1. Use the graph of y = f(x) to estimate the answer to the following questions.

a. Find the local extreme values and where they occur.

min: -2 at x = 2
max: 2 at x = 0

b. For what values of x is f'(x) > 0?

f(x) is inc (-∞, 0) ∪ (2, ∞)

c. For what values of x is f'(x) < 0?

f(x) is dec (0, 2)

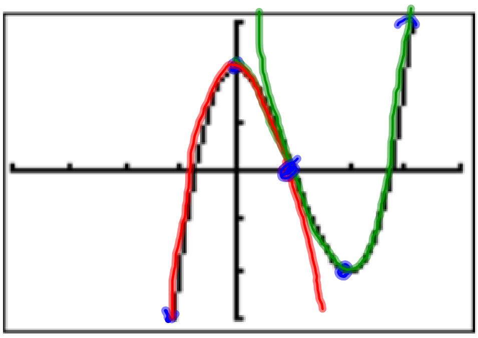
d. For what values of x is f''(x) > 0?

f(x) is concave up (1, ∞)

e. For what values of x is f''(x) < 0?

f(x) is concave down (-∞, 1)

f(x)



[- 4, 4] by [-3,3]

2. For $y = x^4 - 72x^2 - 17$, use analytic methods to find the exact intervals on which the function is

a. increasing b. decreasing

$f'(x) > 0$ $f'(x) < 0$

c. concave up d. concave down

$f''(x) > 0$ $f''(x) < 0$

Then find any

e. local extreme values, f. inflection points.

$y' = 0$ (x_1, y_1)

$y = x^4 - 72x^2 - 17$

$y' = 4x^3 - 144x$

$y' = 4x(x^2 - 36)$

$y' = 4x(x-6)(x+6)$

$4x = 0$ $x = 0$ $x = 6$ $x = -6$

	$(-\infty,-6)$	$(-6,0)$	$(0,6)$	$(6,\infty)$
y'	−	+	−	+
y	dec	inc	dec	inc

2a. $(-6,0) \cup (6,\infty)$

2b. $(-\infty,-6) \cup (0,6)$

2e min -1313 at $x = 6, -6$

max -17 at $x = 0$

$0^4 - 72(0)^2 - 17$

min at $x = -6$

at $x = 6$

$y(6) = 6^4 - 72(6)^2 - 17$

$y(-6) = (-6)^4 - 72(-6)^2 - 17$

max at $x = 0$

$y'' = 12x^2 - 144$

$y'' = 12(x^2 - 12)$

$y'' = 12(x - \sqrt{12})(x + \sqrt{12})$

$0 \neq 12$ $x - \sqrt{12} = 0$ $x = -\sqrt{12}$

$x = \sqrt{12}$

	$(-\infty,-\sqrt{12})$	$(-\sqrt{12},\sqrt{12})$	$(\sqrt{12},\infty)$
y''	+	−	+
y	cup	cd	cup

2c concave up $(-\infty,-\sqrt{12}) \cup (\sqrt{12},\infty)$

2d Concave down $(-\sqrt{12},\sqrt{12})$

$(-\sqrt{12},$

$(\sqrt{12},$

2f. $(-\sqrt{12}, -737)$

$(\sqrt{12}, -737)$

$y = (-\sqrt{12})^4 - 72(-\sqrt{12})^2 - 17$

$y(\sqrt{12}) = (\sqrt{12})^4 - 72(\sqrt{12})^2 - 17$

3. The derivative of a function is $f'(x) = (x - 1)^2 (x+3)$

Find the value of x at each point where f has a

a) Local maximum $f'(x)=0$ b) local minimum, or

c) point of inflection

sign change $-\vee+$

$f''(x)=0$ sign change

$f'(x) = (x-1)^2(x+3)$

$0 = (x-1)^2(x+3)$

$x=1 \quad x=-3$

	$(-\infty,-3)$	min -3	$(-3,1)$	1	$(1,\infty)$
$f'(x)$	$-$	$\vee$	$+$		$+$

a no max

b at $x=-3$

$f''(x) = (x-1)^2(1) + (x+3)\cdot 2(x-1)^1(1)$

$(x-1)(x-1) + (2x+6)(x-1)$

$x^2-2x+1+2x^2-2x+6x-6$

$0 = 3x^2+2x-5$

$3x^2+5x-3x-5$

$-15 \mid 2$

$5 \cdot -3$

$x(3x+5)-1(3x+5)$

$0 = (3x+5)(x-1)$

$3x+5=0 \quad x=1$

$3x=-5$

$x=-\frac{5}{3}$

	$(-\infty,-\frac{5}{3})$	$-\frac{5}{3}$	$(-\frac{5}{3},1)$	1	$(1,\infty)$
$f''(x)$	$+$		$-$		$+$

$f''(x) = (3x+5)(x-1)$

3c) points of inflection occur at $x=-\frac{5}{3}$ and $x=1$

4. The derivative of a function is $g'(x) = 4x + 1$ **and the graph of g** passes through the point **P (5, 3)**. Find $g(x)$.

$x_1 \; y_1$

$g'(x) = 4x+1$

$g(x) = \frac{4x^{1+1}}{(1+1)} + 1x + C$

$g(x) = \frac{4x^2}{2}$

$g(x) = 2x^2 + 1x + C$

$3 = 2(5)^2 + 1(5) + C$

$3 = 50 + 5 + C$

$3 = 55 + C$

$-52 = C$

$\boxed{g(x) = 2x^2 + x - 52}$

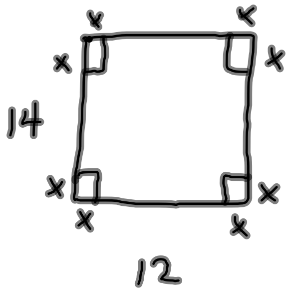
calc.

5 You are planning to make an open rectangular box from a **12- by 14- cm** piece of cardboard by cutting congruent squares from the corners and folding up the sides.

a) Write a single variable function you would use to maximize the volume.

b) What are the dimensions of the box of **largest volume** you can make this way?

c) What is the largest volume?



$V = LWH$

$V = (12-2x)(14-2x)(x)$

$V = 168x - 28x^2 - 24x^2 + 4x^3$

Domain $(0,6)$ $\quad V = 4x^3 - 52x^2 + 168x$

a

$V'(x) = 12x^2 - 104x + 168$

b find zero between $(0,6)$

height $x \approx 2.1475$ cm

$l \quad 12-2x \approx 9.705$ cm

$w \quad 14-2x \approx 7.705$ cm

c $lwh \approx 160.621\ cm^3$

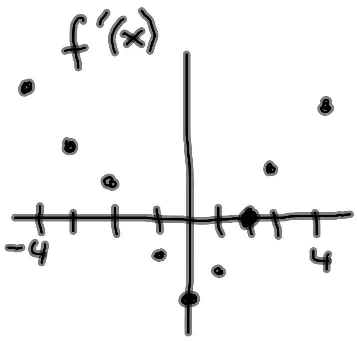
$\boxed{V \approx 160.621\ cm^3}$

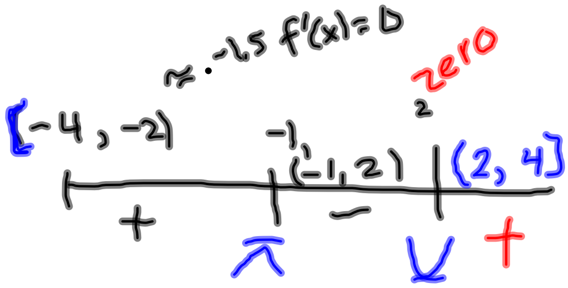
6. Assume that f is continuous on [-4, 4] and differentiable on (-4, 4).
The table gives some values of **f ′(x)**.

x	- 4	- 3	- 2	- 1	0	1	2	3	4
f ′(x)	74	39	10	- 6	-14	- 12	0	22	55

f′(x) +   f′(x)

a) Estimate where f is increasing, decreasing, and has local extrema.





inc [-4, -1.5) ∪ (2, 4]

dec (-1.5, 2)

local min $x \approx 2$

local max $x \approx -1.5$

7. Let $f(x) = x^3 + ax^2$. What is the value of a if

a) f has a local minimum at $x = 4$?

b) f has a point of inflection at $x = 6$?

$f(x) = x^3 + ax^2$

a) $f'(x) = 3x^2 + 2ax$

$0 = 3x^2 + 2ax$

$0 = 3(4)^2 + 2a(4)$

$0 = 48 + 8a$

$\frac{-8a}{-8} = \frac{48}{-8}$

$\boxed{a = -6}$

b) $f(x) = x^3 + ax^2$

$f'(x) = 3x^2 + 2ax$

$f''(x) = 6x + 2a$

$0 = 6x + 2a$

$0 = 6(6) + 2a$

$0 = 36 + 2a$

$-2a = 36$

$a = \frac{36}{-2}$

$\boxed{a = -18}$

8. Find the linearization L (x) of $f(x) = 3x - \frac{2}{x^2}$ at x = 2.

$a = 2$

$L(x) = f(a) + f'(a)(x-a)$

$f(2) + f'(2)(x-2)$

$f(2) = 3(2) - \frac{2}{2^2}$

$6 - \frac{2}{4}$

$6 - \frac{1}{2}$

$5\frac{1}{2}$

$\frac{11}{2}$

$$\boxed{\begin{array}{l} L(x) = \frac{11}{2} + \frac{7}{2}(x-2) \\ \text{OR } L(x) = 5.5 + 3.5(x-2) \end{array}}$$

need $f(x) = 3x - \frac{2}{x^2}$

$f(x) = 3x - 2x^{-2}$

$f'(x) = 3 - 2 \cdot -2 \cdot x^{-2-1}$

$f'(x) = 3 + 4x^{-3}$

$f'(x) = 3 + \frac{4}{x^3}$

$f'(2) = 3 + \frac{4}{2^3}$

$3 + \frac{4}{8}$

$3\frac{1}{2}$

9. Let $y = x^3 - 3x$
a) Find dy and b) estimate dy for x = 2 and dx = 0.05

$$\frac{dy}{dx} = 3x^2 - 3$$

a) $$\boxed{dy = (3x^2 - 3)dx}$$

b) $$\boxed{dy = (3 \cdot 2^2 - 3)(.05)}$$

10. Let y = ln (5x – 2)

a) Find dy and b) estimate dy for x = 2 and dx = 0.03

$$\frac{dy}{dx} = \frac{1}{5x-2} \cdot 5$$

a

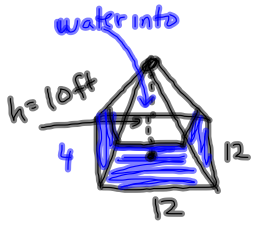
$$\boxed{dy = \frac{5}{5x-2} \cdot dx}$$

b

$$\boxed{dy = \frac{5}{5(2)-2} \cdot (.03)}$$

11. The base of a pyramid-shaped tank is a square with sides of length 12 feet, and the vertex of the pyramid is 10 feet above the base. The tank is filled to a depth of 4 feet, and water is flowing into the tank at the rate of 2 cubic feet per minute. Find the rate of change of the depth of water in the tank

(Hint: The volume of a pyramid is given by $= \frac{1}{3}Bh$, where B is the base area and h is the height of the pyramid.)



$\frac{dV}{dt} = 2\ \frac{ft^3}{min}$

Find $\frac{dh}{dt}$

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}$ square$\cdot h$

$V = \frac{1}{3} \cdot s^2 h$

$\frac{s}{h} = \frac{12}{10}$

$s = \frac{12}{10}h$

$s = \frac{6}{5}h$

$V = \frac{1}{3} \cdot \left(\frac{6}{5}h\right)^2 \cdot h$

$V = \frac{1}{3} \cdot \frac{36}{25}h^2 \cdot h$

$V = \frac{12}{25}h^3$

$\frac{dV}{dt} = \frac{12}{25} \cdot 3h^2 \cdot \frac{dh}{dt}$

$2 = \frac{36h^2}{25}\frac{dh}{dt}$

$\frac{2 \cdot 25}{36h^2} = \frac{dh}{dt}$

what is h?

h is 10-4

$\frac{50}{36h^2} = \frac{dh}{dt}$

$\frac{50}{36(6)^2} = \frac{dh}{dt}$

$\frac{25}{648}$

$\boxed{\approx .039\ ft/min}$

12. Two cars are approaching an intersection along roads that make a 90° angle. The first car is 100 feet from the intersection and is traveling at a rate of 30 feet per second. The second car is 150 feet from the intersection and is traveling at a rate of 40 feet per second. Find the rate of change of the distance between the cars.

$\frac{dy}{dt} = -40$

$x = 100$ ft

$\frac{dx}{dt} = -30$ ft/sec

$$D = \sqrt{x^2+y^2}$$

$$\frac{dD}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}}$$

$$\frac{dD}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

$$\frac{dD}{dt} = \frac{100(-30) + 150(-40)}{\sqrt{100^2+150^2}}$$

$$\frac{-3000 + -6000}{\sqrt{32500}}$$

$$\frac{-9000}{10\sqrt{325}}$$

$$\boxed{\approx -49.923 \text{ ft/sec}}$$