Chapter 6 Calculus Practice Exam

Calculator

1. Consider the region enclosed between the graph of $f(x) = x^2 - \ln x$ and the x-axis for $1 \le x \le 5$.

a) Find the MRAM$_4$, and the area estimate obtained using the 4 midpoint rectangles.

b) Use fnint to find the area.

$y_1 = x^2 - \ln x$

x	$x^2 - \ln x$
1.5	
2.5	
3.5	
4.5	

$n=4$ $\quad \frac{b-a}{n} \quad \frac{5-1}{4} = \frac{4}{4} \quad h=1$

$[1,2]\ [2,3]\ [3,4]\ [4,5]$

1.5 2.5 3.5 4.5

MRAM $\quad h \cdot b$

$1\left[\underbrace{1.8445}_{y(1.5)} + \underbrace{5.3337}_{y(2.5)} + \underbrace{10.997}_{y(3.5)} + \underbrace{18.746}_{y(4.5)}\right]$

18.7459

36.9212 $\quad \boxed{36.921}$

1b) fnint$(x^2 - \ln x, x, 1, 5)$

$$\int_1^5 x^2 - \ln x \, dx \approx 37.286$$

2. A solid is formed by revolving the curve

$y = x^{\frac{2}{3}} + 1, \quad 0 \le x \le 2.5$ about the x-axis.

Estimate the volume of the solid by partitioning [0, 2.5] into five subintervals of equal length, slicing the solid with planes perpendicular to the x-axis at the subintervals' left endpoints, and constructing cylinders of height 0.5 based on cross sections at these points, as shown at the right.

$n=5 \qquad \frac{b-a}{n} \quad \frac{2.5-0}{5} = \frac{2.5}{5} \qquad .5 = \Delta x$

$h = .5$

$V_{cylinder} = \pi r^2 h$

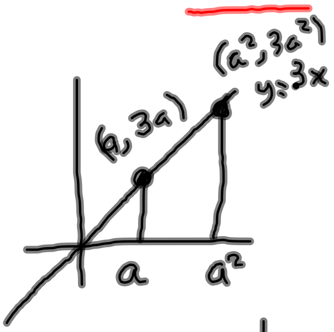
$\pi h (r^2) \qquad r = y \quad r = x^{\frac{2}{3}} + 1$

$[0, .5] \quad [.5, 1] \quad [1, 1.5] \quad [1.5, 2] \quad [2, 2.5]$

need r^2

$$\pi(.5)\left[(0^{\frac{2}{3}}+1)^2 + (.5^{\frac{2}{3}}+1)^2 + (1^{\frac{2}{3}}+1)^2 + (1.5^{\frac{2}{3}}+1)^2 + (2^{\frac{2}{3}}+1)^2\right]$$

3. Use an area to evaluate $\int_a^{a^2}(3x)\,dx$, *where* $a > 1$.



trapezoid

$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}(b-a)(b_1+b_2)$

$\frac{1}{2}(a^2-a)(3a+3a^2)$

$\frac{1}{2}[\cancel{3a^3}+3a^4-3a^2-\cancel{3a^3})$

$\boxed{\frac{1}{2}(3a^4-3a^2)}$

$\cancel{\frac{3x^2}{2}\Big|_a^{a^2}}$

$\cancel{\frac{3(a^2)^2}{2}-\frac{3a^2}{2}}$

$\cancel{\frac{3a^4-3a^2}{2}}$

calculator problem

4. Use fnint to evaluate $\int_3^{7.2}\frac{e^x-\sin(x)}{x}\,dx$.

$y_1 = (e \wedge (x) - \sin(x)) \div x$

fnint$(y_1, x, 3, 7.2)$

$\boxed{216.128}$

5. Suppose that f and g are continuous functions and that

$\int_3^5 f(x)dx = 7, \int_3^5 g(x)dx = 2, \text{ and } \int_0^5 g(x)dx = 4$

Which of the following must be true?

I. $\int_0^3 g(x)dx = 2$

II. $\int_3^5 f(x)g(x)dx = 14$

III. $\int_3^5 [f(x) - g(x)]dx = 5$

A. I and II B. I and III C. II and III

D. III only E. I, II, and III

I $\int_0^3 g(x)\,dx = \int_0^5 g(x)dx + \int_5^3 g(x)\,dx$

True $4 + -2 = \boxed{2}$

II False not multiplication property

III $\int_3^5 f(x)\,dx - \int_3^5 g(x)\,dx$

True $7 - 2 = 5$

$\boxed{B}$

6. Evaluate $\int_2^7 (4x - 10)\,dx$.

Antidifferentiation

$$\frac{4x^2}{2} - 10x \Big|_2^7$$

$$2x^2 - 10x \Big|_2^7$$

$$\boxed{(2(7)^2 - 10(7)) - (2(2)^2 - 10(2))}$$

$$(98 - 70) - (8 - 20)$$

$$28 - -12$$

$$\boxed{40}$$

7. Evaluate $\int_0^{\frac{\pi}{3}} \sec(x)\tan(x)\ dx$

using Part 2 of the Fundamental Theorem of Calculus.

On the test there will be a problem that asks you to write the Fundamental Theorem of Calculus.

part I

if $F(x) = \int_a^x f(t)\,dt$

then $F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt$

$= f(x)$

Part II

if $F'(x) = f(x)$

then $\int_a^b f(x)\,dx = F(b) - F(a)$

$\int_0^{\frac{\pi}{3}} \sec(x)\tan(x)\,dx$

$\sec x \Big|_0^{\frac{\pi}{3}}$

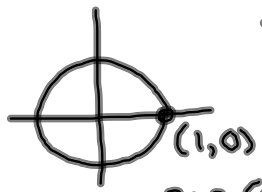
$\sec\left(\frac{\pi}{3}\right) - \sec(0)$

$2 - 1$

$\boxed{1}$

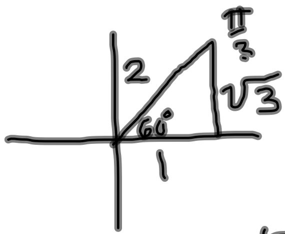
PSST

$\sec x = \frac{1}{\cos(x)}$



$\cos(0) = 1$

$\sec(0) = 1$



$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

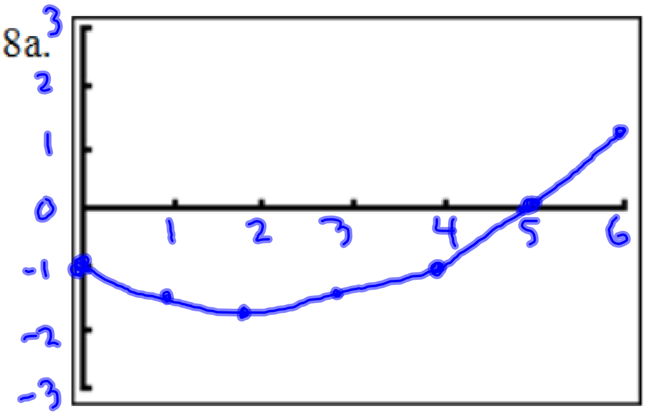
$\sec\left(\frac{\pi}{3}\right) = 2$

8. (a) Graph the function $y = 0.2x^2 - 0.8x - 1$ over the interval [0, 6].
(b) Integrate $y = 0.2x^2 - 0.8x - 1$ over [0, 6].
(c) Find the area of the region between the graph in part (a) and the x-axis Total area

$y_1 = .2x^2 - .8x - 1$

x	y
0	-1
1	-1.6
2	-1.8
3	-1.6
4	-1
5	0
6	1.4

8a.



[0, 6] by [-3, 3]

b) $\int_0^6 0.2x^2 - .8x - 1 \, dx = \boxed{-6}$

c) $-\int_0^5 0.2x^2 - .8x - 1 \, dx + \int_5^6 0.2x^2 - .8x - 1 \, dx$

OR

$\int_0^6 |.2x^2 - .8x - 1| \, dx \approx 7.333$

or $\frac{22}{3}$

9. Let $f(t) = 4 - 3t$.

a)Find $\frac{d}{dx}\int_{-1}^{3x^2} f(t)dt$

b) Find $\frac{d}{dx}\int_{-1}^{x} f(t)\, dt$

a) $\frac{d}{dx}\int_{-1}^{3x^2} f(t)\, dt$

$\frac{d}{dx}\int_{-1}^{3x^2} 4-3t\, dt$

$(4-3(3x^2))\cdot \frac{d}{dx}(3x^2)$

$\boxed{(4-9x^2)(6x)}$

or $\boxed{24x-54x^3}$

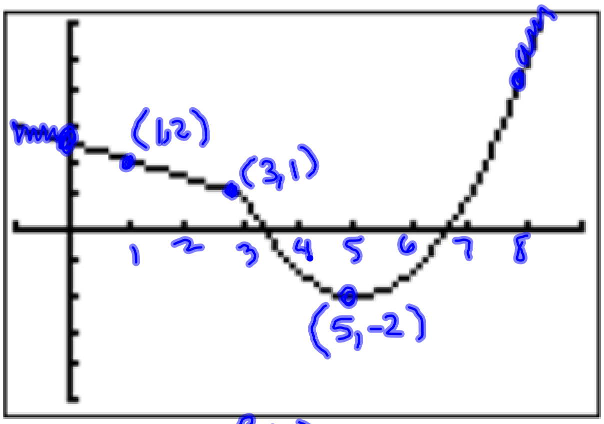
b) $\frac{d}{dx}\int_{-1}^{x} f(t)\, dt$

$\frac{d}{dx}\int_{-1}^{x} 4-3t\, dt$

$\boxed{4-3x}$

10. A particle moves along a coordinate axis. Its position at time t (sec) is $s(t) = \int_0^t f(x)dx$ *cm*, where f(x) is the function whose graph is shown.

$s(t) = \int_0^t f(x)\,dx$



$f(x)$

a) What is the particle's position at t = 0? $s(0) = \int_0^0 f(x)dx$ $\boxed{0\text{ cm}}$

b) What is the particle's position at t = 3?

c) What is the particle's velocity at t = 5?

d) Approximately when is the acceleration zero?

e) At what time during the first 7 sec does s have its largest value?

b) $s(3) = \int_0^3 f(x)\,dx$

area under the curve

$\frac{1}{2}h(b_1+b_2)$

$\boxed{\frac{1}{2}(3)(2.5+1)\text{ cm}}$

$\frac{1}{2}(3)(3.5)$

5.25 cm

$\frac{21}{4}$ cm

c) $s'(t) = \frac{d}{dx}\int_0^t f(x)\,dx$

$s'(t) = f(t)$

$s'(5) = f(5)$ $\boxed{-2\text{ cm/sec}}$

velocity

d) $s'(t) = f(t)$

$s''(t) = f'(t)$

slope of graph

$s''(t) = 0$ $f'(t) = 0$ $\boxed{t = 5\text{ sec}}$

e) when in [0,7] is net area the greatest $\approx$ $\boxed{t = 3.5\text{ sec}}$

11. Use the Trapezoidal Rule with n = 4 to approximate the value of $\int_0^2 (x^2 - 2x + 2)\ dx$.

$\frac{b-a}{n}$ $\frac{2-0}{4}$ $\frac{2}{4}$ $h = \frac{1}{2}$ $\Delta x = \frac{1}{2}$

uniform

x	x^2-2x+2
0	2
$\frac{1}{2}$	$(\frac{1}{2})^2 - 2(\frac{1}{2}) + 2 = \frac{5}{4}$
1	$1 - 2(1) + 2 = 1$
$\frac{3}{2}$	$(\frac{3}{2})^2 - 2(\frac{3}{2}) + 2 = \frac{5}{4}$
2	$2^2 - 2(2) + 2 = 2$

$\frac{1}{2} h [y_0 + 2y_1 + 2y_2 + 2y_3 + y_4]$

$\frac{1}{2} \cdot \frac{1}{2} [2 + 2(\frac{5}{4}) + 2(1) + 2(\frac{5}{4}) + 2]$

$\frac{1}{4}(11) = \boxed{\frac{11}{4}}$

calculator

12. The function f is continuous on the closed interval [1, 7] and has the values that are given in the table below. Use the subintervals [1, 4], [4,6], and [6, 7], what is the trapezoidal approximation of $\int_1^7 f(x)dx$?

x	1	4	6	7
$f(x)$	10	30	40	20

[1, 4] [4, 6] [6, 7]

$\frac{1}{2}h(b_1+b_2) + \frac{1}{2}h(b_1+b_2) + \frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}(4-1)(10+30) + \frac{1}{2}(6-4)(30+40) + \frac{1}{2}(7-6)(40+20)$

$\frac{1}{2}(3)(40) + \frac{1}{2}(2)(70) + \frac{1}{2}(1)(60)$

$60 + 70 + 30$

$\boxed{160}$