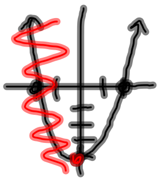
Chapter 8 Calculus Practice Exam

1. The function $v(t) = t^2 - 4$ is the velocity in m/sec for a particle moving along the x-axis, where t is measured in seconds ($t \geq 0$). Use analytic methods to do each of the following.

(a) Determine when the particle is moving to the right, to the left, and stopped.

1. Right: $(2, \infty)$
Left: $\boxed{[0,2)}$
Stopped: $t=2$



$t \geq 0$

$(t-2)(t+2)=0$

Stopped when $t^2-4=0$
$t=\pm 2$
$t=2$

right $v(t)>0$
$t-2>0$
$t>2$
$t^2-4>0$
~~$t<-2$~~ $t>2$

left $v(t)<0$
$t-2<0$
$t<2$

(b) Find the particle's displacement for $0 \leq t \leq 5$

1b. $\frac{65}{3}$ m

displacement = net area

$$\int_0^5 t^2-4 \, dt$$

$$\left.\frac{t^3}{3}-4t\right|_0^5$$

$$\boxed{\left[\left(\frac{5^3}{3}-4(5)\right)-\left(\frac{0^3}{3}-4(0)\right)\right]}$$

$$\boxed{\frac{125}{3}-20} \text{ or } \frac{65}{3} \text{m}$$

(c) Find the total distance traveled by the particle for $0 \leq t \leq 5$

$$\int_0^5 |v(t)| \, dt$$

$$-\int_0^2 t^2-4 \, dt + \int_2^5 t^2-4 \, dt$$

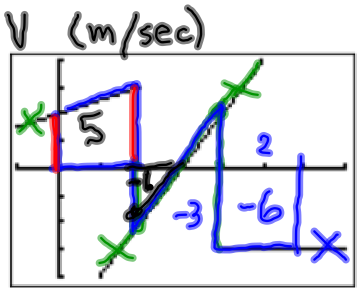
$$-\left(\left.\frac{t^3}{3}-4t\right|_0^2\right) + \left(\left.\frac{t^3}{3}-4t\right|_2^5\right)$$

$$\boxed{-\left(\left(\frac{8}{3}-8\right)-(0)\right)+\left(\left(\frac{5^3}{3}-20\right)-\left(\frac{8}{3}-8\right)\right)}$$

$$-\frac{8}{3}+8+\frac{125}{3}-20-\frac{8}{3}+8$$

$$\frac{97}{3} \text{ m}$$

2. The graph shows the velocity of a particle moving on the x-axis.
The particle starts at x = -3 when t = 0.
(a) Find where the particle is at the end of the trip (t = 6).
(b) Find the total distance traveled by the particle.



Starts at $x=-3$ when $t=0$

(a) $-3+\int_0^6 v(t)\,dt$

$\int_0^2 v(t)\,dt$ trapezoid

$\frac{1}{2}(2)(2+3) = 5$

$\int_2^3 v(t)\,dt$ triangle

$\frac{1}{2}(-2)(1) = -1$

$\int_3^4 v(t)\,dt$ triangle

$\frac{1}{2}(2)(1) = 1$

$\int_4^6 v(t)\,dt$ rectangle

$-3(2) = -6$

$-3+(5+-1+1+-6) = \boxed{-4}$

(b) $\int_0^6 |v(t)|\,dt$

$|5|+|-1|+|1|+|-6|$

$5+1+1+6 = \boxed{13\text{ m}}$

3. The rate of expenditures on public elementary and secondary schools (in billions of dollars a year) in the United States can be modeled by the function $\boldsymbol{S = 6.22e^{0.086t}}$, where t is the number of years after January 1, 1950.
Find the total expenditures from January 1, 1950 to January 1, 1990.

$$\int_{1950-1950}^{1990-1950}$$

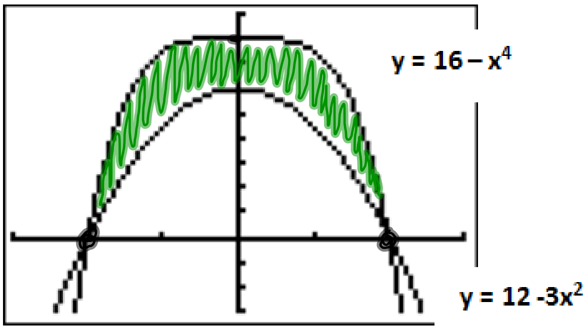
$$\int_0^{40} 6.22e^{0.086t}\, dt$$

$$\left.\frac{6.22e^{0.086t}}{0.086}\right|_0^{40}$$

$$\boxed{\frac{6.22e^{0.086(40)}}{0.086} - \frac{6.22e^{0.086(0)}}{0.086} \quad \text{billions of dollars}}$$

$$\frac{6.22e^{.086(40)}}{0.086} - \frac{6.22}{0.086}$$

4. Find the area of the shaded region analytically.



[-3, 3] by [-6, 18]

symmetrical

top - bottom

intersection

$$16-x^4=12-3x^2$$

$$0=x^4-3x^2-4$$

$$0=(x^2-4)(x^2+1)$$

imag

$$0=x-2$$

$$0=x+2 \qquad \pm 2$$

$$2\int_0^2 16-x^4-(12-3x^2)\,dx$$

$$2\int_0^2 16-x^4-12+3x^2\,dx$$

$$2\int_0^2 4-x^4+3x^2\,dx$$

$$2\left(4x-\frac{x^5}{5}+\frac{\cancel{3}x^3}{\cancel{3}}\Big|_0^2\right)$$

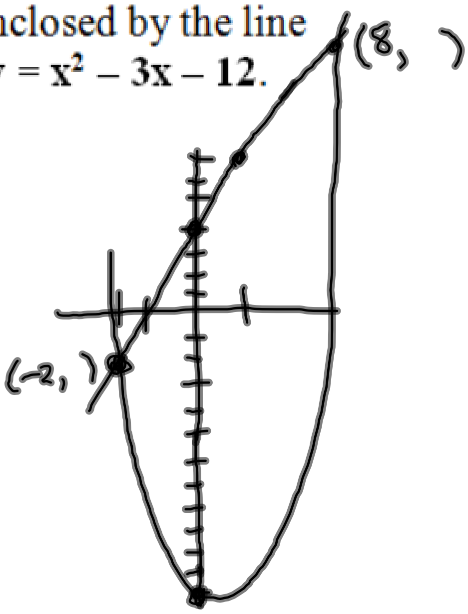
$$\boxed{2\left[\left(8-\frac{2^5}{5}+8\right)-0\right]}$$

$$2\left(16-\frac{32}{5}\right)$$

$$32-\frac{64}{5} \qquad \frac{160-64}{5}$$

$$\frac{96}{5}$$

5. Find the area of the region enclosed by the line $y = 3x + 4$ and the parabola $y = x^2 - 3x - 12$.



intersection

$x^2 - 3x - 12 = 3x + 4$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8, -2$

$$\int_{-2}^{8} 3x+4-(x^2-3x-12)\,dx$$

$$\int_{-2}^{8} 3x+4-x^2+3x+12\,dx$$

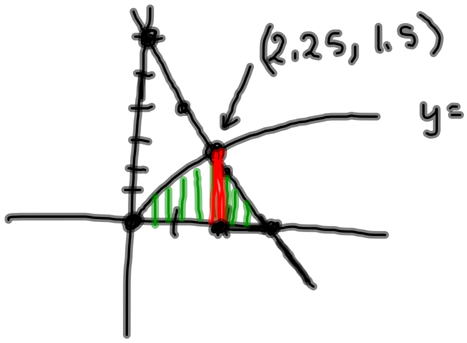
$$\int_{-2}^{8} -x^2+6x+16\,dx$$

$$\frac{-x^3}{3}+\frac{6x^2}{2}+16x\Big|_{-2}^{8}$$

$$\boxed{\left[\left(\frac{-(8)^3}{3}+3(8)^2+16(8)\right)-\left(\frac{-(-2)^3}{3}+3(-2)^2+16(-2)\right)\right]}$$

simplify: $\frac{500}{3}$

6. Find the area enclosed by $y = \sqrt{x}$, y = 6 – 2x and the x-axis.



intersection

$\sqrt{x} = 6-2x$

$x = (6-2x)^2$

$x = 36-24x+4x^2$

$0 = 4x^2-25x+36$

intersection

2.25, 1.5

$\int_0^{2.25} \sqrt{x}\, dx$ + triangle

$\int_0^{2.25} x^{\frac{1}{2}}\, dx$

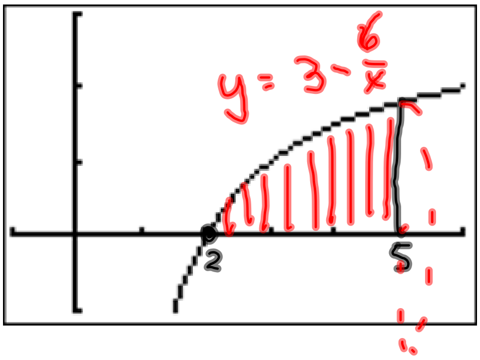
$\frac{2}{3}x^{\frac{3}{2}}\Big|_0^{2.25}$

$\left(\frac{2}{3}(2.25)^{\frac{3}{2}} - 0\right) +$ triangle $\frac{1}{2}bh$

$\frac{1}{2}(3-2.25)(1.5)$

$$\boxed{\frac{2}{3}(2.25)^{\frac{3}{2}} + \frac{1}{2}(3-2.25)(1.5)}$$

7. Find the volume generated by revolving the shaded region about the x-axis



disc method

$$\int_2^5 \pi r^2 \, dx$$

$$\pi \int_2^5 \left(3 - \frac{6}{x}\right)^2 dx$$

$$\left(3 - \frac{6}{x}\right)\left(3 - \frac{6}{x}\right)$$

$$\pi \int_2^5 9 - \frac{36}{x} + \frac{36}{x^2} \, dx$$

$$\pi \int_2^5 9 - 36\left(\frac{1}{x}\right) + 36x^{-2} \, dx$$

$$\pi \left[9x - 36 \ln x + -36x^{-1} \Big|_2^5 \right]$$

$$\boxed{\pi \left[\left(45 - 36 \ln 5 + \frac{-36}{5}\right) - \left(18 - 36 \ln 2 - \frac{36}{2}\right) \right]}$$

$$\frac{189}{5} + 36 \ln \frac{2}{5}$$

8. A curve is given by $y = \left(9 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$ for $1 \le x \le 8$.
Find the exact length of the curve analytically by antidifferentiation.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$y = \left(9 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot -\frac{2}{3}x^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = -x^{-\frac{1}{3}}\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

$$\int_1^8 \sqrt{1 + \left(-x^{-\frac{1}{3}}\left(9 - x^{\frac{2}{3}}\right)^{\frac{1}{2}}\right)^2}\,dx$$

$$\int_1^8 \sqrt{1 + x^{-\frac{2}{3}}\left(9 - x^{\frac{2}{3}}\right)}\,dx$$

$$\int_1^8 \sqrt{1 + 9x^{-\frac{2}{3}} - x^{-\frac{2}{3} + \frac{2}{3}}}\,dx$$

x^0

$$\int_1^8 \sqrt{\cancel{1} + 9x^{-\frac{2}{3}} \cancel{-1}}$$

$$\int_1^8 \left(9x^{-\frac{2}{3}}\right)^{\frac{1}{2}}\,dx$$

$$\int_1^8 3x^{-\frac{1}{3}}\,dx$$

$$\frac{3}{2} \cdot 3x^{\frac{2}{3}}\Big|_1^8$$

$$\boxed{\frac{9}{2}(8)^{\frac{2}{3}} - \frac{9}{2}(1)^{\frac{2}{3}}}$$

$$\frac{27}{2}$$

9. A curve is given by $\int_0^y \sqrt{9t^2+6t}\ dt$ for $1 \le y \le 5$. Find the exact length of the curve analytically by antidifferentiation.

$$f = \int_0^y \sqrt{9t^2+6t}\, dt$$

$$L = \int_1^5 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dy}{dt}\int_0^y \sqrt{9t^2+6t}\, dt = \sqrt{9y^2+6y}$$

$$L = \int_1^5 \sqrt{1+\left(\sqrt{9y^2+6y}\right)^2}\, dy$$

$$\int_1^5 \sqrt{1+9y^2+6y}\, dy$$

$$L = \int_1^5 \sqrt{(3y+1)^2}\, dy$$

$$\int_1^5 3y+1\, dy$$

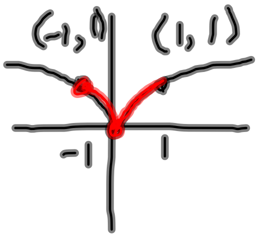
$$\left.\frac{3y^2}{2}+y\right|_1^5$$

$$\boxed{\left(\frac{3(5)^2}{2}+5\right)-\left(\frac{3(1)^2}{2}+1\right)}$$

simplified 40

10. Find the length of the <u>nonsmooth</u> curve $y = (x)^{\frac{2}{5}}$ for $-1 \le x \le 1$. (Round your answer to the nearest 0.001.)

CALCULATOR



cusp

symmetric

$$2\int_0^1 \sqrt{1+\frac{25}{4}y^3}$$

use fnint

$2(1.51565934)$

$\boxed{3.031}$

$y = x^{\frac{2}{5}}$ $\quad \frac{dy}{dx}$ dne $x=0$

$y^{\frac{5}{2}} = \left(x^{\cancel{\frac{2}{5}}}\right)^{\cancel{\frac{5}{2}}}$

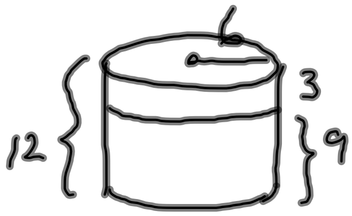
$y^{\frac{5}{2}} = x$

$x = y^{\frac{5}{2}}$

$\frac{dx}{dy} = \frac{5}{2}y^{\frac{3}{2}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{25}{4}y^3$

11. A right cylindrical tank is filled with sea water. The tank has a radius of 6 feet and a height of 12 feet. If the water level is now 3 feet below the top of the tank, how much work will be required to pump the sea water to the top of the tank? (The weight-density of seawater is 64 lb/ft³.)



$$\int w \cdot A \cdot h \cdot dy$$

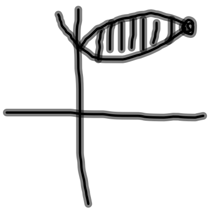
$$\int_0^9 64(\pi \cdot 6^2) \cdot (12-y)\,dy$$

488580.4895 ft-lb

12. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of y = 2 + sin (x) and y = sec (x).

(a) Find the area of R.

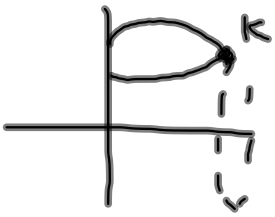
intersection $k = 1.2237831$



$$A = \int_0^k (2+\sin x) - \sec x \, dx$$

fnint $A \approx 1.366$

(b) Find the volume of the solid generated when R is revolved about the x – axis.



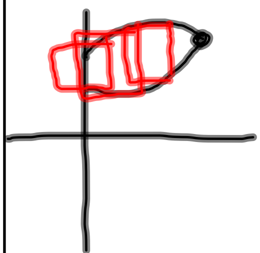
$$\int_0^k \pi(2+\sin x)^2 - \pi \sec^2 x \, dx$$

fnint 16.404

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

side = $2 + \sin x - \sec x$

$A = \text{side}^2$



$$V = \int_0^k (2+\sin x - \sec x)^2 \, dx$$

fnint 1.629