Chapter 7 Calculus Practice Exam

1. Use the Fundamental Theorem of Calculus to evaluate
$\int e^{x^2+\sin(x)}\, dx$

A. $\int_0^x (t^2 + \sin t\ )\ dt\ + C$

B. $\int_x^0 e^{t^2+\sin(t)}\ dt\ + C$

C. $(2x\ + \cos x)e^{x^2+\sin(x)} + C$

D. $\int_0^x e^{t^2+\sin(t)}\ dt\ + C$

E. $\int_0^x (2t + \cos t)e^{t^2+\sin(t)}\ dt\ + C$

Not u-sub
Not $\int u\,dv$

$$\int_0^x e^{t^2+\sin t}\, dt + C$$

2. Evaluate $\int \boldsymbol{e^{3x} - 4\cos x\, dx}$

$$\int e^{3x}\,dx - \int 4\cos x\,dx$$

$$\boxed{\frac{e^{3x}}{3} - 4\sin x + C}$$

$$\int e^{kx}\,dx$$

$$\frac{e^{kx}}{k}$$

3. Solve the initial value problem.

$\frac{dy}{dx} = 5x^2 - 7, \ y(0) = 1$

$dy = (5x^2 - 7)\,dx$

$\int dy = \int (5x^2 - 7)\,dx$

$y = \frac{5x^3}{3} - 7x + C$

$1 = \frac{5(0)^3}{3} - 7(0) + C$

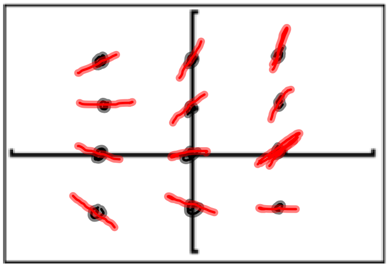
$1 = C$

$\boxed{y = \frac{5x^3}{3} - 7x + 1}$

4. Construct a slope field for the differential equation through the twelve lattice points shown in the graph.

$$\frac{dy}{dx} = x + y$$

4.



[-2,2] by [-2,3]

x	y	$\frac{dy}{dx} = x+y$
-1	-1	$-1+-1=-2$
-1	0	$-1+0=-1$
-1	1	$-1+1=0$
-1	2	$-1+2=1$
0	-1	-1
0	0	0
0	1	1
0	2	2

x	y	$\frac{dy}{dx}$ $x+y$
1	-1	0
1	0	1
1	1	2
1	2	3

5. Use substitution to evaluate $\int \frac{(\ln x)^5 dx}{x}$

$\int (\ln x)^5 \cdot \frac{1}{x}\, dx$ $u = \ln x$

$du = \frac{1}{x}\, dx$

$\int u^5 \cdot du$

$\frac{u^6}{6} + C$

$\boxed{\frac{(\ln x)^6}{6} + C}$

6. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b)

$\int_0^{\frac{\pi}{2}} (e^{\sin x} \cos x)\, dx.$

$u = \sin x$

$\int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \cos x \, dx \rightarrow du = \cos x \, dx$

$\int_{u(0)}^{u(\frac{\pi}{2})} e^u \, du$

$u(0) = \sin(0)$

$u(0) = 0$

$u(\frac{\pi}{2}) = \sin(\frac{\pi}{2})$

$u(\frac{\pi}{2}) = 1$

$\int_0^1 e^u \, du$

$e^u \Big|_0^1$

$e^1 - e^0$

$\boxed{e^1 - 1}$

7. Use separation of variables to solve the initial value problem.
$\frac{dy}{dx} = \frac{\cos x}{3y^2}$; y (π) = 5

$3y^2\,dy = \cos x\,dx$

$\int 3y^2\,dy = \int \cos x\,dx$

$\frac{\cancel{3}y^3}{\cancel{3}} = \sin x + C$

$y^3 = \sin x + C$

$5^3 = \sin \pi + C$

$125 = 0 + C$

$125 = C$

$y^3 = \sin x + 125$

$\boxed{y = \sqrt[3]{\sin x + 125}}$

8. Use integration by parts to evaluate $\int \cos^{-1} 2x \; dx$

$\int u \; dv = uv - \int v\,du$

$\int \cos^{-1}(2x)\,dx$

$u = \cos^{-1}(2x)$ $dv = dx$

$du = \frac{-1}{\sqrt{1-(2x)^2}} \cdot 2\,dx$ $\frac{dv}{dx} = 1$

$v = x$

$du = \frac{-2}{\sqrt{1-4x^2}}\,dx$

$uv - \int v\,du$

$x\cos^{-1}(2x) - \int x \cdot \frac{-2}{\sqrt{1-4x^2}}\,dx$

now let
$u = 1-4x^2$
$du = -8x\,dx$

$x\cos^{-1}(2x) - \int \frac{-2x}{\sqrt{1-4x^2}}\,dx \Rightarrow \frac{1}{4}du = -2x\,dx$

$x\cos^{-1}(2x) - \int \frac{1}{\sqrt{u}} \cdot \frac{1}{4}\,du$

$x\cos^{-1}(2x) - \frac{1}{4}\int u^{-\frac{1}{2}}\,du$

$x\cos^{-1}(2x) - \frac{1}{4} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$\div \frac{1}{2}$ is same as mult by 2

$x\cos^{-1}(2x) - \frac{1}{4} \cdot \frac{2}{1} \cdot (1-4x^2)^{\frac{1}{2}} + C$

$\boxed{x\cos^{-1}(2x) - \frac{1}{2}(1-4x^2)^{\frac{1}{2}} + C}$

9. Evaluate $\int (4x^2 - 3x)e^x \, dx$

Tabular product - 1 factor is polynomial

derive	$\int$
$4x^2-3x$	e^x
$+$	
$8x-3$	e^x
$-$	
8	e^x
$+$	
0	e^x

$$\boxed{(4x^2-3x)e^x - (8x-3)e^x + 8e^x + C}$$

$$e^x(4x^2-3x-8x+3+8) + C$$

$$e^x(4x^2-11x+11) + C$$

10. Evaluate $\int 2\cos(\ln t)\ dt$ by using a substitution **prior** to integration by parts.

$\int 2\cos(\ln t)\,dt$

$w = \ln t$ $\quad w = \log_e t$

$dw = \frac{1}{t}dt$ $\quad e^w = t$

$t\,dw = dt$

$\int 2\cos(w)\cdot t\,dw$

$\int 2\cos(w)\cdot e^w dw$

$u = 2\cos w$ $\quad dv = e^w dw$

$du = -2\sin w\,dw$ $\quad \frac{dv}{dw} = e^w$

$v = e^w$

product

$\int u\,dv = uv - \int v\,du$

$\int 2\cos(w)\cdot e^w dw = 2\cos(w)\cdot e^w - \int e^w\cdot -2\sin w\,dw$

$\int 2\cos(w)\cdot e^w dw = 2\cos(w)\cdot e^w + \underbrace{\int 2\sin w\cdot e^w dw}_{\int u\,dv}$

$u = 2\sin w$ $\quad dv = e^w dw$

$du = 2\cos w\,dw$ $\quad v = e^w$

$uv - \int v\,du$

$\underline{\int 2\cos(w)e^w dw} = 2\cos(w)e^w + 2\sin w\cdot e^w - \underline{\int e^w\cdot 2\cos w\,dw}$

same

$$\frac{2\int 2\cos(w)e^w dw}{2} = \frac{2\cos(w)\cdot e^w + 2\sin(w)e^w}{2} + C$$

$\int 2\cos(w)e^w dw = \cos w\cdot e^w + \sin w\,e^w + C$

$\cos(\ln t)\cdot e^{\ln t} + \sin(\ln t)e^{\ln t} + C$

$\cos(\ln t)\cdot t + \sin(\ln t)\cdot t + C$

$\boxed{t\cos(\ln t) + t\sin(\ln t) + C}$

11. Find the solution of the differential equation $\frac{dy}{dt} = ky$, k is a constant, that satisfies the given conditions. k = 1.5, y(0) = 100. Show your work.

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$|y| = e^{kt+C}$$

$$|y| = e^{kt} \cdot e^{C}$$

$$y = \pm e^{C} \cdot e^{kt}$$

$$y = Ce^{kt}$$

$$A = A_0 e^{kt}$$

$$\boxed{A(t) = 100e^{1.5t}}$$

$$100 = Ce^{1.5(0)}$$

$$100 = C$$

$$\boxed{y = 100e^{1.5t}}$$

12. Find the partial fraction decomposition.

$$\frac{2x+16}{x^2+x-6}$$

$$\frac{2x+16}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$2x+16 = A(x-2) + B(x+3)$$

Let $x=2$

$$2(2)+16 = \cancel{A(2-2)} + B(2+3)$$

$$20 = 5B$$

$$4 = B$$

Let $x=-3$

$$2(-3)+16 = A(-3-2) + \cancel{B(-3+3)}$$

$$10 = -5A$$

$$-2 = A$$

$$\boxed{\frac{-2}{x+3} + \frac{4}{x-2}}$$

13. Evaluate the integral.

$$\int \frac{2x+16}{x^2+x-6}\,dx$$

use from #12

$$\int \frac{-2}{x+3} + \frac{4}{x-2}\,dx$$

$$\boxed{-2\ln|x+3| + 4\ln|x-2| + C}$$

$$4\ln|x-2| - 2\ln|x+3| + C$$

$$\boxed{\ln\left(\frac{|x-2|^4}{|x+3|^2}\right) + C}$$

14. A population of wild horses is represented by the logistic differential equation $\frac{dP}{dt} = 0.08P - 0.00004P^2$, where t is measured in years.

a.) Find k and the carrying capacity for the population.

b.) The initial population is P(0) = 10 horses. Find a formula for the population in terms of t.

c.) When is the size of the population growing the fastest?

$$\frac{dP}{dt} = kP(M-P)$$

$$\frac{dP}{dt} = 0.08P - 0.00004P^2$$

$$\frac{dP}{dt} = .08P(1 - .0005P)$$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

$$.08P\left(1 - \frac{5}{10000}P\right)$$

$$.08P\left(1 - \frac{1}{2000}P\right)$$

$\boxed{k = .08}$ $\boxed{M = 2000}$

Remember: $P(t) = \frac{M}{1 + Ae^{-kt}}$

$$P(0) = \frac{2000}{1 + Ae^{-.08(0)}}$$

$$10 = \frac{2000}{1 + A(1)}$$

$$10(1+A) = 2000$$

$$1 + A = 200$$

$$A = 199$$

$$\boxed{P(t) = \frac{2000}{1 + 199e^{-.08t}}}$$

(c) Find t when P = 1,000

$$1000 = \frac{2000}{1 + 199e^{-.08t}}$$

$$1000(1 + 199e^{-.08t}) = 2000$$

$$1 + 199e^{-.08t} = 2$$

$$199e^{-.08t} = 1$$

$$e^{-.08t} = \frac{1}{199}$$

$\log_e\left(\frac{1}{199}\right) = -.08t$ $\quad \ln e^{-.08t} = \ln\left(\frac{1}{199}\right)$

$\frac{\ln\left(\frac{1}{199}\right)}{-.08} = t$ $\quad -.08t = \ln\left(\frac{1}{199}\right)$

$$t = \ln\left(\frac{1}{199}\right) \div (-.08)$$

$$\boxed{\approx 66.166 \text{ yrs}}$$

15. Suppose Euler's method, with increment dx, is used to numerically solve the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition (x_0, y_0) lies on the solution curve. Let (x_1, y_1), (x_2, y_2), and so on denote the points generated by Euler's method, and let y = y(x) denote the exact solution to the initial value problem. Which of the following must be true?

I. $y_3 = y(x_3)$ F does Euler's y_3 = exact $y(x_3)$

II. $y_2 = y_1 + f(x_1, y_1)\,dx$ True

III. $x_3 = x_0 + 3dx$ True

A. II only ~~B.~~ I and II ~~C.~~ I and III

D. II and III ~~E.~~ I, II, and III

$dx =$

$x_0 = x_0$ $y_0 = y_0$

$x_1 = x_0 + dx$ $y_1 =$

$x_2 = x_0 + 2dx$ $y_2 =$

$x_3 = x_0 + 3dx$ $y_3 =$

16. Use Euler's method to numerically solve the initial value problem $y' = e^x - 10y$, y(2) = 3.5. Using dx = 0.1 find y (2.3). Show all steps leading to your answer and round y-values to the nearest 0.001.

$y_1 = y_0 + y'(x_0, y_0)dx$

$y_1 = 3.5 + (e^{x_0} - 10(y_0))(.1)$

$y_1 = 3.5 + (e^2 - 10(3.5))(.1)$

$y_1 = .739$

$dx = .1$

$x_0 = 2 \quad y_0 = 3.5$

$x_1 = 2.1 \quad y_1 = .739$

$x_2 = 2.2 \quad y_2 = .817$

$x_3 = 2.3 \quad y_3 = .903$

$y_2 = .739 + (e^{2.1} - 10(.739))(.1)$

$y_2 = .817$

$y_3 = .817 + (e^{2.2} - 10(.817))(.1)$

$y_3 = .903$