Name _____ Date _____ Period ___ Score _____

Calculus Chapter 10 Practice Exam

1. Write the first four terms of the series $\sum_{n=2}^{\infty} \frac{x^n}{3n-1}$

2. Tell whether the series $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$ converges or diverges. If it converges, find the sum.

3. Given that $\mathbf{1 - x + x^2 + \cdots + (-x)^n}$ is a power series representation for $\frac{1}{1+x}$ **,** find a power series representation for $\frac{x^3}{1+x^2}$ **.**

4. Find the Taylor polynomial of order 3 generated by ***f(x) = sin(2x)*** at $\boldsymbol{x = \frac{\pi}{4}}$

5. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$,

$f'(0) = -3, f''(0) = 8, f'''(0) = 24$. Write the third order Taylor polynomial for f at x = 0 and use it to approximate $f(0.4)$

6. The Maclaurin series for f(x) is $\mathbf{1 + 2x + \frac{3x^2}{2} + \frac{4x^3}{6} + \cdots + \frac{(n+1)x^n}{n!} + \cdots}$

a. Find $f''(0)$.

b. Let g(x) = $xf'(x)$. Write the Maclaurin series for g(x).

c. Let $h(x) = \int_0^x f(t)dt$. Write the Maclaurin series for h(x).

7. Find the Taylor polynomial of order 4 for $f(x) = \ln(1 - x^2)$ at x = 0 and use it to approximate f(0.3)

8. The polynomial $1 + 7x + 21x^2$ is used to approximate $f(x) = (1 + x)^7$ on the interval $-0.01 \leq x \leq 0.01$. Use the Remainder Estimation Theorem to estimate the maximum absolute error.

9. Determine the convergence or divergence of each series. Identify the test(s) you use.

a. $\sum_{n=2}^{\infty} \frac{(2n)!}{(n-1)3^n}$

b. $\sum_{n=1}^{\infty} \frac{(n^2+3n-4)}{n!}$

c. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{2n}\right)^{3n}$

10. Find the radius of convergence of each power series.

a. $\sum_{n=0}^{\infty} \frac{(5x)^n}{3^n}$

b. $\sum_{n=1}^{\infty} \frac{n^2(2x-3)^n}{6^n}$

11. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(4x-3)^{3n}}{8^n}$ and within this interval, the sum of the series as a function of x.

12. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{\sqrt{n+2}\cdot 2^n}$