Notes 2.8 pre-calc

Solving Inequalities in One Variable

Polynomial Inequalities: Every polynomial inequality can be written in the form $f(x) > 0$, $f(x) \geq 0$, $f(x) < 0$, $f(x) \leq 0$ where $f(x)$ is a polynomial. There is a fundamental connection between inequalities and the positive or negative sign of the corresponding expression $f(x)$:

1. To solve the inequality $f(x) > 0$ is to find the values of x that make $f(x)$ positive.
2. To solve the inequality $f(x) < 0$ is to find the values of x that make $f(x)$ negative.
3. If the expression $f(x)$ is a product, determine its sign by determining the sign of each of its factors.

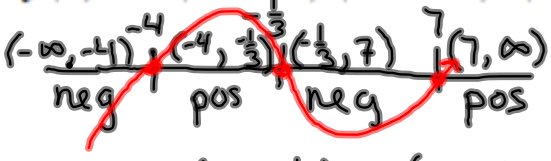
Note: To sketch a graph of a polynomial use end behavior, multiplicity of real zeros, and a chart of signs.

Examples: Determine the x values that cause the polynomial to be zero, positive and negative.

a) $f(x) = (x-7)(3x+1)(x+4)$

Zeros $x=7$ $x=-\frac{1}{3}$ $x=-4$

Cubic



positive $(-4,-\frac{1}{3}) \cup (7,\infty)$

negative $(-\infty,-4) \cup (-\frac{1}{3},7)$

zeros: $x=7$, $x=-\frac{1}{3}$, $x=-4$

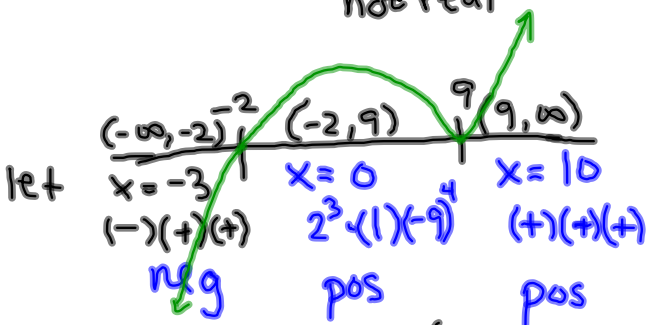
b) $f(x) = (x+2)^3(4x^2+1)(x-9)^4$

Zeros	Multiplicity	Touch / cross
$x=-2$	3	C
$x=9$	4	T

$x=-2$ $x=9$ imag zero not real

END BEH. $x^3 \cdot 4x^2 \cdot x^4$

$4x^9$



positive $(-2,9) \cup (9,\infty)$

negative $(-\infty,-2)$

zeros $x=-2$ $x=9$

Solve the polynomial inequality using a sign chart. Support graphically.

$(2x+1)(x-2)(3x-4) \leq 0$ ← below x-axis negative; ← include zeros

$2x+1=0$

zeros: $x=-\frac{1}{2}$ $x=2$ $x=\frac{4}{3}$

END Behavior $2x(x)(3x)$

$6x^3$ ↗

below and zeros: $(-\infty,-\frac{1}{2}] \cup [\frac{4}{3},2]$

$x^3-4x^2+x+6 \geq 0$ cubic

need above x-axis and zeros

-1	1	-4	1	6
		-1	5	-6
	1	-5	6	0

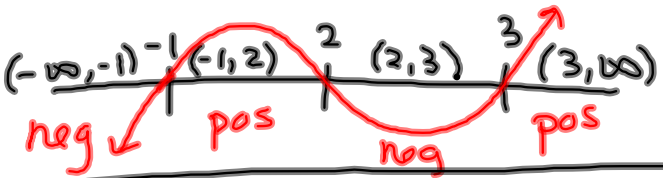
x^2-5x+6 factors to $(x-2)(x-3)$

$x-2=0$ $x-3=0$

$x=2$ $x=3$

zero: $x=-1$

zeroes, $x=-1, x=2, x=3$



answer is: $[-1,2] \cup [3,\infty)$

$(3-2x)^2(x^2+4) \le 0$ below and zero

Zeros $3-2x=0$ $x^2+4=0$

$3=2x$ $x^2=-4$

$\frac{3}{2}=x$ $x=\pm 2i$ not real

nothing below zero is answer

$[\frac{3}{2}]$ or $\{\frac{3}{2}\}$

$(3-2x)^2(x^2+4) \ge 0$

above and zero

All reals

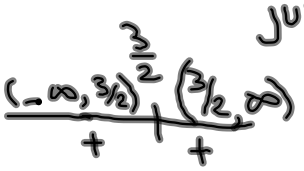
$(3-2x)^2(x^2+4) < 0$

just below x-axis

no solution

$(3-2x)^2(x^2+4) > 0$

just above positive



don't include $\frac{3}{2}$

$(-\infty,\frac{3}{2}) \cup (\frac{3}{2},\infty)$

Determine the x values that cause the function to be zero, undefined, positive and negative

$$\frac{x^2-4}{x^2+4} \qquad \frac{(x-2)(x+2)}{x^2+4}$$

zeros $(x-2)(x+2)=0$

denominator never 0.

zero: $x=2$ $x=-2$

undefined: never (no V.A.)

$(-\infty,-2)$ | $(-2,2)$ | $(2,\infty)$

pos | neg | pos

positive $(-\infty,-2)\cup(2,\infty)$

negative $(-2,2)$

$$\frac{x+2}{x^2-9} \qquad (x+3)(x-3)$$

Set $x+2=0$ to find zeros.

undefined set denominator $=0$

zero: $x=-2$

undefined $x=-3$ $x=3$

$(-\infty,-3)$ | $(-3,-2)$ | $(-2,3)$ | $(3,\infty)$

Let $x=-4$ | Let $x=-2.5$ | Let $x=0$ | let $x=4$

$\frac{-2}{7}$ neg | pos | $\frac{2}{-9}$ neg | $\frac{6}{7}$ pos

positive $(-3,-2)\cup(3,\infty)$

negative $(-\infty,-3)\cup(-2,3)$

$$\frac{x^2+3x-10}{x^2-6x+9} \Rightarrow \frac{(x+5)(x-2)}{(x-3)(x-3)} \text{ or } \frac{(x+5)(x-2)}{(x-3)^2}$$

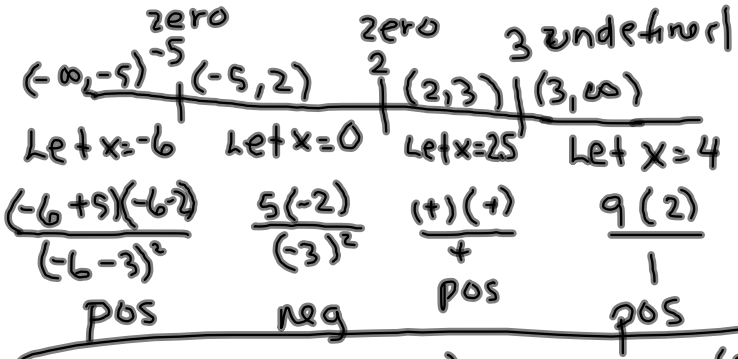
factor

set numerator = 0 to find zeros

set denominator = 0 to find undefined

zeros: $x=-5$ $x=2$

undefined: $x=3$



positive: $(-\infty,-5) \cup (2,3) \cup (3,\infty)$

negative: $(-5,2)$

Solve the inequality using a sign chart.

$$\frac{x^2-4}{x^2+4} \leq 0$$

negative and zeros or below x-axis

$$\frac{(x-2)(x+2)}{x^2+4}$$

never undefined

zeros $x=2$ $x=-2$

$(-\infty,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
Let $x=-3$		Let $x=0$		Let $x=3$
$\frac{(-5)(-1)}{13}$		$\frac{-4}{4}$		$\frac{5}{13}$
pos		neg		pos

answer: $[-2,2]$

$$\frac{x+2}{x^2-9} \geq 0$$

above and zero need positive and zero

$(x-3)(x+3)$

zero $x+2=0$ $x=-2$

undefined $x=3$ $x=-3$

$(-\infty,-3)$	-3	$(-3,-2)$	-2	$(-2,3)$	3	$(3,\infty)$
Let $x=-4$		Let $x=-2.5$		Let $x=0$		Let $x=4$
$\frac{-4+2}{16-9}$		$\frac{-2.5+2}{6.25-9}$		$\frac{2}{-9}$		$\frac{6}{7}$
neg		$\frac{neg}{neg}$ pos		neg		pos

answer: $(-3,-2] \cup (3,\infty)$

Solve the inequality using a sign chart.

$$\frac{x^2+3x-10}{x^2-6x+9}<0$$

only negative or below x-axis

factor $\frac{(x+5)(x-2)}{(x-3)^2}$

zeros: $x+5=0$ $x=-5$ $x-2=0$ $x=2$

Undefined at $x=3$

zero -5, zero 2, 3

$(-\infty,-5)$ | $(-5,2)$ | $(2,3)$ | $(3,\infty)$

Let $x=-6$ Let $x=0$ Let $x=2.5$ Let $x=4$

$\frac{(-1)(-8)}{(-9)^2}$ pos $\frac{(5)(-2)}{(-3)^2}$ neg $\frac{(7.5)(.5)}{(2.5-3)^2}$ pos $\frac{(9)(2)}{7^2}$ pos

need below not zeros

answer $(-5,2)$