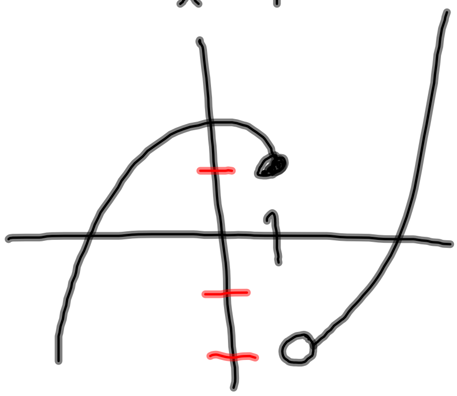
Use the graph to estimate the limits and value of the function, or explain why the limits do not exist.

1.a. $\lim\limits_{x \to 1^-} g(x)$



$\lim\limits_{x \to 1^+} (g(x))$

$\lim\limits_{x \to 1} g(x)$

$g(1)$

1(a) 1

1b. -2

1c. DNE

1d 1

Determine the limit by substitution.

2. $\lim\limits_{x \to -2} (5x^2 + 4x - 2)$

$5(-2)^2 + 4(-2) - 2$

$20 + -8 - 2$

$\boxed{10}$

3. $\lim_{x\to b} f(x) = 5 \qquad \lim_{x\to b} g(x) = -2$

Find value of $\lim_{x\to b} (f(x) - g(x))$

$\lim_{x\to b} f(x) - \lim_{x\to b} g(x)$

$5 - -2$

$\boxed{7} \qquad \boxed{E}$

4. Find the limit graphically. Show how the Sandwich Theorem can be used to confirm your answer.

$\lim_{x\to 0} \left(3+x^2\sin\left(\frac{1}{x}\right)\right)$ $\underline{\quad 3 \quad}$

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$-1x^2 \le x^2\sin\left(\frac{1}{x}\right) \le x^2$ mult by x^2

$3+-1x^2 \le 3+x^2\sin\left(\frac{1}{x}\right) \le 3+x^2$ add 3

take $\lim_{x\to 0}$

$\lim_{x\to 0} 3+-1x^2 \le \lim_{x\to 0} 3+x^2\sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} 3+x^2$

Find limit by substitution

$3 \le \lim_{x\to 0} 3+x^2\sin\left(\frac{1}{x}\right) \le 3$

confirmed by sandwich theorem

5. For $f(x) = \frac{2x+5}{|3x-4|}$, use graphs and tables to find

a) $\lim_{x \to \infty} f(x)$ and (b) $\lim_{x \to -\infty} f(x)$

and (c) Identify any horizontal asymptotes.

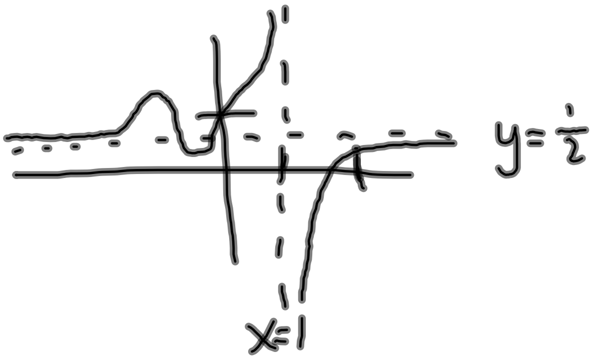
$$\lim_{x \to \infty} \frac{2x+5}{|3x-4|} \Rightarrow \lim_{x \to \infty} \frac{2x+5}{3x-4} \Rightarrow \frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{2x+5}{|3x-4|} \quad \lim_{x \to \infty} \frac{2x+5}{-3x+4} \Rightarrow -\frac{2}{3}$$

$y = \frac{2}{3}$ and $y = -\frac{2}{3}$

Consider the function f(x) given below. Which of the following appear to be true for f(x)

6. I The line $y = \frac{1}{2}$ is a horizontal asymptote



II $\lim_{x \to 2} f(x) = 2$

III $x = 1$ The line is a vertical asymptote

IV $\lim_{x \to +\infty} f(x) = \lim_{x \to -\infty} f(x)$

I true

II false $\lim_{x \to 2} f(x) = \frac{1}{2}$

III true

IV true

I, III, IV $\boxed{C}$

7. (a.) Find the vertical asymptotes of the graph of the function below.

$$f(x) = \frac{3-x}{x^2-16} \qquad \frac{3-x}{(x-4)(x+4)}$$

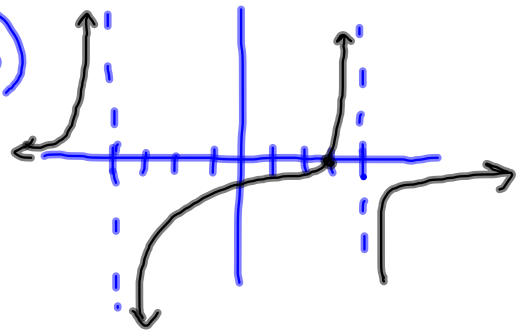
zero is 3

V.A. ~~4, -4~~

(b) Describe the behavior of f(x) to the left and right of each vertical asymptote.

a) Vertical asymptotes are $x=4$ $x=-4$

b)



$$\lim_{x \to -4^-} f(x) = \infty$$

$$\lim_{x \to -4^+} f(x) = -\infty$$

$$\lim_{x \to 4^-} f(x) = \infty$$

$$\lim_{x \to 4^+} f(x) = -\infty$$

8. For the function $y = e^x - 2x^3 + 15x$

(a) Find a simple basic function right end behavior model, and
(b) Find a simple basic function left end behavior model.

a) $$\lim_{x\to\infty} \frac{\cancel{e^x}}{\cancel{e^x}} + \cancel{\frac{-2x^3}{e^x}}^{\,0} + \cancel{\frac{15x}{e^x}}^{\,0} = 1$$

REBM is e^x

b) $$\lim_{x\to-\infty} \cancel{\frac{e^x}{-2x^3}}^{\,0} + \frac{\cancel{-2x^3}}{\cancel{-2x^3}} + \cancel{\frac{15x}{-2x^3}}^{\,0} = 1$$

9. Find the points of discontinuity of the function

$$y = \frac{x^2+x-2}{x^2+5x+6}$$

For each discontinuity, identify the type of discontinuity (removable, jump, infinite, or oscillating.

factor

$$\frac{\cancel{(x+2)}(x-1)}{(x+3)\cancel{(x+2)}} \longrightarrow x = -2 \quad \text{removable (hole)}$$

$x = -3$ V.A infinite disc.

10. Find a value for a so that the function is continuous.

$$f(x) = \begin{cases} 2x-5, & x<2 \\ ax^2, & x \geq 2 \end{cases}$$

To be continuous (for this function) each expression must equal the same y value when $x=2$

$$2x-5 = ax^2$$

set expressions =

$$2(2)-5 = a(2)^2$$

replace x with 2

$$4-5 = 4a$$

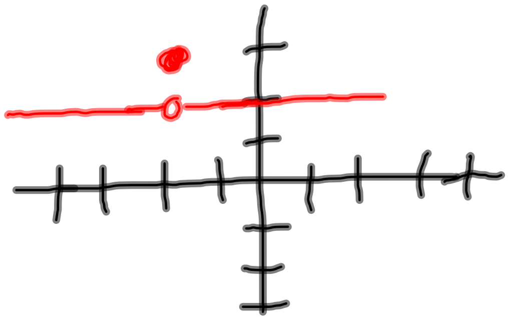
solve for a

$$-1 = 4a$$

$$\frac{-1}{4} = a$$

11. Sketch a possible graph for a function f, where f(-2) exists,

$\lim_{x \to -2} f(x) = 2$, and f(x) is not continuous at x = - 2



12. Use the concept of composite functions to explain why

$h(x) = |x^2 - 4x - 6|$ is a continuous function.

$f(x) = |x|$ abs value functions are continuous.

$g(x) = x^2 - 4x - 6$ poly functions are continuous

$h(x) = f(g(x))$

or $h(x) = f \circ g$

h(x) is continuous because composite functions of continuous functions are continuous.

13. Find the average rate of change of the function $f(x) = x^3 - 2x + 4$ over the interval [-3,5]

$(-3, -17)$ $(-3)^3 - 2(-3) + 4$ $-27 + 6 + 4$

$(5, 119)$ $5^3 - 2(5) + 4$

$\frac{119 - -17}{5 - -3} = \boxed{\frac{136}{8}}$ or $\boxed{17}$

14. For the function $f(x) = 5x^2$ at the point (2,20), find
(a) the slope of the curve
(b) an equation of the tangent line
(c) an equation of the normal line

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(2,20)

$(2+h, 5(2+h)^2)$

$$\lim_{h \to 0} \frac{5(2+h)^2 - 20}{h}$$

$$\lim_{h \to 0} \frac{5(4+4h+h^2) - 20}{h}$$

$$\lim_{h \to 0} \frac{\cancel{20}+20h+5h^2\cancel{-20}}{h}$$

a) $$\lim_{h \to 0} \frac{\cancel{h}(20 + 5\cancel{h}^{\,0})}{\cancel{h}} = 20$$

b) $y - y_1 = m(x - x_1)$

$$\boxed{y - 20 = 20(x-2)}$$

c) $y - 20 = -\frac{1}{20}(x-2)$

15. The equation for free fall at the surface of the planet Quixon is $s = 3.8t^2$ m with t in sec. Assume a rock is dropped from the top of a 400-m cliff. Find the speed of the rock at t = 6 sec.

find average rate of change

$$\lim_{h\to 0} \frac{f(6+h) - f(6)}{h}$$

$$\lim_{h\to 0} \frac{3.8(6+h)^2 - 3.8(6)^2}{h}$$

work $3.8(6^2 + 12h + h^2) - 3.8(6)^2$

$$\lim_{h\to 0} \frac{\cancel{3.8(6^2)} + 3.8(12h) + 3.8h^2 - \cancel{3.8(6)^2}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}[3.8(12) + 3.8h]}{\cancel{h}}$$

speed $3.8(12)$ m/sec

$\boxed{45.6 \text{ m/sec}}$