Chapter 4 Calculus Practice Exam

1. Find $\frac{dy}{dx}$ *for* $y = \sin(x^2 - 1)$

$$\cos(x^2-1)\cdot\frac{d}{dx}(x^2-1)$$

$$\cos(x^2-1)\cdot 2x$$

$$\boxed{2x\cos(x^2-1)}$$

2. A curve is parametrized by the equations $x = \sqrt{t}$ and $y = (t-3)^2$.

Find an equation of the line tangent to the curve at the point defined by t = 9.

$x=\sqrt{t}$ $\quad x(9)=\sqrt{9}$ $\quad x_1=3$ $\quad (3, 36)$

$y=(t-3)^2$ $\quad y(9)=(9-3)^2$ $\quad y_1=36$ $\quad x_1 \; y_1$

Eq of tangent line $y-y_1=m(x-x_1)$

$y-36=m(x-3)$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dy}{dt}=2(t-3)(1)$$

$$\frac{dx}{dt}=\frac{1}{2\sqrt{t}} \quad \text{or} \quad x=t^{\frac{1}{2}} \quad \frac{dx}{dt}=\frac{1}{2}t^{-\frac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{t=9} \frac{2(9-3)}{\frac{1}{2\sqrt{9}}} \Rightarrow \frac{12}{\frac{1}{2(3)}}$$

$$\frac{12}{1}\cdot\frac{6}{1}=72$$

$$\left.\frac{dy}{dx}\right|_{t=9} \frac{2(9-3)}{\frac{1}{2}(9)^{-\frac{1}{2}}} \quad \frac{2(6)}{\frac{1}{2}\cdot\frac{1}{3}} \quad \frac{12}{\frac{1}{6}} = 72$$

$$\boxed{y-36=72(x-3)}$$

OR

$$y=36+72(x-3)$$

3. Which of the following could be true if $f''(t) = t^{\frac{-2}{3}}$?

True

I. $f(t) = \frac{9}{4} t^{\frac{4}{3}}$

$f'(t) = \cancel{\frac{4}{3}} \cdot \cancel{\frac{9}{4}} t^{\frac{4}{3}-1}$ $3t^{\frac{1}{3}}$

$f''(t) = \cancel{\frac{1}{3}(3)} t^{\frac{1}{3}-1}$ $t^{\frac{-2}{3}}$

False

II. $f'(t) = 7 - 3t^{\frac{1}{3}}$

$f''(t) = 0 - 3 \cdot \frac{1}{3} t^{\frac{1}{3}-1}$

$-t^{-2/3}$

True

III. $f'''(t) = \frac{-2}{3} t^{\frac{-5}{3}}$?

given $f''(t) = t^{\frac{-2}{3}}$ $f'''(t) = \frac{-2}{3} t^{\frac{-2}{3}-1}$ $f'''(t) = \frac{-2}{3} t^{\frac{-5}{3}}$

~~(A)~~ I and II (B) I and III (C) I only

~~(D)~~ III only ~~(E)~~ I, II, and III

4. Use implicit differentiation to find $\frac{dy}{dx}$ *if* $x^2 + 5xy + y^5 = 8$.

$x^2 + 5x(y) + (y)^5 = 8$

find $\frac{dy}{dx}$ so y is a function of x

$2x + \left[5x \cdot \frac{d}{dx}(y)' + y\frac{d}{dx}(5x)\right] + 5(y)^4 \cdot \frac{d}{dx}(y) = 0$

$2x + \left[5x\frac{dy}{dx} + 5y\right] + 5y^4\frac{dy}{dx} = 0$

$2x + 5x\frac{dy}{dx} + 5y + 5y^4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(5x + 5y^4) = -2x - 5y$

$\frac{dy}{dx} = \frac{-2x-5y}{5x+5y^4}$

5. Find $\frac{dy}{dx}$ *if* $y = \tan^{-1}\left(\frac{1}{2x}\right)$.

$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx}$

$u = \frac{1}{2x}$ $(2x)^{-1}$

$\frac{du}{dx} = -1(2x)^{-2} \cdot 2$

$\frac{-2}{(2x)^2}$ $\frac{-2}{4x^2} = \frac{-1}{2x^2}$

$\boxed{\frac{1}{1+\left(\frac{1}{2x}\right)^2} \cdot \frac{-1}{2x^2}}$

or simplify

$\frac{1}{1+\frac{1}{4x^2}} \cdot \frac{-1}{2x^2}$

$\frac{1}{\frac{4x^2}{4x^2}+\frac{1}{4x^2}} \cdot \frac{-1}{2x^2}$

$\frac{1}{\frac{4x^2+1}{\cancel{4x^2}\,2}} \cdot \frac{-1}{\cancel{2x^2}}$

$\boxed{\frac{-2}{4x^2+1}}$

6. Find $\frac{dy}{dx}$ *if* $y = 4^{-x+3}$

$a=4$

$u=-x+3$

$\frac{du}{dx}=-1$

$\frac{d}{dx}a^u = a^u \ln a \cdot \frac{du}{dx}$

$\boxed{\frac{dy}{dx}=4^{-x+3}\cdot \ln 4 \cdot -1}$

7. Which of the following expression has the same derivative as $y = \log x$?

(A) $\log_6 x$ (B) $\log 5x$ (C) $\log x^2$

(D) $3 \log x$ (E) $\log \frac{1}{x}$

$y=\log x$

$y=\log_{10} x$

$\frac{dy}{dx}=\frac{1}{x \ln 10}$

$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\cdot\frac{du}{dx}$

$u=x$ $a=10$ $\frac{du}{dx}=1$

A $\frac{dy}{dx}=\frac{1}{x\ln 6}$

B $\log_{10} 5x$

$\frac{1}{\cancel{5}x\cdot\ln 10}\cdot\cancel{5}$

$\frac{1}{x\ln 10}$

C $\log_{10} x^2$

$\frac{1}{x^2\ln 10}\cdot 2x$

$\frac{2}{x\ln 10}$

D $3\log_{10} x$

$\frac{3}{x\ln 10}$

E $\log_{10}\frac{1}{x}$

$\frac{1}{\frac{1}{x}\cdot\ln 10}\cdot\frac{-1}{x^2}$

$\frac{-1}{\frac{\ln 10}{x}\cdot x^2}$

$\frac{-1}{x\ln 10}$

8. Find $\frac{dy}{dx}$. $\quad log_5(e^x - 4x)$

$a=5 \quad u=e^x-4x \quad \frac{du}{dx}=e^x-4$

$\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$

$\frac{1}{(e^x-4x)\ln 5} \cdot e^x-4$ or $\boxed{\frac{e^x-4}{\ln 5(e^x-4x)}}$

9. Find the derivative of the function $f(x)g(h(x))$

product composite chain rule

$f(x) \cdot g(h(x))$

$f(x) \cdot \frac{d}{dx}[g(h(x))] + g(h(x)) \cdot f'(x)$

$\boxed{f(x)\, g'(h(x)) \cdot h'(x) + g(h(x)) \cdot f'(x)}$

10. Find $\frac{dV}{dt}$ $\quad V = 3000\left(1 - \frac{t}{20}\right)^2$

$\frac{dV}{dt} = 3000 \frac{d}{dt}\left(1 - \frac{1}{20}t\right)^2$

blob^2

$3000 \cdot 2\left(1-\frac{1}{20}t\right)^1 \cdot \frac{d}{dt}\left(1-\frac{1}{20}t\right)$

$\boxed{6000\left(1-\frac{1}{20}t\right) \cdot -\frac{1}{20}}$

$-300\left(1-\frac{1}{20}t\right)$

$-300 + 15t$

11. Find $\frac{dh}{dr}$ $\quad h(r) = \sqrt[3]{3r} \cdot g(r)$

$(3r)^{\frac{1}{3}} \cdot g(r)$ product

$(3r)^{\frac{1}{3}} \cdot g'(r) + g(r) \cdot \frac{d}{dr}(3r)^{\frac{1}{3}}$

$\boxed{(3r)^{\frac{1}{3}} \cdot g'(r) + g(r) \cdot \frac{1}{3}(3r)^{-\frac{2}{3}} \cdot 3}$

$\sqrt[3]{3r} \cdot g'(r) + g(r) \cdot (3r)^{-\frac{2}{3}}$

12. Find $\frac{df}{dy}$ $\quad f(y) = \frac{\pi}{y^8} + 3e^{\sin(-2y)}$

variable is y $\quad \pi y^{-8} + 3e^{\sin(-2y)}$

e^u

$\frac{df}{dy} = -8\pi y^{-9} + 3e^{\sin(-2y)} \cdot \frac{d}{dx}\sin(-2y)$

$\frac{df}{dy} = -8\pi y^{-9} + 3e^{\sin(-2y)} \cdot \cos(-2y) \cdot \frac{d}{dx}(-2y)$

$\boxed{\frac{df}{dy} = -8\pi y^{-9} + 3e^{\sin(-2y)} \cdot \cos(-2y) \cdot -2}$

$-8\pi y^{-9} + -6e^{\sin(-2y)} \cdot \cos(-2y)$

13. Find $\frac{dy}{dx}$ $\quad y = \sin^{-1}(4x^3)$

$u = 4x^3$

$\frac{d}{dx} \sin^{-1} u \quad \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \quad \frac{du}{dx} = 12x^2$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x^3)^2}} \cdot 12x^2$$

$$\boxed{\frac{12x^2}{\sqrt{1-(4x^3)^2}}}$$

14. Find $\frac{dy}{dx}$ $\quad y = (\ln(\tan^{-1}(\pi x)))^2$

$$\frac{dy}{dx} = 2(\ln(\tan^{-1}(\pi x)))^1 \cdot \frac{d}{dx} \ln(\tan^{-1}(\pi x))$$

$$\frac{dy}{dx} = 2(\ln(\tan^{-1}(\pi x)))^1 \cdot \frac{1}{\tan^{-1}(\pi x)} \cdot \frac{d}{dx} \tan^{-1}(\pi x)$$

$$\frac{2(\ln(\tan^{-1}(\pi x)))^1}{\tan^{-1}(\pi x)} \cdot \frac{1}{1+(\pi x)^2} \cdot \frac{d}{dx}(\pi x)$$

$$\boxed{\frac{2(\ln(\tan^{-1}(\pi x)))^1}{\tan^{-1}(\pi x)} \cdot \frac{1}{1+(\pi x)^2} \cdot \pi}$$

15. Find $\frac{dy}{dx}$ $\quad y = x^{\ln(x)}$

$\ln y = \ln x^{\ln(x)}$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{2\ln x}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x} \cdot y$

$\boxed{\frac{dy}{dx} = \frac{2\ln x}{x} \cdot x^{\ln(x)}}$

16. Find $\frac{dy}{dx}$ if $3xy = 0$

product

$3x \cdot \frac{dy}{dx} + y(3) = 0$

$3x\frac{dy}{dx} = -3y$

$\boxed{\frac{dy}{dx} = \frac{-3y}{3x}}$ or $\frac{-y}{x}$

17. Find $\frac{dy}{dx}$ if $(y)^4 + \underline{\underline{2x^2}}\,\underline{\underline{(y)^2}} - 3x^2 = 10$

implicit

$$4y^3\frac{dy}{dx} + \left[2x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 4x\right] - 6x = 0$$

$$4y^3\frac{dy}{dx} + 4x^2y\frac{dy}{dx} + 4xy^2 - 6x = 0$$

$$\frac{dy}{dx}(4y^3 + 4x^2y) = -4xy^2 + 6x$$

$$\boxed{\frac{dy}{dx} = \frac{-4xy^2 + 6x}{4y^3 + 4x^2y}}$$

18. Find $\frac{df}{dy}$ if $f(y) = \left(\frac{y-3}{y+2}\right)^3$ chain quotient

not implicit
y is variable

blob^3

$$3\left(\frac{y-3}{y+2}\right)^2 \cdot \frac{d}{dy}\left(\frac{y-3}{y+2}\right)$$

$$\boxed{3\left(\frac{y-3}{y+2}\right)^2 \cdot \left(\frac{(y+2)(1) - (y-3)(1)}{(y+2)^2}\right)}$$

simplified $\frac{15(y-3)^2}{(y+2)^4}$

19. Find $\frac{dx}{d\theta}$ if $x(\theta) = \cos\left(\tan\left(\sqrt{2\theta}\right)\right)$

$$\frac{dx}{d\theta} = -\sin\left(\tan\sqrt{2\theta}\right) \cdot \frac{d}{d\theta}\left(\tan\sqrt{2\theta}\right)$$

$$-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{d}{d\theta}\left(\sqrt{2\theta}\right)$$

$$-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{1}{2\sqrt{2\theta}} \cdot \frac{d}{d\theta}(2\theta)$$

$$\boxed{-\sin\left(\tan\sqrt{2\theta}\right) \cdot \sec^2\left(\sqrt{2\theta}\right) \cdot \frac{1}{2\sqrt{2\theta}} \cdot 2}$$

20. Find $\frac{df}{dt}$ if $f(t) = 5\sec(3t) - \tan(3t)$

$u = 3t$

$$\frac{df}{dt} = 5\sec(3t)\tan(3t) \cdot 3 - \sec^2(3t) \cdot 3$$

$$\boxed{15\sec(3t)\tan(3t) - 3\sec^2(3t)}$$