Chapter 7 Calculus Practice Exam

1. Use the Fundamental Theorem of Calculus to evaluate
$\int e^{x^2+\sin(x)}\,dx$

A. $\int_0^x (t^2 + \sin t\,)\;dt\; + C$

B. $\int_x^0 e^{t^2+\sin(t)}\;dt\; + C$

C. $(2x\; + \cos x)e^{x^2+\sin(x)} + C$

D. $\int_0^x e^{t^2+\sin(t)}\;dt\; + C$

E. $\int_0^x (2t + \cos t)e^{t^2+\sin(t)}\;dt\; + C$

$\int_0^x e^{t^2+\sin t}(dt)\;\; + C$

2. Evaluate $\int e^{3x} - 4\cos x\, dx$

$\int e^{3x}\,dx - \int 4\cos x\,dx$

Memorize From table

$\int e^{kx}\,dx$

$\boxed{\frac{e^{3x}}{3} - 4\sin x + C}$

$\frac{e^{kx}}{k} + C$

3. Solve the initial value problem.

$$\frac{dy}{dx} = 5x^2 - 7,\ \ y(0) = 1$$

$$dy = (5x^2 - 7)dx$$

$$\int dy = \int 5x^2 - 7\ dx$$

$$y = \frac{5x^3}{3} - 7x + C$$

$$1 = \frac{5(0)^3}{3} - 7(0) + C$$

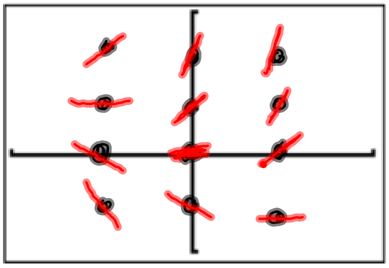
$$1 = C$$

$$\boxed{y = \frac{5x^3}{3} - 7x + 1}$$

4. Construct a slope field for the differential equation through the twelve lattice points shown in the graph.

$$\frac{dy}{dx} = x + y$$

4.



[-2,2] by [-2,3]

x	y	$\frac{dy}{dx} = x+y$
-1	-1	$-1+-1 = -2$
-1	0	$-1+0 = -1$
-1	1	$-1+1 = 0$
-1	2	$-1+2 = 1$
0	-1	-1
0	0	0
0	1	1
0	2	2

x	y	$\frac{dy}{dx} = x+y$
1	-1	0
1	0	1
1	1	2
1	2	3

5. Use substitution to evaluate $\int \frac{(\ln x)^5 dx}{x}$

$$\int (\ln x)^5 \cdot \frac{1}{x} dx \qquad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u^5 \, du$$

$$\frac{u^6}{6} + C$$

$$\boxed{\frac{(\ln x)^6}{6} + C}$$

6. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b)

$\int_0^{\frac{\pi}{2}} \left(e^{\sin x} \cos x\right) dx.$

$u = \sin x$

$du = \cos x\, dx$

$$\int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \cos x\, d\cancel{x}$$

$$\int_{u(0)}^{u(\frac{\pi}{2})} e^u \cdot du$$

$u(0) = \sin(0)$

$u(0) = 0$

$u\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2}$

$u\left(\frac{\pi}{2}\right) = 1$

$$\int_0^1 e^u \cdot du$$

$$e^u \Big|_0^1$$

$$e^1 - e^0$$

$$\boxed{e - 1}$$

7. Use separation of variables to solve the initial value problem.
$\frac{dy}{dx} = \frac{\cos x}{3y^2}$; $y(\pi) = 5$

$$3y^2 \cdot dy = \cos x \, dx$$

$$\int 3y^2 \cdot dy = \int \cos x \, dx$$

$$\frac{3y^3}{3} = \sin x + C$$

$$y^3 = \sin x + C$$

$$5^3 = \sin(\pi) + C$$

$$125 = 0 + C$$

$$125 = C$$

$$y^3 = \sin x + 125$$

$$\boxed{y = \sqrt[3]{\sin x + 125}}$$

8. Use integration by parts to evaluate $\int \cos^{-1} 2x \; dx$

$\int u\,dv = uv - \int v\,du$

$\int \cos^{-1}(2x) \cdot dx$

$u = \cos^{-1}(2x)$ $\quad dv = dx$

$du = \frac{-1}{\sqrt{1-(2x)^2}} \cdot 2$ $\quad \frac{dv}{dx} = 1$

$du = \frac{-2}{\sqrt{1-4x^2}}dx$ $\quad V = x$

$uv - \int v\,du$

$x\cos^{-1}(2x) - \int x \cdot \frac{-2}{\sqrt{1-4x^2}}dx$

u substitution

$u = 1-4x^2$

$du = -8x\,dx$

$\frac{1}{4}du = -2x\,dx$

I have a $-2x\,dx$

$x\cos^{-1}(2x) - \int \frac{1}{\sqrt{u}} \cdot \frac{1}{4}du$

$x\cos^{-1}(2x) - \frac{1}{4}\int u^{-\frac{1}{2}}\,du$

$x\cos^{-1}(2x) - \frac{1}{4}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$\div$ by $\frac{1}{2}$ means multiply by 2

$x\cos^{-1}(2x) - \frac{1}{4} \cdot 2\,(1-4x^2)^{\frac{1}{2}} + C$

$$\boxed{x\cos^{-1}(2x) - \frac{1}{2}(1-4x^2)^{\frac{1}{2}} + C}$$

9. Evaluate $\int (4x^2 - 3x)e^x \ dx$

use Tabular → product and one of factors is a polynomial

take derivative	$\int$ integrate
$4x^2-3x$	e^x
$8x-3$	e^x
8	e^x
0	e^x

$$\boxed{(4x^2-3x)e^x - (8x-3)e^x + 8e^x + C}$$

$$e^x(4x^2-3x-8x+3+8) + C$$

$$e^x(4x^2-11x+11) + C$$

10. Evaluate $\int 2\cos(\ln t)\ dt$ by using a substitution **prior** to integration by parts.

Let $w = \ln t$ $\quad dw = \frac{1}{t}dt \quad w = \ln t$

$\frac{dw}{dt} = \frac{1}{t} \quad t\,dw = dt \quad w = \log_e t$

$dw = \frac{1}{t}dt \quad e^w = t$

$\int 2\cos(w)\ dt$

$\int 2\cos(w)\cdot t\,dw$

$\int 2\cos(w)\cdot e^w\,dw$ $\quad u = 2\cos w \quad du = -2\sin w\,dw \quad dv = e^w dw \quad \frac{dv}{dw} = e^w \quad v = e^w$

$u\,dv = uv - \int v\,du$

$\int 2\cos w\cdot e^w dw = 2\cos w\cdot e^w - \int e^w\cdot -2\sin w\,dw$

$\int 2\cos w\cdot e^w dw = 2\cos w\cdot e^w + \underbrace{\int 2\sin w\cdot e^w dw}$

$u = 2\sin w \quad du = 2\cos w\,dw \quad dv = e^w dw \quad \frac{dv}{dw} = e^w \quad V = e^w$

$uv - \int v\,du$

$\int 2\cos w\cdot e^w dw = 2\cos(w)e^w + 2\sin w\cdot e^w - \int e^w\cdot 2\cos w\,dw$

same

$2\int 2\cos w\cdot e^w dw = 2\cos(w)e^w + 2\sin w\cdot e^w$

$\int 2\cos w\cdot e^w dw = \frac{2\cos(w)e^w + 2\sin(w)\cdot e^w}{2} + C$

$\int 2\cos w\cdot e^w dw = \cos(w)e^w + \sin(w)e^w + C$

need "t" not "w"

$\cos(\ln t)e^{\ln t} + \sin(\ln t)e^{\ln t} + C$

$\boxed{\cos(\ln t)\cdot t + \sin(\ln t)\cdot t + C}$

$t\cos(\ln t) + t\sin(\ln t) + C$

11. Find the solution of the differential equation $\frac{dy}{dt} = ky$, k is a constant, that satisfies the given conditions. k = 1.5, y(0) = 100. Show your work.

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k \, dt$$

$$\ln|y| = kt + C$$

$$|y| = e^{kt+C}$$

$$|y| = e^{kt} \cdot e^{C}$$

$$y = \pm e^{C} \cdot e^{kt}$$

$$y = Ce^{kt}$$

$$100 = Ce^{1.5(0)}$$

$$100 = C$$

$$\boxed{y = 100e^{1.5t}}$$

12. Find the partial fraction decomposition.

$$\frac{2x+16}{x^2+x-6}$$

$$\frac{2x+16}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$2x+16 = A(x-2) + B(x+3)$$

Let $x=2$ $\quad 2(2)+16 = \cancel{A(2-2)} + B(2+3)$

$$20 = 5B$$

$$4 = B$$

Let $x=-3$

$$2(-3)+16 = A(-3-2) + \cancel{B(-3+3)}$$

$$-6+16 = -5A$$

$$10 = -5A$$

$$-2 = A$$

Answer $\boxed{\frac{-2}{x+3} + \frac{4}{x-2}}$

13. Evaluate the integral.

$$\int \frac{2x+16}{x^2+x-6}\,dx$$

from #12

$$\int \frac{-2}{x+3} + \frac{4}{x-2}\,dx$$

$$\boxed{-2\ln|x+3| + 4\ln|x-2| + C}$$

$$\ln|x-2|^4 - 2\ln|x+3| + C$$

$$\ln\left(\frac{|x-2|^4}{|x+3|^2}\right) + C$$

14. A population of wild horses is represented by the logistic differential equation $\frac{dP}{dt} = 0.08P - 0.00004P^2$, where t is measured in years.

a.) Find k and the carrying capacity for the population.

b.) The initial population is P(0) = 10 horses. Find a formula for the population in terms of t.

c.) When is the size of the population growing the fastest?

$\frac{dP}{dt} = .08P - .00004P^2$

$\frac{dP}{dt} = KP(M-P)$ one of formulas for logistics

factor out a .08P

$\frac{dP}{dt} = .08P(1-.0005P)$

use this formula

$\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right)$

$\frac{dP}{dt} = .08P\left(1-\frac{5}{10000}P\right)$

$\frac{dP}{dt} \quad .08P\left(1-\frac{1}{2000}P\right)$

$k = .08 \quad M = 2000$

b) $P = \frac{M}{1+Ae^{-kt}}$

$P(0) = 10$

$10 = \frac{2000}{1+Ae^{-.08(0)}}$

$10 = \frac{2000}{1+A(1)}$

$10(1+A) = 2000$

$1+A = 200$

$A = 199$

$\boxed{P = \frac{2000}{1+199e^{-.08t}}}$

c) growing fastest when $P = 1000$

when

$1000 = \frac{2000}{1+199e^{-.08t}}$

$1000(1+199e^{-.08t}) = 2000$

$1+199e^{-.08t} = 2$

$199e^{-.08t} = 1$

$e^{-.08t} = \frac{1}{199}$

$-.08t\cancel{\ln e} = \ln\left(\frac{1}{199}\right)$

$t = \ln(1 \div 199) \div -.08$

$\boxed{t \approx 66.166 \text{yrs.}}$

15. Suppose Euler's method, with increment dx, is used to numerically solve the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition (x_0, y_0) lies on the solution curve. Let (x_1, y_1), (x_2, y_2), and so on denote the points generated by Euler's method, and let y = y(x) denote the exact solution to the initial value problem. Which of the following must be true?

is Euler's y_3 same as exact $y(x_3)$

dx=

F I. $y_3 = y(x_3)$

T II. $y_2 = y_1 + f(x_1, y_1)\,dx$

T III. $x_3 = x_0 + 3dx$

A. II only B. I and II C. I and III

D. II and III E. I, II, and III

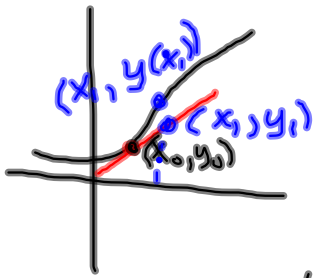
$X_0 = x_0 \quad y_0 = y_0$

$X_1 = X_0 + dx \quad y_1 = \_\_$

$X_2 = X_0 + dx + dx$

$X_2 = X_0 + 2dx \quad y_2 = \_\_$

$X_3 = X_0 + 3dx \quad y_3 = \_\_$



Slope

$y_n = y_{n-1} + \frac{dy}{dx}(x_{n-1}, y_{n-1})\,dx$

16. Use Euler's method to numerically solve the initial value problem $\boxed{y' = e^x - 10y}$, $y(2) = 3.5$. Using $dx = 0.1$ find y (2.3). Show all steps leading to your answer and round y-values to the nearest 0.001.

$dx = 0.1$ Find $y(2.3)$

$y_1 = y_0 + (e^{x_0} - 10y_0)dx$

$y_1 = 3.5 + (e^2 - 10(3.5))(.1)$

do this first then add 3.5

$y_1 = .739$

$y_2 = y_1 + (e^{x_1} - 10(y_1))\,dx$

$.739 + (e^{2.1} - 10(.739))(.1)$

$y_2 = .817$

$y_3 = y_2 + (e^{x_2} - 10(y_2))(dx)$

$.817 + (e^{2.2} - 10(.817))(.1)$

$y_3 = .903$

$x_0 = 2 \quad y_0 = 3.5$

$x_1 = 2.1 \quad y_1 = .739$

$x_2 = 2.2 \quad y_2 = .817$

$x_3 = 2.3 \quad \boxed{y_3 = .903}$

(2.3, ≈

when x is 2.3

they (2.3) ≈ .903