Mathematical Models: Building Functions

Functions from data –

Given a set of data points of the form (x, y) construct a formula that approximates y as a function of x

1) Make a scatter plot of the data points.
2) Determine from the shape of the plot whether the points seem to follow the graph of a familiar type of function (line, parabola, cubic, sine, cosine, etc.)
3) Transform a basic function of that type to fit the points as closely as possible.

Line of Regression -- line of best fit

The effectiveness of a data-based model is highly dependent on the number of data points and on the way they were selected.

On the graphing calculator- r is the correlation coefficient

r^2 is the coefficient of determination. The closer the absolute value of this number is to 1, the better the curve fits the data.

Functions from Formulas

Formulas can be solved to give one variable explicitly as a function of the other. We will use a variety of formulas to pose and solve problems algebraically.

Functions from Graphs

Given a graph -- the graph provides valuable information about a function. It is sometimes actually easier to start with the graphical model than it is to go straight to the algebraic formula.

Functions from Verbal Descriptions

This is difficult. Creating a function from verbal cues requires reading the problem carefully. Understanding what the words say is critical if you hope to model the situation they describe.

Real-world problems often result in mathematical models that involve functions. These functions need to be constructed or built based on the information given. In constructing functions, we must translate the verbal description into the language of math. We do this by assigning symbols to represent the independent and dependent variables and then by finding the function or rule that relates these variables.

Examples:

Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$.

a) Express the distance *d* from point *P* to the point $0, -1$ as a function of *x*.

b) What is d if $x = 0$? If $x = -1$?

c) Use a graphing utility to graph $d = d \;\; x$. For what values of x is d smallest?

A rectangle has one corner in quadrant I on the graph of $y = 9 - x^2$, another at the origin, a third on the positive *x*-axis, and a fourth on the positive *y*-axis.

a) Express the area *A* of the rectangle as a function of *x*.

b) What is the domain of *A*?

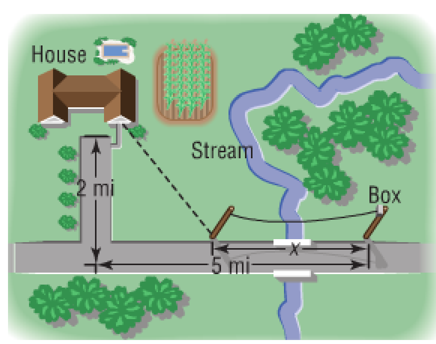
c) Graph $A = A \;\; x$. For what value of x is the area largest?

An isosceles triangle has its base along the positive *x*-axis with one vertex at the origin, another on the *x*-axis, and a third on the graph of $y = 4\sqrt{x}$. Express the area, *A*, of the triangle as a function of the *length of the base*.

Suppose two planes flying at the same altitude are headed toward each other. One plane is flying due south at a groundspeed of 400 miles per hour and is 600 miles from the potential intersection point of the planes. The other plane is flying due east with a groundspeed of 250 miles per hour and is 400 miles from the potential intersection point of the planes.

a) Build a model that expresses the distance *d* between the planes as a function of the time *t*.
b) Use a graphing utility to graph $d = d(t)$. How close do the planes come to each other? At what time are they closest to each other?

A cable company needs to provide service to a customer whose house is located 2 miles from the road along which the cable is buried. The nearest connection box for the cable is located 5 miles down the road. See the figure.



a) If the installation cost is \$500 per mile along the road and \$700 per mile off the road, build a model that expresses the total cost *C* of the installation as a function of the distance *x* (in miles) from the connection box to the point where the cable installation turns off the road.
b) Give the domain.
c) Compute the cost if $x = 1$ and if $x = 3$ miles.
d) Graph the function $C = C(x)$. What value of x results in the least cost? What is the least cost?