Chapter 4 Calculus Practice Exam

1. Find $\frac{dy}{dx}$ $for\ y = \sin(x^2 - 1)$

CHAIN RULE

$$\frac{dy}{dx} = \cos(x^2-1)\frac{d}{dx}(x^2-1)$$

$$\cos(x^2-1)\cdot(2x)$$

$$\boxed{2x\cos(x^2-1)}$$

2. A curve is parametrized by the equations $x = \sqrt{t}$ and $y = (t-3)^2$.

Find an equation of the line tangent to the curve at the point defined by t = 9.

$$y - y_1 = m(x - x_1)$$

(x_1, y_1)

$x(9) = \sqrt{9}$

$x_1 = 3$

$y(t) = (t-3)^2$

$y(9) = (9-3)^2$

$y_1 = 36$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dt} = 2(t-3)^1 \cdot 1$$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \cdot 1 \Rightarrow \frac{1}{2\sqrt{t}}$$

or $x = \sqrt{t}$ $\quad t^{\frac{1}{2}}$

$$\frac{dx}{dt} = \frac{1}{2}t^{-1/2}$$

$$\frac{dy}{dx} = \frac{2(t-3)}{\frac{1}{2\sqrt{t}}}$$

$$\left.\frac{dy}{dx}\right|_{t=9} \frac{2(9-3)}{\frac{1}{2\sqrt{9}}} \Rightarrow \frac{12}{\frac{1}{2\cdot 3}}$$

$$12 \div \frac{1}{6}$$

$$\frac{12}{1} \cdot \frac{6}{1} = 72$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 36 = 72(x-3)}$$

3. Which of the following could be true if $f''(t) = t^{\frac{-2}{3}}$?

If $f(t) = \frac{9}{4}t^{\frac{4}{3}}$ $f'(t) = \cancel{\frac{9}{4}}\cdot\cancel{\frac{4}{3}}t^{\frac{1}{3}}$

$3t^{\frac{1}{3}}$

$f''(t) = 3\cdot\frac{1}{3}t^{\frac{-2}{3}}$

$f''(t) = t^{\frac{-2}{3}}$

True

I. $f(t) = \frac{9}{4}t^{\frac{4}{3}}$

II. $f'(t) = 7 - 3t^{\frac{1}{3}}$

III. $f'''(t) = \frac{-2}{3}t^{\frac{-5}{3}}$?

~~(A)~~ I and II

(B) I and III

(C) I only

~~(D)~~ III only

~~(E)~~ I, II, and III

II If $f'(t) = 7 - 3t^{\frac{1}{3}}$

$f''(t) = -3\cdot\frac{1}{3}t^{\frac{-2}{3}}$ False

$-t^{\frac{-2}{3}}$

III $f'''(t) = -\frac{2}{3}t^{\frac{-5}{3}}$

If $f''(t) = t^{\frac{-2}{3}}$

$f'''(t) = \frac{-2}{3}t^{\frac{-2}{3}-1}$

$-\frac{2}{3}t^{\frac{-5}{3}}$

True

4. Use implicit differentiation to find $\frac{dy}{dx}$ *if* $x^2 + 5xy + y^5 = 8$.

$$x^2 + \underbrace{(5x)y}_{\text{Product}} + (y)^5 = 8$$

$$2x + 5x \cdot \frac{dy}{dx} + y(5) + 5(y)^4 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(5x + 5y^4) = -2x - 5y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-5y}{5x+5y^4}}$$

5. Find $\frac{dy}{dx}$ *if* $y = \tan^{-1}\left(\frac{1}{2x}\right)$.

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$u = \frac{1}{2x}$

$u = (2x)^{-1}$

$\frac{du}{dx} = -1(2x)^{-2} \cdot 2$

$\frac{-2}{(2x)^2} = \frac{-2}{4x^2}$

$\frac{-1}{2x^2}$

$$\boxed{\frac{1}{1+\left(\frac{1}{2x}\right)^2} \cdot -2(2x)^{-2}}$$

OR

$$\boxed{\frac{1}{1+\left(\frac{1}{2x}\right)^2} \cdot \frac{-1}{2x^2}}$$

OR

$$\boxed{\frac{1}{1+\left(\frac{1}{2x}\right)^2} \cdot \frac{-2}{(2x)^2}}$$

simplified $\boxed{\frac{-2}{4x^2+1}}$

6. Find $\frac{dy}{dx}$ *if* $y = 4^{-x+3}$

$a = 4$ a is base

$u = -x+3$ u is exponent

$\frac{du}{dx} = -1$

$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot \frac{du}{dx}$

$\boxed{4^{-x+3} \cdot \ln 4 \cdot -1}$

7. Which of the following expression has the same derivative as $y = \log x$?

(A) $\log_6 x$ (B) $\log\ 5x$ (C) $\log\ x^2$

(D) $3 \log\ x$ (E) $\log \frac{1}{x}$

$y = \log_{10} x$

$y = \frac{\ln x}{\ln 10}$

$* \boxed{\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{1}{x}}$

Look for this

A) $\log_6 x = \frac{\ln x}{\ln 6}$

$\frac{dy}{dx} = \frac{1}{\ln 6} \cdot \frac{1}{x}$

✓B) $\log_{10} 5x = \frac{\ln 5x}{\ln 10}$

$\frac{1}{\ln 10} \cdot \frac{1}{\cancel{5}x} \cdot \cancel{5}$

$\frac{1}{\ln 10} \cdot \frac{1}{x}$

C) $\log_{10} x^2$

$\frac{\ln x^2}{\ln 10}$

$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{1}{x^2} \cdot 2x = \frac{1}{\ln 10} \cdot \frac{2}{x}$

D) $3 \log_{10} x$

$\frac{3 \ln x}{\ln 10}$ $\frac{dy}{dx} = \frac{3}{\ln 10} \cdot \frac{1}{x}$

E) $\log_{10} \frac{1}{x} = \frac{\ln \frac{1}{x}}{\ln 10}$

$\frac{\ln x^{-1}}{\ln 10}$

$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{1}{x^{-1}} \cdot -1x^{-2}$

$\frac{1}{\ln 10} \cdot x \cdot \frac{-1}{x^2}$

$\frac{-1}{\ln 10} \cdot \frac{1}{x}$

8. Find $\frac{dy}{dx}$. $\log_5(e^x - 4x)$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$

$$\boxed{\frac{1}{(e^x - 4x)\ln 5} \cdot (e^x - 4)}$$

9. Find the derivative of the function $f(x)g(h(x))$

product $g(h(x))$ composite chain rule needed!

first times deriv of 2nd + 2nd times deriv of 1st

chain rule

$$f(x) \cdot \frac{d}{dx} g(h(x)) + g(h(x)) \cdot f'(x)$$

$$\boxed{f(x) \cdot g'(h(x)) \cdot h'(x) + g(h(x)) \cdot f'(x)}$$

10. Find $\frac{dV}{dt}$ $V = 3000\left(1 - \frac{t}{20}\right)^2$

chain rule

$$3000 \cdot 2\left(1 - \frac{t}{20}\right)^1 \cdot \frac{d}{dt}\left(1 - \frac{t}{20}\right)$$

$$\boxed{6000\left(1 - \frac{t}{20}\right) \cdot \frac{-1}{20}}$$

simplified $\frac{-6000}{20}\left(1 - \frac{t}{20}\right)$

$$-300\left(1 - \frac{t}{20}\right)$$

$$-300 + 15t$$

11. Find $\frac{dh}{dr}$ $\quad h(r) = \sqrt[3]{3r} \cdot g(r)$

$h(r) = (3r)^{\frac{1}{3}} \cdot g(r)$

$\frac{dh}{dr} = (3r)^{\frac{1}{3}} \cdot g'(r) + g(r) \cdot \frac{d}{dr}(3r)^{\frac{1}{3}}$

$\boxed{(3r)^{\frac{1}{3}} \cdot g'(r) + g(r) \cdot \frac{1}{3}(3r)^{-\frac{2}{3}} \cdot 3}$

$(3r)^{\frac{1}{3}} \cdot g'(r) + g(r) \cdot (3r)^{-\frac{2}{3}}$

12. Find $\frac{df}{dy}$ $\quad f(y) = \frac{\pi}{y^8} + 3e^{\sin(-2y)}$

Not implicit $\quad f(y) = \pi y^{-8} + 3e^{\sin(-2y)}$ $\boxed{e^u}$

$\frac{df}{dy} = -8\pi y^{-9} + 3e^{\sin(-2y)} \cdot \frac{d}{dy}(\sin(-2y))$

$\boxed{-8\pi y^{-9} + 3e^{\sin(-2y)} \cdot \cos(-2y) \cdot -2}$

$\frac{-8\pi}{y^9} + -6e^{\sin(-2y)} \cos(-2y)$

13. Find $\frac{dy}{dx}$ $\quad y = \sin^{-1}(4x^3)$

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{dy}{dx}$$

$$\boxed{\frac{1}{\sqrt{1-(4x^3)^2}} \cdot 12x^2}$$

14. Find $\frac{dy}{dx}$ $\quad y = (\ln(\tan^{-1}(\pi x)))^2$

u^2

$$\frac{dy}{dx} = 2(\ln(\tan^{-1}(\pi x)))^1 \cdot \frac{d}{dx}(\ln(\tan^{-1}(\pi x)))$$

$$\boxed{2(\ln(\tan^{-1}(\pi x)))^1 \cdot \frac{1}{\tan^{-1}(\pi x)} \cdot \frac{1}{1+(\pi x)^2} \cdot \pi}$$

15. Find $\frac{dy}{dx}$ $y = x^{\ln(x)}$

$$\ln y = \ln x^{\ln(x)}$$

$$\ln y = \ln x \cdot \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{2\ln x}{x} \cdot y$$

$$\boxed{\frac{dy}{dx} = \frac{2\ln x}{x} \cdot x^{\ln x}}$$

16. Find $\frac{dy}{dx}$ if $3xy = 0$

implicit and product

$$(3x)y = 0$$

$$3x\frac{dy}{dx} + y(3) = 0$$

$$3x\frac{dy}{dx} = -3y$$

$$\frac{dy}{dx} = \boxed{\frac{-3y}{3x}}$$

or $\boxed{\frac{-y}{x}}$

17. Find $\frac{dy}{dx}$ if $y^4 + 2x^2y^2 - 3x^2 = 10$

implicit

$$4y^3\frac{dy}{dx} + 2x^2 \cdot 2y^1\frac{dy}{dx} + y^2 \cdot 4x - 6x = 0$$

$$\frac{dy}{dx}(4y^3 + 4x^2y) = -4xy^2 + 6x$$

$$\boxed{\frac{dy}{dx} = \frac{-4xy^2 + 6x}{4y^3 + 4x^2y}}$$

18. Find $\frac{df}{dy}$ if $f(y) = \left(\frac{y-3}{y+2}\right)^3$

Not implicit

CHAIN!

$$3\left(\frac{y-3}{y+2}\right)^2 \cdot \frac{d}{dy}\left(\frac{y-3}{y+2}\right)$$

Quotient

$$\boxed{3\left(\frac{y-3}{y+2}\right)^2 \cdot \frac{(y+2)(1) - (y-3)(1)}{(y+2)^2}}$$

simplified

$$\frac{3(y-3)^2 \cdot (y+2) - (y-3)}{(y+2)^2 \cdot (y+2)^2}$$

$$\frac{3(y-3)^2 \cdot 5}{(y+2)^4} \Rightarrow \frac{15(y-3)^2}{(y+2)^4}$$

19. Find $\frac{dx}{d\theta}$ if $x(\theta) = \cos\left(\tan\left(\sqrt{2\theta}\right)\right)$

chain rule

$$-\sin\left(\tan\sqrt{2\theta}\right) \cdot \frac{d}{d\theta}\left(\tan\sqrt{2\theta}\right)$$

$$-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{d}{d\theta}\sqrt{2\theta}$$

$$-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{1}{2\sqrt{2\theta}} \cdot \frac{d}{d\theta}2\theta$$

$$\boxed{-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{1}{2\sqrt{2\theta}} \cdot 2}$$

20. Find $\frac{df}{dt}$ if $f(t) = 5\sec(3t) - \tan(3t)$

$$5\sec(3t)\tan(3t) \cdot 3 - \sec^2(3t) \cdot 3$$

$$\boxed{15\sec(3t)\tan(3t) - 3\sec^2(3t)}$$