

**Solving Inequalities in One Variable**

**Polynomial Inequalities:** Every polynomial inequality can be written in the form  $f(x) > 0$ ,  $f(x) \geq 0$ ,  $f(x) < 0$ ,  $f(x) \leq 0$  where  $f(x)$  is a polynomial. There is a fundamental connection between inequalities and the positive or negative sign of the corresponding expression  $f(x)$ :

1. To solve the inequality  $f(x) > 0$  is to find the values of  $x$  that make  $f(x)$  positive.
2. To solve the inequality  $f(x) < 0$  is to find the values of  $x$  that make  $f(x)$  negative.
3. If the expression  $f(x)$  is a product, determine its sign by determining the sign of each of its factors.

**Note:** To sketch a graph of a polynomial use end behavior, multiplicity of real zeros, and a chart of signs.

**Examples:** Determine the  $x$  values that cause the polynomial to be zero, positive and negative.

$$a) f(x) = (x - 7)(3x + 1)(x + 4)$$

$$b) f(x) = (x + 2)^3(4x^2 + 1)(x - 9)^4$$

Solve the polynomial inequality using a sign chart. Support graphically.

$$(2x + 1)(x - 2)(3x - 4) \leq 0$$

$$x^3 - 4x^2 + x + 6 \geq 0$$

$$(3 - 2x)^2(x^2 + 4) \leq 0$$

$$(3 - 2x)^2(x^2 + 4) \geq 0$$

$$(3 - 2x)^2(x^2 + 4) < 0$$

$$(3 - 2x)^2(x^2 + 4) > 0$$

Determine the x values that cause the function to be zero, undefined, positive and negative

$$\frac{x^2 - 4}{x^2 + 4}$$

$$\frac{x + 2}{x^2 - 9}$$

$$\frac{x^2 + 3x - 10}{x^2 - 6x + 9}$$

Solve the inequality using a sign chart.

$$\frac{x^2 - 4}{x^2 + 4} \leq 0$$

$$\frac{x + 2}{x^2 - 9} \geq 0$$

$$\frac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$$