{\frac{2+\sqrt{2}}{-\sqrt{2}}}$

Use the given information to find the exact value of the trigonometric function(s).

25. Find $\sin(A+B)$ if $\sin A = -5/13$ and $\cos B = 3/5$, with A in quadrant III and B in quadrant I.

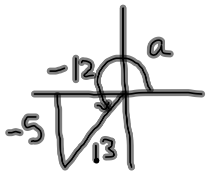
$\sin(A+B)$

$\sin A\cos B + \cos A\sin B$

$-\frac{5}{13}\cdot\frac{3}{5} + \frac{-12}{13}\cdot\frac{4}{5}$

$\frac{-15}{65} + \frac{-48}{65}$

$\boxed{\frac{-63}{65}}$



$\sqrt{13^2-(-5)^2}$

$\sqrt{144}$

$\sqrt{25-3^2}$

$\sqrt{16}$

4

Use the given information to find the exact value of the trigonometric function(s).

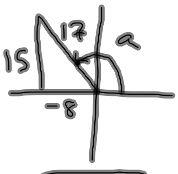
26. Find $\cos(\alpha+\beta)$ if $\sin\alpha = 15/17$ and $\sin\beta = -1/3$, with α in quadrant II and β in quadrant IV.

$\cos(a+\beta)$

$\cos a \cos\beta - \sin a \sin\beta$

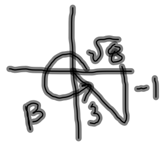
$\frac{-8}{17}\cdot\frac{2\sqrt{2}}{3} - \frac{15}{17}\cdot\frac{-1}{3}$

$\boxed{\frac{-16\sqrt{2}+15}{51}}$



$\sqrt{17^2-15^2}$

8



$\sqrt{9-1}$

27. Find $\cos(\alpha/2)$ if $\sin\alpha = -1/4$, and α is in quadrant IV.

$\cos\left(\frac{a}{2}\right) = -\sqrt{\frac{1+\cos a}{2}}$

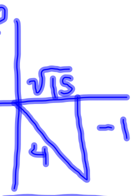
$= \boxed{-\sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}}}$

Simplified $\boxed{-\sqrt{\frac{4+\sqrt{15}}{8}}}$

$\frac{270}{2} < \frac{a}{2} < \frac{360}{2}$

$135 < \frac{a}{2} < 180$

QII



$\sqrt{4^2-(-1)^2}$

$\cos a = \frac{\sqrt{15}}{4}$

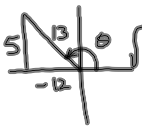
28. Find $\sin(2\theta)$ if $\cos(\theta) = -12/13$, and θ is in quadrant II.

$\sin(2\theta)$

$2\sin\theta\cos\theta$

$\frac{2}{1}\cdot\frac{5}{13}\cdot\frac{-12}{13}$

$\boxed{\frac{-120}{169}}$



$\sqrt{13^2-(-12)^2}$

5

29. Find $\sin\beta$, $\cos\beta$, and $\tan\beta$ if $\cos(2\beta) = 24/25$ and $180° < 2\beta < 360°$.

$\frac{180°}{2} < \frac{2\beta}{2} < \frac{360°}{2}$

$90° < \beta < 180°$

Q2

$\cos(2\beta) = \frac{24}{25}$

$2\cos^2\beta - 1 = \frac{24}{25}$

$2\cos^2\beta = \frac{24}{25} + 1$

$2\cos^2\beta = \frac{24}{25} + \frac{25}{25}$

$\frac{1}{\cancel{2}}\cdot\cancel{2}\cos^2\beta = \frac{49}{25}\cdot\frac{1}{2}$

$\cos^2\beta = \frac{49}{50}$

$\cos\beta = \sqrt{\frac{49}{50}}$

QII neg

$\frac{adj}{hyp}$ $\frac{x}{r}$ $\boxed{\cos\beta = -\frac{7}{5\sqrt{2}}}$

$\frac{y}{r}$ $\boxed{\sin\beta = \frac{1}{5\sqrt{2}} \text{ or } \frac{1}{\sqrt{50}}}$

$\frac{y}{x}$ $\boxed{\tan\beta = \frac{1}{-7}}$

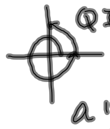
$\sqrt{(\sqrt{50})^2-(-7)^2}$

$\sqrt{50-49}$

1

30. Find $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$ if $\sin(\alpha/2) = -2/7$ and $\pi < \alpha/2 < 5\pi/4$.

$2\pi < a < \frac{5\pi}{2}$



a is in QI

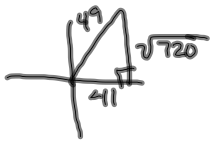
$\sin\left(\frac{a}{2}\right) = \frac{-2}{7}$

$\sin\left(\frac{a}{2}\right) = -\sqrt{\frac{1-\cos a}{2}}$

$\left(\frac{-2}{7}\right)^2 = \left(-\sqrt{\frac{1-\cos a}{2}}\right)^2$

$2 \cdot \frac{4}{49} = \frac{1-\cos a}{2} \cdot 2$

$\frac{8}{49} = 1 - \cos a$



$\frac{8}{49} - 1 = -\cos a$

$\frac{8-49}{49} = -\cos a$

$\sqrt{49^2 - 41^2}$

$\sqrt{720}$

$\frac{-41}{49} = -\cos a$

$\boxed{\frac{41}{49} = \cos a}$ $\tan a = \boxed{\frac{\sqrt{720}}{41}}$

$\boxed{\frac{\sqrt{720}}{41} = \sin a}$