Chapter 3 Calculus Practice Exam

1. Use the definition of the derivative to find the derivative of $f(x) = x^2 - 3$ at $x = 2$. Show your work.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$(2+h, (2+h)^2-3)$

$(2, 2^2-3)$

$$\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$\lim_{h\to 0} \frac{[(2+h)^2-3] - (2^2-3)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{4}+4h+h^2\cancel{-3}\cancel{-1}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$\lim_{h\to 0} 4+h = \boxed{4}$$

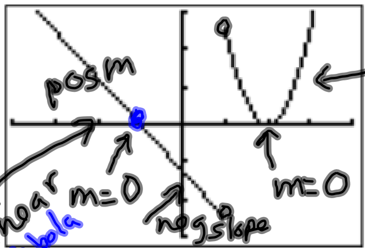
check it by short cut

x^2-3

$2x$ at $x=2$

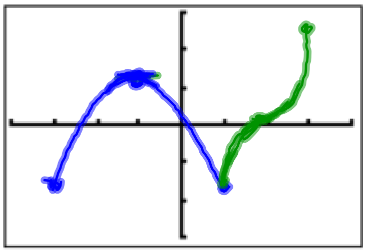
$2(2)=4$

2. Sketch a possible graph of a continuous function f that has domain [-3,3], where $\boxed{f(-1) = 1}$ and the graph of $y = f'(x)$ is shown below. 2.



[-4,4] by [-3,3]

positive slope
m=0 at x=2
not a max or min
f(x) is cubic



[-4,4] by [-3,3]

graph of f'(x)

slope of f(x) is positive from x=-3 to x=-1

slope of f(x) is 0 at x=-1
means (maximum at x=-1)

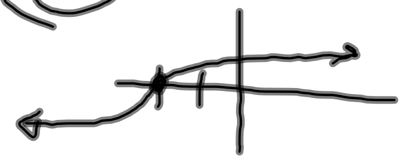
slope of f(x) is negative from x=-1 to x=1

no slope at x=1 (but f(x) continuous)

3. Which of the following describes the behavior of $y = \sqrt[3]{x+2}$ at x = -2?

(A) differentiable (B) corner (C) cusp

(D) vertical tangent (E) discontinuity

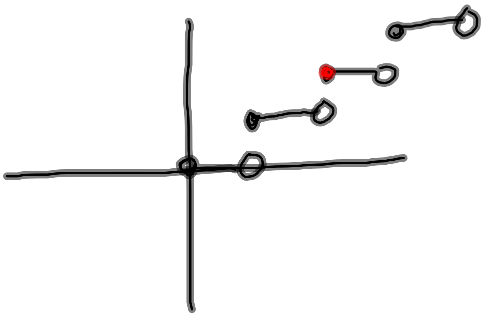


4. Let $f(x) = int(x)$.

(a) Using a calculator find NDeriv $(f(x), 3)$. 500

(b) Is your answer to part (a) a meaningful estimate of a derivative of $f(x)$? No

Explain your answer.



it's a jump disc at x=3 and the derivative dne at x=3.

5. Find

(a) $\frac{dy}{dx}$ **and** **(b)** $\frac{d^2y}{dx^2}$ **if** $y = 3x^4 - 9x^3 + 5x$.

$\frac{dy}{dx} = 12x^3 - 27x^2 + 5$

$\frac{d^2y}{dx^2} = 36x^2 - 54x$

6. Suppose that u and v are differentiable at **x = 5** and that

u(5) = 7, v(5) = 2, u′(5) = -3, and v′(5) = 6.

Find (a) $\frac{d}{dx}\left(\frac{u}{v}\right)$ **and (b)** $\frac{d}{dx}(10uv)$ **at x = 5.**

a) $\frac{vu' - uv'}{v^2}$

$\frac{v(5)\cdot u'(5) - u(5)\cdot v'(5)}{(v(5))^2}$

$\frac{2(-3) - 7(6)}{2^2}$

$\frac{-6-42}{4}$

$\boxed{\frac{-48}{4}}$

$\boxed{-12}$

b) $10\frac{d}{dx}uv$

$10[uv' + vu']$

$10[7(6) + 2(-3)]$

$10[42 + -6]$

$10[36]$

$\boxed{360}$

7. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $\boxed{s(t) = t^3 - 8t + 1}$ where s is measured in feet and t is measured in seconds.

(a) Find the displacement during the first 3 seconds.

$s(0) = 1$ $s(3) = 3^3 - 8(3) + 1$ $s(3) = 4$

$s(3) - s(0) = 3$

$\boxed{3 \text{ ft}}$

(b) Find the average velocity during the first 3 seconds. find slope of secant line.

(c) Find the instantaneous velocity when t = 3.

(d) Find the acceleration of the particle when t = 3.

(e) At what value or values of t does the particle change direction?

b) $\dfrac{s(3) - s(0)}{3 - 0} = \dfrac{4 - 1}{3 - 0} = \dfrac{3}{3} = \boxed{1 \text{ ft/sec}}$

c) $s(t) = t^3 - 8t + 1$

$s'(t) = 3t^2 - 8$

$s'(3) = 3(3)^2 - 8$

$27 - 8 \Rightarrow \boxed{19 \text{ ft/sec}}$

d) $s''(t) = 6t$

$s''(3) = 6(3)$ $\boxed{18 \text{ ft/sec}^2}$

e) Can Changes direction when $v(t) = 0$

$0 = 3t^2 - 8$

$8 = 3t^2$

$\frac{8}{3} = t^2$

$\pm\sqrt{8/3} = t$

$\boxed{t = \sqrt{\frac{8}{3}} \text{ sec}}$

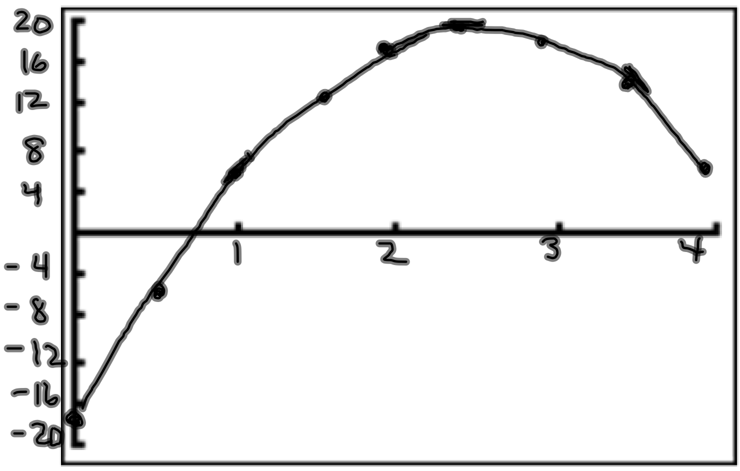
Remember $t \geq 0$

8. The coordinates s of a moving body for various values of t are given.

t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	- 19.5	- 6	4.5	12	16.5	18	16.5	12	4.5

Plot s versus t, and sketch a smooth curve through the given points.

Assuming this smooth curve represents the motion of the body, estimate the velocity at t = 1.0, t = 2.5, and t = 3.5



[0,4] by [-20,20]

$$\frac{12 - -6}{1.5 - .5} \quad \frac{18}{1}$$

$t = 1.0 \approx 18$ ft/sec

$$\frac{16.5 - 16.5}{3 - 2} = \frac{0}{1} = 0$$

$t = 2.5 \approx \boxed{0 \text{ ft/sec}}$

$$\frac{4.5 - 16.5}{4 - 3} = \frac{-12}{1}$$

$t = 3.5 \approx \boxed{-12 \text{ ft/sec}}$

9. **Find $\frac{dy}{dx}$ if $y = \frac{\cos(x)}{1+\tan(x)}$**

quotient rule

$$\frac{(1+\tan x)\cdot\frac{d}{dx}\cos(x) - \cos(x)\cdot\frac{d}{dx}(1+\tan x)}{(1+\tan(x))^2}$$

$$\boxed{\frac{(1+\tan(x))\cdot(-\sin x) - \cos(x)(\sec^2 x)}{(1+\tan(x))^2}}$$

10. Find the points on the graph of $y = \sec(x)$, $0 \le x \le 2\pi$, where the tangent is parallel to the line $3y - 2x = 4$.

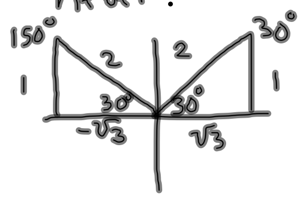
parallel lines have same slope

$$3y - 2x = 4$$
$$\frac{3y}{3} = \frac{2x}{3} + \frac{4}{3}$$
$$y = \frac{2}{3}x + \frac{4}{3} \qquad \text{slope is } \frac{2}{3}$$

$$y = \sec x$$
$$y' = \sec x \tan x$$
$$\frac{2}{3} = \sec x \cdot \tan x$$
$$\frac{2}{3} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$
$$\frac{2}{3} = \frac{\sin x}{\cos^2 x}$$

$$2\cos^2 x = 3\sin x$$
$$2(1 - \sin^2 x) - 3\sin x = 0$$
$$2 - 2\sin^2 x - 3\sin x = 0$$
$$2\sin^2 x + 3\sin x - 2 = 0 \qquad \text{mult by } -1$$
$$(2\sin x - 1)(1\sin x + 2) = 0$$

$$2\sin x - 1 = 0 \qquad \sin x + 2 = 0$$
$$2\sin x = 1 \qquad \sin x = -2$$
$$\sin x = \frac{1}{2} \qquad \text{never!}$$



QI and QII

$30°$ $\quad$ $150°$

$x = \frac{\pi}{6}$ $\quad$ $x = \frac{5\pi}{6}$

found where parallel

points on sec x

$$\boxed{\left(\frac{\pi}{6}, \frac{2}{\sqrt{3}}\right) \quad \left(\frac{5\pi}{6}, \frac{-2}{\sqrt{3}}\right)}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$$
$$\cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$$
$$\sec\frac{5\pi}{6} = \frac{-2}{\sqrt{3}}$$

Find the derivatives of the following functions.

11. $y = x^6 - 10x^5 + 3x^3 + 7$

$y' = 6x^5 - 50x^4 + 9x^2$

12. $f(x) = \dfrac{x^3 - 3\sqrt{x}}{x}$

$f(x) = \dfrac{x^3}{x} - \dfrac{3x^{\frac{1}{2}}}{x^1}$

$f(x) = x^2 - 3x^{-\frac{1}{2}}$

$2x - 3\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$\boxed{2x + \frac{3}{2}x^{-\frac{3}{2}}}$

OR Quotient Rule

$\boxed{\dfrac{x\left(3x^2 - \frac{3}{2\sqrt{x}}\right) - \left(x^3 - 3\sqrt{x}\right)(1)}{x^2}}$

13. $f(x) = \dfrac{x^3 - x + 5}{x^2 - 2}$

$$\boxed{\frac{(x^2-2)(3x^2-1) - (x^3-x+5)(2x)}{(x^2-2)^2}}$$

14. $y = e^3x^5 - 6x + 2$

e^3 is a constant

e^3x^5 is not a product

$\boxed{y' = 5e^3x^4 - 6}$