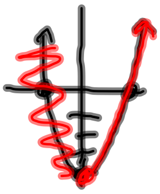
Chapter 8 Calculus Practice Exam

1. The function $v(t) = t^2 - 4$ is the velocity in m/sec for a particle moving along the x-axis, where t is measured in seconds ($t \geq 0$). Use analytic methods to do each of the following.

(a) Determine when the particle is moving to the right, to the left, and stopped.

1. Right: $(2, \infty)$ — $t > 2$ sec
Left: $[0, 2)$ — $0 \leq t < 2$
Stopped: 2 sec



Stopped
$v(t) = 0$
$t^2 - 4 = 0$
$t = \pm 2$
$t = 2$

Right
$v(t) > 0$
$t^2 - 4 > 0$
$t > 2; \cancel{t < -2}$
-2 not in domain

left
$v(t) < 0$
$t^2 - 4 < 0$
$-2 < t < 2$

(b) Find the particle's displacement for $0 \leq t \leq 5$ 1b.________

net area

$$\int_0^5 t^2 - 4 \, dt$$

$$\left. \frac{t^3}{3} - 4t \right|_0^5$$

$$\boxed{\left[\left(\frac{5^3}{3} - 4(5)\right) - 0\right]}$$

$$\frac{125}{3} - 20 \qquad \frac{65}{3} \text{ m}$$

(c) Find the total distance traveled by the particle for $0 \leq t \leq 5$

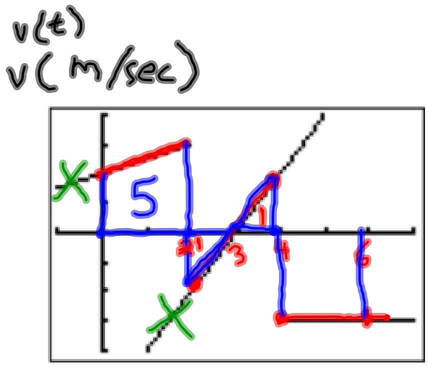
$$\int_0^5 |v(t)| \, dt$$

$$-\int_0^2 t^2 - 4 \, dt + \int_2^5 t^2 - 4 \, dt$$

$$-\left[\left. \frac{t^3}{3} - 4t \right|_0^2\right] + \left(\left. \frac{t^3}{3} - 4t \right|_2^5\right)$$

$$\boxed{-\left[\left(\frac{8}{3} - 8\right) - (0)\right] + \left[\left(\frac{5^3}{3} - 20\right) - \left(\frac{2^3}{3} - 8\right)\right]}$$

simplified $\boxed{\frac{97}{3} \text{ m}}$



2. The graph shows the velocity of a particle moving on the x-axis. The particle starts at $x = -3$ when $t = 0$.
(a) Find where the particle is at the end of the trip ($t = 6$).
(b) Find the total distance traveled by the particle.

a) final position

$$-3 + \int_0^6 v(t)\,dt$$

$\int_0^2 v(t)\,dt$ trapezoid

$$\tfrac{1}{2}(2)(2+3) = 5$$

$\int_2^3 v(t)\,dt$ triangle

$$\tfrac{1}{2}(1)(-2) = -1$$

$\int_3^4 v(t)\,dt$ triangle

$$\tfrac{1}{2}(1)(2) = 1$$

$\int_4^6 v(t)\,dt$ rectangle

$$(-3)(2) = -6$$

$$-3 + [5 + -1 + 1 + -6] \Rightarrow \boxed{-4}$$

b) $\int_0^6 |v(t)|\,dt$

$$5 + |-1| + 1 + |-6|$$

$$5 + 1 + 1 + 6 = \boxed{13\text{ m}}$$

3. The rate of expenditures on public elementary and secondary schools (in billions of dollars a year) in the United States can be modeled by the function $S = 6.22e^{0.086t}$, where t is the number of years after January 1, 1950.
Find the total expenditures from January 1, 1950 to January 1, 1990.

$$\int_{1950-1950}^{1990-1950}$$

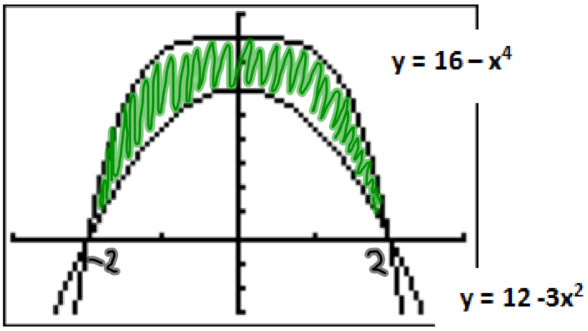
$$\int_{0}^{40} 6.22e^{0.086t}\,dt$$

$$\left.\frac{6.22e^{.086t}}{0.086}\right|_{0}^{40}$$

$$\boxed{\frac{6.22e^{.086(40)}}{0.086} - \frac{6.22e^{.086(0)}}{0.086}}$$

$$\frac{6.22e^{.086(40)}}{0.086} - \frac{6.22}{0.086}$$ billions of dollars

4. Find the area of the shaded region analytically.



[-3, 3] by [-6, 18]

find intersection

$16-x^4=12-3x^2$

$0=x^4-3x^2-4$

$0=(x^2-4)(x^2+1)$

$(x-2)(x+2)=0$ ↑ imag

± 2

top - bottom

$$2\int_0^2 16-x^4-(12-3x^2)\,dx$$

$$2\int_0^2 16-x^4-12+3x^2\,dx$$

$$2\int_0^2 -x^4+4+3x^2\,dx$$

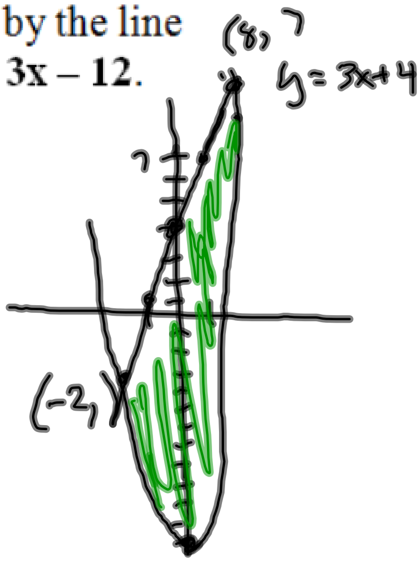
$$2\left(-\frac{x^5}{5}+4x+\frac{\cancel{3}x^3}{\cancel{3}}\Big|_0^2\right)$$

$$\boxed{2\left[\left(-\frac{2^5}{5}+4(2)+(2)^3\right)-(0)\right]}$$

$$\boxed{2\left(-\frac{32}{5}+16\right)}$$

$$\frac{96}{5}$$

5. Find the area of the region enclosed by the line $y = 3x + 4$ and the parabola $y = x^2 - 3x - 12$.



find the intersection

$$x^2 - 3x - 12 = 3x + 4$$

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

$$x = 8 \quad x = -2$$

$$\int_{-2}^{8} 3x+4-(x^2-3x-12)\,dx$$

$$\int_{-2}^{8} 3x+4-x^2+3x+12\,dx$$

$$\int_{-2}^{8} 6x+16-x^2\,dx$$

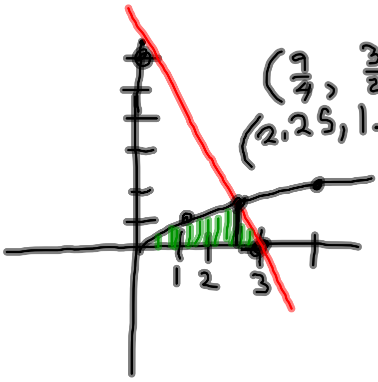
$$\frac{6x^2}{2} + 16x - \frac{x^3}{3}\Big|_{-2}^{8}$$

$$\boxed{\left(3(8)^2+16(8)-\frac{8^3}{3}\right)-\left(3(-2)^2+16(-2)-\frac{(-2)^3}{3}\right)}$$

$$\boxed{\frac{500}{3}}$$

6. Find the area enclosed by $y = \sqrt{x}$, y = 6 – 2x and the x-axis.

Intersection



$\sqrt{x} = 6-2x$

$x = (6-2x)^2$

$x = 36-24x+4x^2$

$0 = 36-25x+4x^2$

$x = 2.25 \quad \frac{9}{4}$

$\int_0^{2.25} \sqrt{x}\, dx$ + triangle

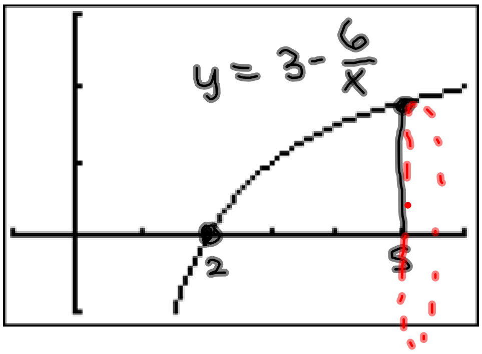
$\int_0^{2.25} x^{\frac{1}{2}}\, dx$ $\quad \frac{1}{2}$ base(height)

$\frac{2}{3} x^{\frac{3}{2}} \Big|_0^{2.25}$ $\quad \frac{1}{2}(3-2.25)(1.5)$

$$\boxed{\frac{2}{3}(2.25)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}} + \frac{1}{2}(3-2.25)(1.5)}$$

Simplified $\boxed{2.8125}$

7. Find the volume generated by revolving the shaded region about the x-axis



disc method

$A_{circle} = \pi r^2 \quad r = 3 - \frac{6}{x}$

$$\int_2^5 \pi \left(3 - \frac{6}{x}\right)^2 dx$$

$$\pi \int_2^5 9 - \frac{18}{x} - \frac{18}{x} + \frac{36}{x^2}\, dx$$

$$\pi \int_2^5 9 - \frac{36}{x} + 36x^{-2}\, dx$$

$$\pi \int_2^5 9 - 36 \cdot \frac{1}{x} + 36x^{-2}\, dx$$

$$\pi \left(9x - 36 \ln x + \frac{36x^{-1}}{-1} \Big|_2^5 \right)$$

$$\boxed{\pi \left[\left(45 - 36 \ln 5 - \frac{36}{5}\right) - \left(18 - 36 \ln 2 - \frac{36}{2}\right) \right]}$$

simplified $\pi \left[\frac{89}{5} + 36 \ln\left(\frac{2}{5}\right) \right]$

8. A curve is given by $y = \left(9 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$ for $1 \le x \le 8$.

Find the exact length of the curve analytically by antidifferentiation.

$$y = \left(9 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$\frac{dy}{dx}$ dne at $x=0$

not in domain.

we're good!

$$\frac{dy}{dx} = \cancel{\frac{3}{2}}\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot -\cancel{\frac{2}{3}}x^{-\frac{1}{3}}$$

$$-\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} x^{-\frac{1}{3}}$$

$$L = \int_1^8 \sqrt{1 + \left(-\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} x^{-\frac{1}{3}}\right)^2}\, dx$$

$$\int_1^8 \sqrt{1 + \left(\left(9 - x^{\frac{2}{3}}\right) x^{-\frac{2}{3}}\right)}\, dx$$

$$\int_1^8 \sqrt{1 + 9x^{-2/3} - x^{\frac{2}{3} + -\frac{2}{3}}}\, dx$$

$$\int_1^8 \sqrt{\cancel{1} + 9x^{-2/3} \cancel{-1}}\, dx$$

$$\int_1^8 \left(9x^{-\frac{2}{3}}\right)^{\frac{1}{2}} dx$$

$$\int_1^8 3x^{-\frac{1}{3}}\, dx$$

$$\frac{3}{2} \cdot 3x^{\frac{2}{3}} \Big|_1^8$$

$$\boxed{\frac{9}{2}(8)^{\frac{2}{3}} - \frac{9}{2}(1)^{\frac{2}{3}}}$$

$$\boxed{\frac{27}{2}}$$

9. A curve is given by $\int_0^y \sqrt{9t^2 + 6t}\ dt$ for $1 \le y \le 5$. Find the exact length of the curve analytically by antidifferentiation.

$$f(t) = \int_0^y \sqrt{9t^2+6t}\ dt$$

$$f' = \frac{d}{dt}\int_0^y \sqrt{9t^2+6t}\ dt$$

$$\frac{dt}{dy} = \sqrt{9y^2+6y}$$

$$\left(\frac{dt}{dy}\right)^2 = 9y^2+6y$$

$$L = \int_1^5 \sqrt{1+\left(\frac{dt}{dy}\right)^2}\ dy$$

$$\int_1^5 \sqrt{1+9y^2+6y}\ dy$$

$$\int_1^5 \sqrt{9y^2+6y+1}\ dy$$

$$\int_1^5 \sqrt{(3y+1)^2}\ dy$$

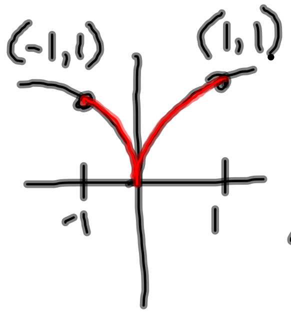
$$\int_1^5 3y+1\ dy$$

$$\left.\frac{3y^2}{2}+y\right|_1^5$$

$$\boxed{\left(\frac{3(5)^2}{2}+5\right)-\left(\frac{3(1)^2}{2}+1\right)}$$

OR $\boxed{40}$

10. Find the length of the nonsmooth curve $y = (x)^{\frac{2}{5}}$ for $-1 \le x \le 1$.
(Round your answer to the nearest 0.001.)



$y_1 = x^{\frac{2}{5}}$

$x = 0$

$\frac{dy}{dx}$ dne $x = 0$

calculator

$y^{\frac{5}{2}} = (x^{\frac{2}{5}})^{\frac{5}{2}}$

$y^{\frac{5}{2}} = x$

$x = y^{\frac{5}{2}}$

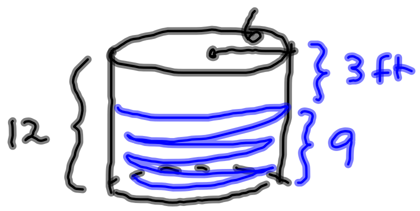
$\frac{dx}{dy} = \frac{5}{2} y^{\frac{3}{2}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{25}{4} y^3$

$$L = 2\int_0^1 \sqrt{1 + \frac{25}{4} y^3}\, dy$$

fnint $\boxed{3.031}$

11. A right cylindrical tank is filled with sea water. The tank has a radius of 6 feet and a height of 12 feet. If the water level is now 3 feet below the top of the tank, how much work will be required to pump the sea water to the top of the tank? (The weight-density of seawater is 64 lb/ft^3.)



circle

$A = \pi r^2$

$r = 6$

$A = 36\pi$

$W = \int$ weight (area of base) height dy

$\int 64 \ \pi r^2$

$\int_0^9 64 (36\pi)(12-y)\,dy$

fnint $2304\pi \int_0^9 12-y \ dy$

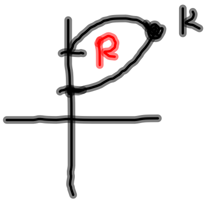
$\rightarrow 155{,}520\pi$

or 488580.489

12. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of $y = 2 + \sin(x)$ and $y = \sec(x)$.

(a) Find the area of R.

$K \approx 1.2237831$



intersection

top − bottom

$$\int_0^K 2+\sin(x) - \sec x \, dx$$

fnint $\boxed{1.366}$

(b) Find the volume of the solid generated when R is revolved about the x – axis.

washer

$$\int \pi R_o^2 - \pi R_i^2$$

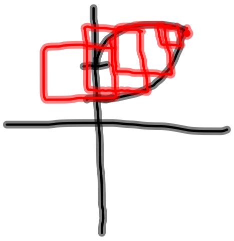
$R_o = 2+\sin x$

$R_i = \sec x$

$$\pi \int_0^K (2+\sin x)^2 - \sec^2 x \, dx$$

fnint $\boxed{16.404}$

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.



side of square = $2+\sin x - \sec x$

$A = \text{side}^2$

$$V = \int_0^K (2+\sin x - \sec x)^2 \, dx$$

$$\boxed{V \approx 1.629}$$