Name _____ Date _____ Period _____ Score_____

Calculus   Practice Exam   Chapter 1 -1 1

Directions: Show all steps leading to your answers, including any intermediate results obtained using a graphing utility.

1. Let $f(x) = \begin{cases} x^2 + 5, & x \le 2 \\ \dfrac{4x-3}{x+3}, & x > 2 \end{cases}$

Find the limit *of* $f(x)$ *as*   1. (a)_____

$a) x \to -\infty;$ $b) \ x \to 2^-;$ $c) \ x \to 2^+;$ $d) \ x \to \infty$

(b)_____

(c)_____

(d)_____

2. Find the equations of all lines tangent to y = x² – 4 that pass through the point (5, 5).

2._____

3. Find $\dfrac{dy}{dx}$, where $y = \dfrac{x^2 - 5}{\cos(x)}$ 3. _____

4. Use implicit differentiation to find $\dfrac{dy}{dx}$, *if* $x^2(3y-5) = y^3 - x$ 4._____

5. Find $\dfrac{dy}{dx}$, if $y = 4^{-x}$

A. $(-x)(4^{-x-1})$ B. -4^{-x} C. $(\ln 4)^{-x}$

D. $(\ln 4)(4^{-x})$ E. $(-\ln 4)(4^{-x})$

5. _____

6. For, $y = x - 3e^{-x^2}$ use graphing techniques with analytical support to find the approximate intervals on which the function is:

(a) increasing;   (b) decreasing

(c) concave up   (d) concave down

Then find any

(e) local extreme values,   (f) inflection points

6. (a) _____

(b) _____

(c) _____

(d) _____

(e) Min: _____ at $x \approx$ _____

Max: _____ at $x \approx$ _____

(f) _____

7. You are planning to make an open box from a 30- by 42-inch piece of sheet metal by cutting congruent squares from the corners and folding up the sides. You want the box to have the largest possible volume. Using calculus determine the dimensions of the box that would give the largest possible volume of your box. Also find the maximum volume. (Remember the units.)

7. dimensions _____
    maximum volume _____

8. Consider the integral $\int_0^4 (x^2 - 3x + 5)\ dx$

(a) Estimate the value of the integral using 4 right-endpoint rectangles (RRAM).   8.(a)_____

(b) Estimate the value of the integral using the Trapezoidal Rule with n = 4.   (b) _____

(c) Integrate to find the exact value of the integral.   (c)_____

9. Find $\frac{d}{dx} \int_0^{2x} (5 - t)\ dt$ 9. _____

10. Use the Fundamental Theorem of Calculus to evaluate $\int \sqrt{5 - \sin(x)}\ dx$ 10. _____

A. $\frac{-\cos(x)}{2\sqrt{5-\sin(x)}} + C$ B. $\int_0^x \sqrt{5 - \sin(t)}\ dt + C$

C. $\int_x^0 \sqrt{5 - \sin(t)}\ dt + C$ D. $\int_0^x \frac{-\cos(x)}{2\sqrt{5-\sin(x)}} + C$

E. $\frac{2}{3}(5 - \sin(x))^{\frac{3}{2}} + C$

11. Solve the initial value problem. $\frac{dy}{dx} = (3x - 4)^3, y(1) = -1$ 11. _____

12. Solve the differential equation by separation of variables.   12._____

$\frac{dy}{dx} = \frac{x\cos(x^2)}{y^2 - 2}$

13. Use integration by parts to evaluate $\int x \sin(2x - 5)\ dx$ 13. _____

14. Suppose a certain element has a half-life of 4.6 weeks. If a sample contains 300 grams of this element, how much of it will remain after 7 weeks?

14. _____

15. Use Euler's method to solve the initial value problem. Do the first 3 iterations by hand.

Start at $x_0 = 0$ with dx = 0.1 . $\frac{-1}{4}(x + y^3), y(0) = 1$

15. $x_0 =$____ $y_0 =$_____

$x_1 =$____ $y_1 =$_____

$x_2 =$____ $y_2 =$_____

$x_3 =$____ $y_3 =$_____

16. The function v(t) = 2t - 4 is the velocity in ft./sec of a particle moving along the x-axis for $0 \le t \le 8$. Use analytic methods to answer the following questions.

(a) Determine when the particle is moving to the right, to the left, and stopped.

16.(a)right:_____

Left:_____

Stopped:_____

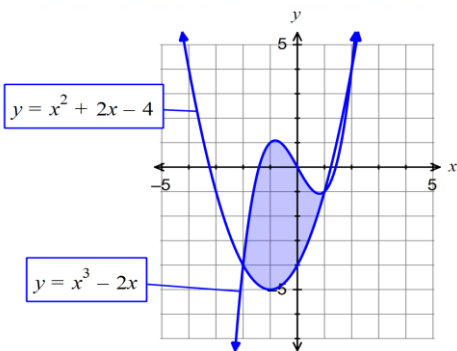(b) Find the particle's displacement for the given time interval.

16.b._____

(c) Find the total distance traveled by the particle.

16.c._____

17. Find the area of the shaded region analytically.   17._____

$y = x^2 + 2x - 4$

$y = x^3 - 2x$

18. A curve is given by $y = f(x)$ for $a \le x \le b$, where $a > 0$ and $f(x) > 0$.   18._____

The integral $\int_a^b \sqrt{1+\left(f'(x)\right)^2}\ dx$ can be used to find which of the following?

A. The length of the curve

B. The volume of the solid generated by revolving the region below the curve about the x-axis

C. The volume of the solid generated by revolving the region below the curve about the y-axis

19. A region is bounded by the line y = x and the parabola y = x² – 6x + 10. Find the volume of the solid generated by revolving the region about the x-axis.

19._____

20. Find the length of the curve describe by $y = \dfrac{4}{3}x^{\frac{3}{2}}$ for $0 \le x \le 6$.   20._____

21. A spring has a natural length of 12 cm. A 40-N force stretches the spring to 16 cm. How much work is done in stretching the spring from 12 cm. to 24 cm.

21._____

22. Use L'Hopital's rule to evaluate each limit.

(a) $\lim_{x \to 3} \dfrac{x^3 - 2x^2 - 5x + 6}{x^2 + 2x - 15}$

22.(a)_____

(b) $\lim_{x \to \infty} \left(1 + \dfrac{2}{5x} \right)^{3x}$

22.(b)_____

23. Order the functions from the slowest-growing to the fastest growing as $x \to \infty$.

$e^{\frac{x}{2}}, \ \ln(x+3), \ 5x^2, x^3$

23. _____

24. Evaluate the improper integral $\displaystyle\int_{2}^{\infty} \dfrac{dx}{x(\ln x)^4}$ or state that it diverges. Show work that justifies your answer.

24._____

25. Express $\dfrac{(x-2)^2}{(x-1)^2(x^2+3)}$ as a sum of partial fractions.　　　　　25._____

26. Evaluate $\displaystyle\int \dfrac{4x^3-69x-83}{(x+1)(x-5)}dx$　　　　　26._____

27. Tell whether the series converges or diverges. If it converges, find the sum.

$$5-\dfrac{15}{4}+\dfrac{45}{16}-\dfrac{135}{64}+...+5\left(-\dfrac{3}{4}\right)^n+...$$

27._____

28. Find the MacLaurin series for xe^{2x}　　　　　28._____

  Include the first three terms and the nth term.

29. Let f(x) = (3x − 2)^8. Use the Remainder Estimation Theorem to estimate the maximum absolute error when f(x) is replaced by 256 − 3072x for $|x|\le 0.04$.

29. _____

30. Determine the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\dfrac{(n-3)}{n^3+\ln(2n)}$. Justify your answer by identifying and using the test (or tests) you use.

30._____

31. Which of the following converge conditionally.   31. _____

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\,\frac{1}{\sqrt{n+3}}$

II. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\,\frac{n^{2}+3n-4}{n!}$

III. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\,\frac{n-2}{n^{2}+2}$

A. I only   B. II only   C. III only   D. I and III   E. II and III

32. Find the interval of convergence of the series   32. _____

$$\sum_{n=1}^{\infty}\frac{(x-3)^{n}}{2^{n}\cdot n}$$

33. Find the length of the curve. X = 2sin(t), y = cos(t), $0 \le t \le \pi$

33._____

34. Let $u=\langle 7,-2\rangle \ and \ v=\langle 3,5\rangle$.

(a) Find 3**u** − 5**v**   34a._____

(b) Find the magnitude of **u**.   34b._____

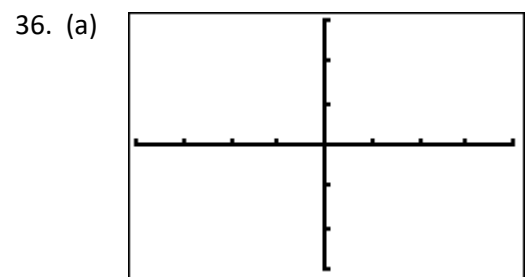35. A particle moves in the plane with position vector $r(t) = \langle 4\cos(t), \sqrt{2}\sin(t) \rangle$.

Find the velocity vector v(t) and the acceleration vector a(t)

35. v(t) _____

a(t) _____

36. (a) Graph the polar curve $r = 3\cos(2\theta)$

(b) What is the shortest length a θ -interval can have and still produce the graph?

36. (a)

[-4,4] by [-3,3]

36 (b) _____

37. Find the area of the region inside the oval limacon r = 2+ cos θ and outside the

oval limacon r = 3 – cos θ.

37._____