Precalculus – Homework #5 section 1.6 Notes

Transformations pages 138-147

Transformations – functions that map real numbers to real numbers

Rigid Transformations – leave the size and shape of a graph unchanged (horizontal and vertical translations, reflections)

Non-rigid transformations – distort shape of graph (horizontal and vertical stretches and shrinks)

Vertical translation shifts graph up or down

Horizontal translation shifts graph left or right.

Y = f(x-c) to right by c units. Y = f(x+c) to left by c units.

Y = f(x) + c up by c units. Y = f(x) - c down by c units

Reflections – (x, y) and (x, -y) reflections of each other across the x-axis. (x, y) and (-x, y) reflections of each other across the y-axis.

Across the x-axis y = f(x) $\rightarrow$ y = - f(x)

Across the y-axis y = f(x) $\rightarrow$ y = f(-x)

Vertical Stretches or shrinks

y = c $\bullet$ f(x) a stretch by a factor of c if c>1

A shrink by a factor of c if 0<c <1

Horizontal stretches or shrinks

y = f $\left(\frac{x}{c}\right)$ a stretch by a factor of c if c >1

a shrink by a factor of c if 0 < c < 1

Parent Graph: $y = f(x)$

Offspring: Transformations of the parent graph.

	$f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = 2^x$	Effect on Parent Graph
$y = -f(x)$				
$y = f(-x)$				
$y = 2f(x)$				
$y = \frac{1}{2}f(x)$				
$y = f(2x)$				
$y = f(\frac{1}{2}x)$				
$y = f(x) + 2$				
$y = f(x) - 2$				
$y = f(x+2)$				
$y = f(x-2)$				

Combining transformations Transformations may be performed in succession – one after another. Pay attention to the order of the transformations….it makes a difference.

When graphing a transformed graph *based on the equation of the function,* apply transformations in the following order:

1.
2.
3.

Examples: List the transformations in the appropriate order:

Parent graph: $y = \sqrt{x}$

a) $y = -\frac{1}{2}\sqrt{x+3}$

b) $y = 5\sqrt{-x} + 3$

c) $y = \sqrt{-2x+9}$

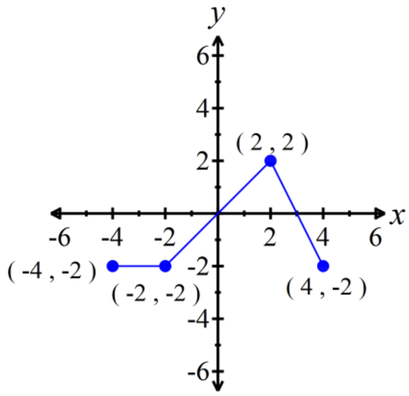
Parent graph: $f(x) = |x|$

a) $f(x) = 4|x-2| + 7$

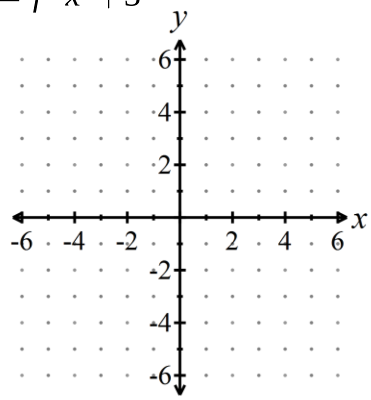
b) $f(x) = -|x+5| - 3$

c) $f(x) = -|\frac{x}{3} + 2|$

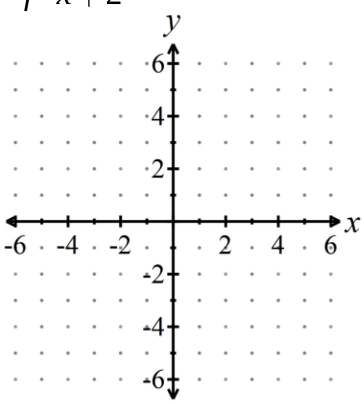
Example: The graph of a function f is illustrated below. Use the graph of f as the first step towards graphing each of the following functions:



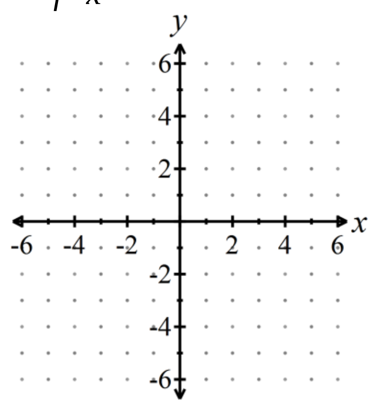
a) $F\ x = f\ x + 3$



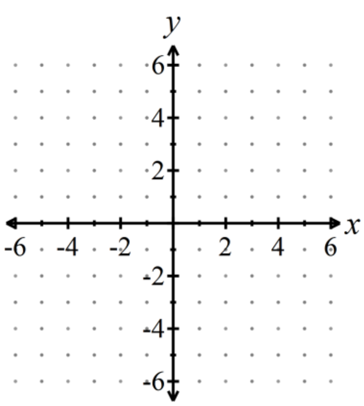
b) $G\ x = f\ x+2$



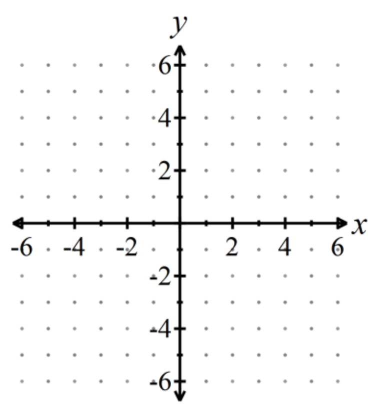
c) $P\ x = -f\ x$



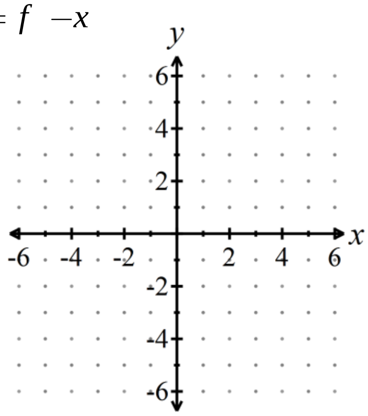
d) $H\ x = f\ x+1 - 2$



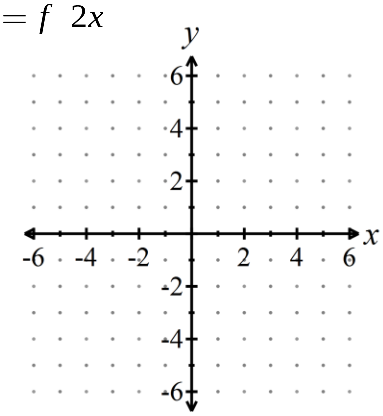
e) $Q\ x = 2f\ x$



f) $g\ x = f\ -x$

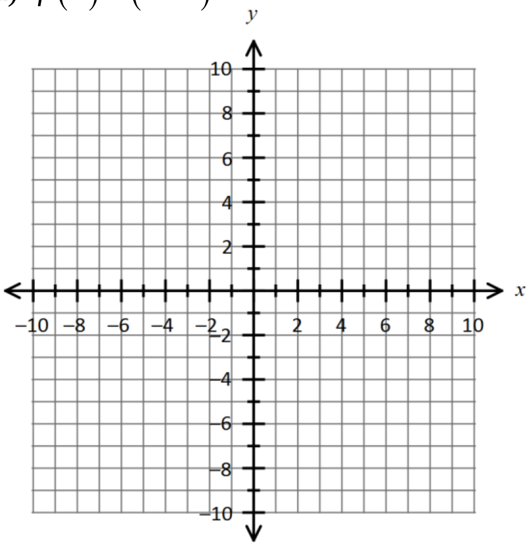


g) $h\ x = f\ 2x$

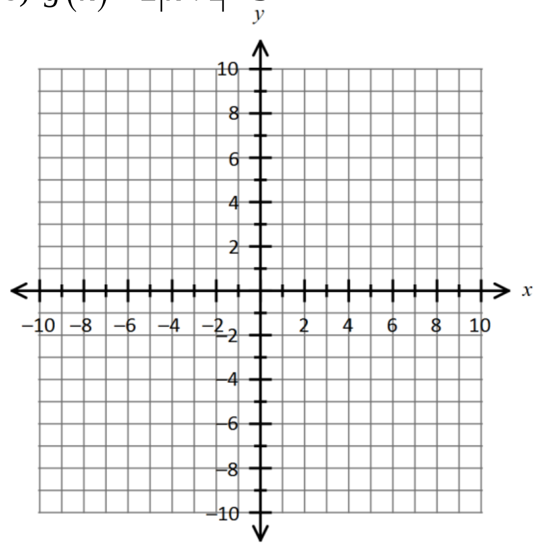


Examples: Graph the following:

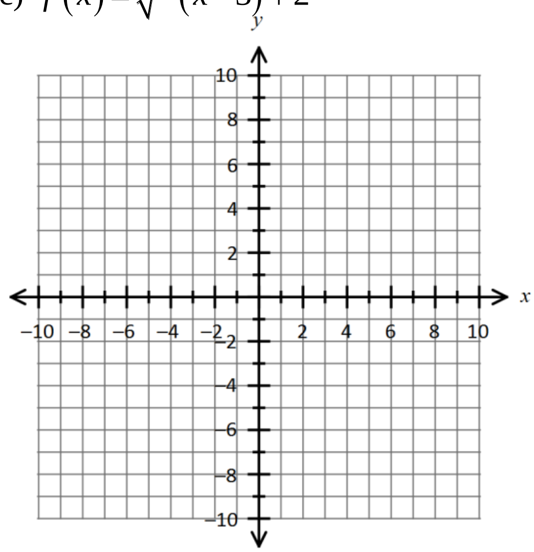
a) $f(x) = (x-1)^3 + 2$



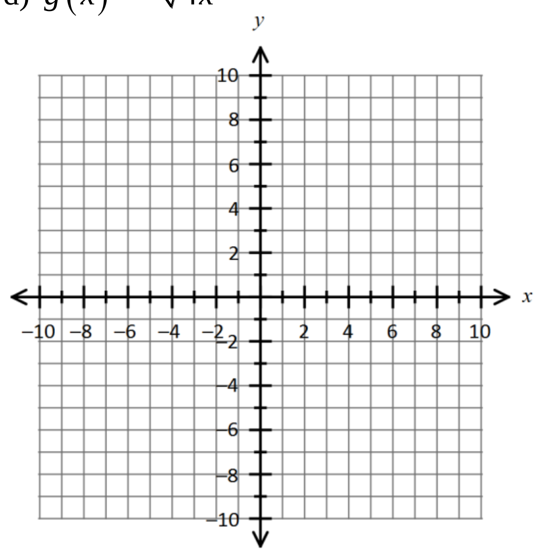
b) $g(x) = 2|x+1| - 3$



c) $f(x) = \sqrt{-(x-3)} + 2$



d) $g(x) = -\sqrt[3]{4x}$



Example: Write the equation of the function that is graphed after the following transformations are applied, in order, to the graph of $g(x) = x^3$.

1. Reflect across *y*-axis
2. Shift right 4 units
3. Vertical compression by a factor of 1/2

Example: Write the equation of the function that is graphed after the following transformations are applied, in order, to the graph of $h(x) = \sqrt{x}$.

1. Vertical stretch by a factor of 3
2. Move left 5 units
3. Reflect across the *y*-axis

Example: Write the equation of the function that is graphed after the following transformations are applied, in order, to the graph of $f(x) = |x|$.

1. Horizontal compression by a factor of 1/2
2. Move up 6 units
3. Reflect across the *x*-axis

Summary of Graphing Transformations:

To Graph:	**Draw the Graph of $y = f(x)$ and:**	**Functional Change to $y = f(x)$:**
Reflection About the *x*-axis		
$y = -f(x)$	Reflect the graph of f about the x-axis.	Multiply $f(x)$ by -1.
Reflection About the *y*-axis		
$y = f(-x)$	Reflect the graph of f about the y-axis.	Replace x by $-x$.
Vertical Stretches & Compressions		
$y = af(x),\ a > 0$	Multiply each y-coordinate of $y = f(x)$ by a. This stretches the graph of f vertically if $a > 1$. This compresses the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a.
Horizontal Stretches & Compressions		
$y = f(bx),\ b > 0$	Divide each x-coordinate of $y = f(x)$ by b. This stretches the graph of f horizontally if $0 < b < 1$. This compresses the graph of f horizontally if $b > 1$.	Replace x by bx.
Vertical Shifts		
$y = f(x) + k,\ k > 0$	Raise the graph of f by k units.	Add k to $f(x)$.
$y = f(x) - k,\ k > 0$	Lower the graph of f by k units.	Subtract k from $f(x)$.
Horizontal Shifts		
$y = f(x - h),\ h > 0$	Shift the graph of f to the right by h units.	Replace x by $x - h$.
$y = f(x + h),\ h > 0$	Shift the graph of f to the left by h units.	Replace x by $x + h$.